Mock Board Exam
Term II
Mathematics

SECTION A
(Attempt all questions.)

Question 1

Choose the correct answers to the questions from the given options. (Do not copy the question, Write the correct answer only.) **[10]**

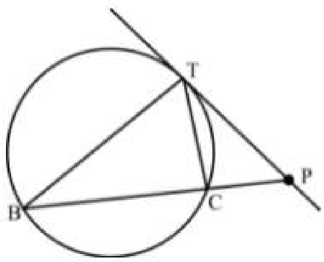
(i) In the above given figure, PB = 9 cm, CP = 4 cm and TP is a tangent at T. Find PT.

(a) 6 cm

(b) 9 cm

(c) 8 cm

(d) 2 cm



Solution: Option (a)

Given: CP = 4 cm, PB = 9 cm and TP is a tangent at point T.

We know that, if a chord and a tangent intersect externally, then the product of the lengths of the segments of the chord is equal to the square of the length of the tangent from the point of contact to the point of intersection

i.e. (PC) x (PB) = PT^2

⇒ 4 x 9 = PT^2

⇒ 36 = PT^2

⇒ $\sqrt{36}$ = PT

⇒ PT = 6

(ii) Perpendicular from the centre of the circle to the chord bisects the chord in what ratio?

(a) 1 : 1

(d) 1 : 2

(c) 2 : 1

(d) 1 : 4

Solution: Option (a)

Perpendicular from the centre of the circle to the chord, bisects the chord in the 1:1 ratio.

(iii) The ratio of volume to the total surface area of a solid sphere is 8. Find its radius.

(a) 6 cm

(b) 12 cm

(c) 24 cm

(d) 48 cm

Solution: Option (c)

Volume of a sphere is given as: $\frac{4}{3}\pi r^3$

Total surface area of a sphere is given as: $4\pi r^2$

Thus, for a sphere,

$\Rightarrow \frac{Volume}{Surface\ area} = \frac{\frac{4}{3}\pi r^3}{4\pi r^2}$

$\Rightarrow\ 8 = \frac{r}{3}$

$\Rightarrow\ r = 24$

(iv) If a tower 30 m high, casts a shadow 103 m long on the ground, then what is the angle of elevation of the sun?

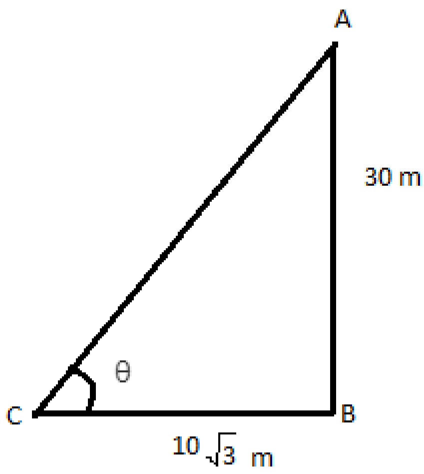
(a) 30°

(b) 45°

(c) 60°

(d) 90°

Solution: Option (c)



$$tan\theta = \frac{30}{10\sqrt{3}} = \frac{3}{\sqrt{3}} = \sqrt{3}$$

$\because$ tan 60° = $\sqrt{3}$

$\therefore \theta$ =60°

So, the angle of elevation of the sun is 60°.

(v) Find the value of $4(1 - sin^2\theta)(1 + tan^2\theta)$

(a) 4

(b) 1

(c) 2

(d) 0

Solution: Option (a)

We know that $1 - sin^2\theta = cos^2\theta$ and $1 + tan^2\theta = sec^2\theta$

4 $[1 - sin^2\theta]$ $[1 + tan^2\theta]$ = 4 $cos^2\theta sec^2\theta$ = 4

Because $sec\theta = \frac{1}{cos\theta}$

(vi) If the cumulative frequency at a particular class interval 30-35 is 19 and the cumulative frequency at the next class -interval i.e 35-40 is 27 , the frequency at 35-40 is ____.

(a) 18

(b) 8

(c) 10

(d) 3

Solution: Option (b)

The cumulative frequency is the sum of all the frequencies till a particular class interval.

If the cumulative frequency at 30-35 is 19 and the cumulative frequency at 35-40 is 27

Hence the frequency at 35-40 is 27 -19 = 8

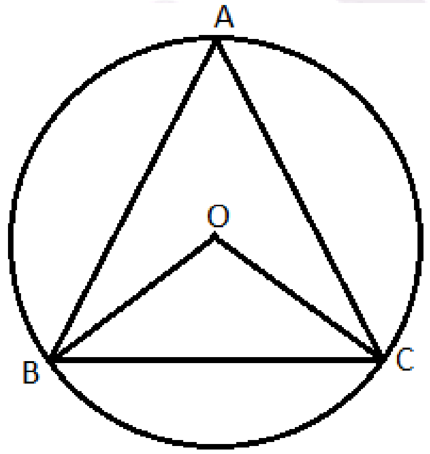
(vii) In the below given figure, an equilateral triangle ABC is inscribed in a circle centered at O. Then find the measure of $\angle BOC$.

(a) 60°

(b) 90°

(c) 110°

(d) 120°



Solution: Option (d)

Since ABC is an equilateral triangle and sides of equilateral triangle are equal, then

AB = BC = CA

In the circle, AB, BC and CA are equal chords and equal chords subtend equal angle, then

$$\angle AOB = \angle AOC = \angle BOC$$

Since, the sum of all angles around a point is 360°.

So,

$$\angle AOB + \angle AOC + \angle BOC = 360°$$

$\angle BOC + \angle BOC + \angle BOC = 360°$ $\quad$ $(\because \angle AOB = \angle AOC = \angle BOC)$

$$3\angle BOC = 360°$$

$\Rightarrow \angle BOC = \frac{360°}{3} = 120°$

(viii) Find the volume of a cone whose height and radius is 6 cm and 5 cm respectively

(a) 50π cm^3

(b) 45π cm^3

(c) 55π cm^3

(d) 40π cm^3

Solution: Option (a)

Volume of the cone $= \frac{1}{3} \times \pi \times r^2 \times h$

Therefore, volume $= \frac{1}{3} \times \pi \times 5^2 \times 6$ cm^3

$= 50\pi$ cm^3

(ix) The following data gives the information on the observed lifetime (in hours) of 225 electrical components:

Lifetime (in hours)	0 – 20	20 – 40	40 – 60	60 – 80	80 - 100	100 - 120
No. of Components	10	35	52	61	38	29

Determine the modal class lifetimes of the components.

(a) 20-40

(b) 40-60

(c) 60-80

(d) 80-100

Solution: Option (c)

As we know, class 60-80 has the highest number of components

Modal class = 60 - 80,

(x) A card is drawn at random from a pack of 52 cards. Find the probability that the card drawn is a king.

(a) $\frac{1}{2}$

(b) $\frac{1}{13}$

(c) $\frac{15}{26}$

(d) $\frac{8}{13}$

Solution: Option (b)

Given,

Total number of outcomes = 52

In a pack of 52 cards,

Number of kings = 4

=> Number of favourable outcomes = 4

Probability of an event, P(E) = $\frac{number\ of\ favourable\ outcomes}{total\ number\ of\ outcomes}$

P(E) = $\frac{4}{52} = \frac{1}{13}$

Therefore, the probability that the card drawn is a king is $\frac{1}{13}$

SECTION B

(Attempt **any three** questions from this Section.)

Question 2

(i) A conical tent is built such that it can accommodate 25 people. If each person, on an average occupies an area of 4 m^2 of ground, and the height of the conical tent is 18 m, then find the volume of the tent. [2]

Solution:

Since, each person on an average occupies an area of 4 m^2 of the ground.

The conical tent is built such that it can accommodate 25 people.

So, area of base = $25 \times 4\ m^2$= 100 m^2 [1]

Since, the height of the conical tent is, $h = 18\ m$

Area of the base $= \pi r^2 = 100\ m^2$

So, volume of the tent = $\frac{1}{3}(area\ of\ base) \times h$

$$= \frac{1}{3} \times \pi r^2 \times h$$

$$= \frac{1}{3} \times 100 \times 18$$

$$= 600\ m^3$$

Therefore, the volume of the tent is $600\ m^3$. [1]

(ii) Find the probability of getting 53 Fridays in a leap year. [2]

Solution:

Leap year has 366 days = (52 x 7 + 2) days = 52 weeks and 2 days

Thus a leap year always has 52 Fridays. [1]

The remaining two days can be:

(i) Sunday and Monday (ii) Monday and Tuesday (iii) Tuesday and Wednesday

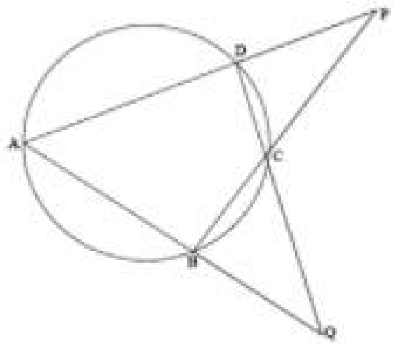
(iv) Wednesday and Thursday (v) Thursday and Friday (vi) Friday and Saturday

(vii) Saturday and Sunday

From above, we have Fridays in two cases:

=> P(53 Fridays) = $\frac{2}{7}$ [1]

(iii) In the given figure, ∠A = 60° and ∠ABC = 80°, then find ∠DPC and ∠BQC. [3]



Solution:

In a cyclic quadrilateral, the exterior angle is equal to the opposite interior angle.

So, in cyclic quadrilateral ABCD, we have,

∠ PDC = ∠ ABC and ∠ DCP = ∠ A

∠ PDC = 80° and ∠ DCP = 60° [Given, ∠ ABC = 80°) and ∠ A = 60°)]

In (△ DPC), we have

∠ DPC = 180° - (∠ PDC +∠ DCP)

⇒ ∠ DPC = 180° - (80° + 60°) = 40° [1]

Similarly, we have

∠ QBC = ∠ ADC and ∠ BCQ = ∠ BAD

[(∠ ADC + ∠ ABC = 180°) (Opposite angle sum of cyclic quadrilateral), (∠ ABC = 80°) and (∠ A = 60°)]

∠ QBC = 180° - ∠ ABC

∠ QBC = 180° - 80° = 100° and ∠ BCQ = ∠ BAD = 60° [1]

Now, in (△ BQC), we have

∠ BQC = 180° - (∠ QBC + ∠ BCQ)

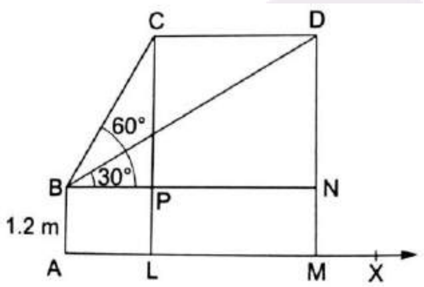
⇒ ∠ BQC = 180° - (100° + 60°) = 20° [1]

(iv) A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60°. After some time, the angle of elevation reduces to 30°. Find the distance traveled by the balloon during the interval. [3]

Solution:

Let AB be the position of the girl and AX be the horizontal ground. Let C and D be the two positions of the balloon.

Draw (CL ⊥ AX, DM ⊥ AX) and (BN ⊥ DM), intersecting CL at P.



Then, (∠ CBP = 60°, ∠ DBN = 30°, AB = PL = NM = 1.2 m) and (CL = DM = 88.2 m)

⇒ CP = 88.2 m - 1.2 m = 87 m [1]

From right angled triangle BPC, we have

⇒ $\frac{BP}{CP}$ = cot60° = $\frac{1}{\sqrt{3}}$

⇒ $\frac{BP}{87\ m}$ = $\frac{1}{\sqrt{3}}$ = BP = $\frac{87}{\sqrt{3}}$

⇒ BP = $\frac{87}{\sqrt{3}}$ x $\frac{\sqrt{3}}{\sqrt{3}}$= 29$\sqrt{3}$ m [1]

From right angled triangle BND, we have

$\frac{BN}{DN}$ = cot 30° = $\sqrt{3}$ => $\frac{BP + PN}{CP}$ = $\sqrt{3}$

$\frac{29\sqrt{3} + CD}{87\ m}$ = $\sqrt{3}$ [PN = CD and DN = CP]

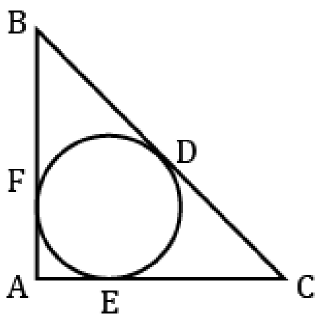
⇒ $29\sqrt{3}$ m + CD = 87$\sqrt{3}$ m

⇒ CD = $87\sqrt{3}\,m\ -\ 29\sqrt{3}\,m\ =\ 58\sqrt{3}\,m$

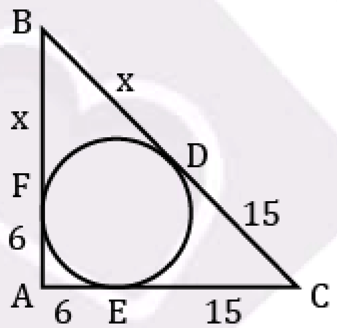
Hence, the required distance traveled by the balloon is $58\sqrt{3}\,m$ [1]

Question 3

(i) In the given diagram, a circle is inscribed in a right angled triangle such that AF = 6 cm and EC = 15 cm. Find the difference between CD and BD. [2]



Solution:



As the lengths of tangents drawn from an external point to a circle are equal

AF = AE = 6

CE = CD = 15 [1]

Let BF = BD = x

Applying Pythagoras theorem to the above figure

$(6+x)^2 + (6+15)^2 = (15+x)^2$

36 + x^2 + 12x + 441 = 225 + x^2 + 30x

18x = 252

x = 14

Difference between CD and BD = 15 – 14 = 1 cm [1]

(ii) How many cylindrical containers of r = 0.5 cm and h = 7 cm are required to completely fill a cylindrical container of radius 5 cm and height 14 cm? [2]

Solution:

Let us assume x cylindrical containers will be required to pour liquid filled in a container of 5 cm radius and 14 cm height

So, $x \times \pi r^2 h = \pi R^2 H$ [1]

$\Rightarrow x\,\pi(0.5\ cm)^2$x (7 cm) = $\pi\ (5\ cm)^2$x (14 cm)

$\Rightarrow x = \frac{\pi\ (5\ cm)^2 x\ (14\ cm)}{\pi(0.5\ cm)^2 x\ (7\ cm)}$

$\Rightarrow x = 200$ [1]

(iii) Prove that $(\sin\theta + \operatorname{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2 = 7 + \tan^2\theta + \cot^2\theta$ [3]

Solution:

We use the reciprocal identities and Pythagorean identities to prove this identity.

LHS = $(\sin\theta + \operatorname{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2$

$\Rightarrow (\sin^2\theta + \operatorname{cosec}^2\theta + 2\sin\theta\operatorname{cosec}\theta) + (\cos^2\theta + \sec^2\theta + 2\cos\theta\sec\theta)$ [1]

$\Rightarrow \sin^2\theta + \cos^2\theta + \operatorname{cosec}^2\theta + \sec^2\theta + 2 + 2$

$\Rightarrow 1 + (1 + \cot^2\theta) + (1 + \tan^2\theta) + 2 + 2$ [1]

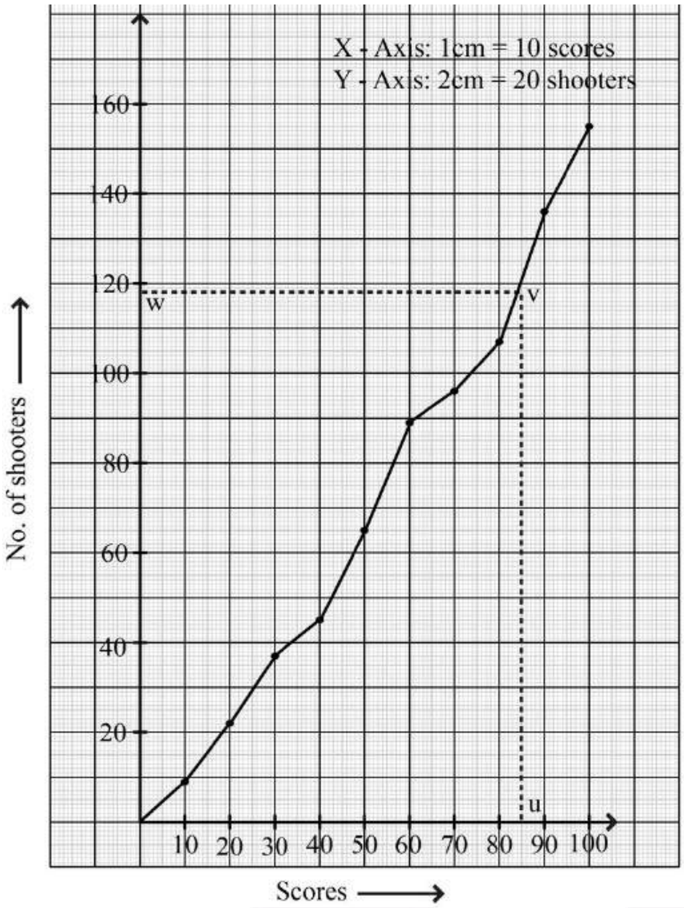
$\Rightarrow 7 + \tan^2\theta + \cot^2\theta$ = RHS.

Hence proved. [1]

(iv)The table shows the Distribution of the Scores obtained by 155 shooters in a shooting competition. Use a graph sheet to draw an ogive for the distribution. Estimate the number of shooters who obtained a score of more than 85%. [3]

score	No of Shooters
0-10	10
10-20	12
20-30	15
30-40	8
40-50	20
50-60	24
60-70	7
70-80	11
80-90	30
90-100	18

Solution:



[1]

To find the number of shooters who obtained a score more than 85%, take a point U at 85 on the x - axis.

From it, draw a vertical line, which touches the ogive at point V and from it, draw a horizontal line which touches the y - axis at point W. At that point W, we are getting those shooters who obtained a score till 85.

No. of shooters at point W = 118 [1]

number of shooters who obtained a score of more than 85% = Total number - No. of shooters who scored till 85

= 155 - 118

= 37

⇒ Number of shooters who obtained a score of more than 85% is 37. [1]

Question 4:

(i) If $A = a\cos\alpha.\cos\beta.\cos\theta, B = a\cos\alpha.\cos\beta.\sin\theta, C = a\sin\alpha.\cos\beta$ and $D = a\sin\beta$, then find the value of $A^2 + B^2 + C^2 + D^2$.
[2]

Solution:
First, find the value of $A^2 + B^2$.
So, $A^2 + B^2 = (a\cos\alpha.\cos\beta.\cos\theta)^2 + (a\cos\alpha.\cos\beta.\sin\theta)^2$

$$= a^2\cos^2\alpha.\cos^2\beta.\cos^2\theta + a^2.\cos^2\alpha.\cos^2\beta.\sin^2\theta = a^2\cos^2\alpha.\cos^2\beta(\cos^2\theta + \sin^2\theta)$$

$= a^2\cos^2\alpha.\cos^2\beta$ [1]

Now, find the value of $A^2 + B^2 + C^2$

$$A^2 + B^2 + C^2 == a^2\cos^2\alpha.\cos^2\beta + (a\sin\alpha.\cos\beta)^2 = a^2\cos^2\alpha.\cos^2\beta + a^2\sin^2\alpha.\cos^2\beta$$

$$= a^2(\cos^2\alpha + \sin^2\alpha)\cos^2\beta = a^2\cos^2\beta$$

Next find the value of $A^2 + B^2 + C^2 + D^2$

$$A^2 + B^2 + C^2 + D^2 = a^2\cos^2\beta + (a\sin\beta)^2 = a^2\cos^2\beta + a^2\sin^2\beta$$

$$= a^2(\cos^2\beta + a^2\sin^2\beta) = a^2$$

Hence, the required value of $A^2 + B^2 + C^2 + D^2$ is a^2. [1]

(ii) The age of the employees in a startup company is shown below. Find the average age of the employes [2]

Age	18 – 26	26 – 34	34 – 42	42 – 50	50 - 58	58 - 66
No of Employees	30	70	50	30	10	10

Solution:

Let the assumed mean 'a' be 38.

Age	xi	fi	di = xi - a	fidi

18 - 26	22	30	-16	-480
26 - 34	30	70	-8	-560
34 - 42	38	50	0	0
42 - 50	46	30	8	240
50 - 58	54	10	16	160
58 - 66	62	10	24	240

[1]

Here,

$\sum fi = 200$

$\sum fidi = -400$

=> Mean = a + $\frac{\sum fidi}{\sum fi}$

= 38 + $\frac{-400}{200}$

= 38 - 2

= 36

The average age of employees is 36. [1]

(iii) The radius of a solid circular cylinder decreases by 20% and its height increases by 10%. Find the percentage change in its:

a. volume

b. curved surface area [3]

Solution:

Let the original dimensions of the solid right cylinder be radius (r) and height (h) units.

Then its volume = $\pi r^2 h$ and curved surface area = $2\pi rh$

Now, after the change the new dimensions are:

New radius (r') = r – 0.2r = 0.8r and

New height (h') = h + 0.1h = 1.1h

So,

The new volume = $\pi(r')^2h'$

= $\pi(0.8r)^2(1.1h)$

= $0.704\ \pi r^2 h$

And, the new curved surface area = $2\pi r'h' = 2\pi(0.8r)(1.1h)$

$= (0.88)\ 2\pi rh$ [1]

(a) Percentage change in its volume $= \frac{Change\ in\ volume}{original\ volume} \times 100\ \%$

$$= \frac{Original\ volume - new\ volume}{original\ volume} \times 100\ \%$$

$$= \frac{\pi r^2 h - 0.704\ \pi r^2 h}{\pi r^2 h} \times 100\ \%$$

$= 0.296 \times 100 = 29.6\ \%$ [1]

(b) Percentage change curved surface area $= \frac{Change\ in\ CSA}{original\ CSA} \times 100\ \%$

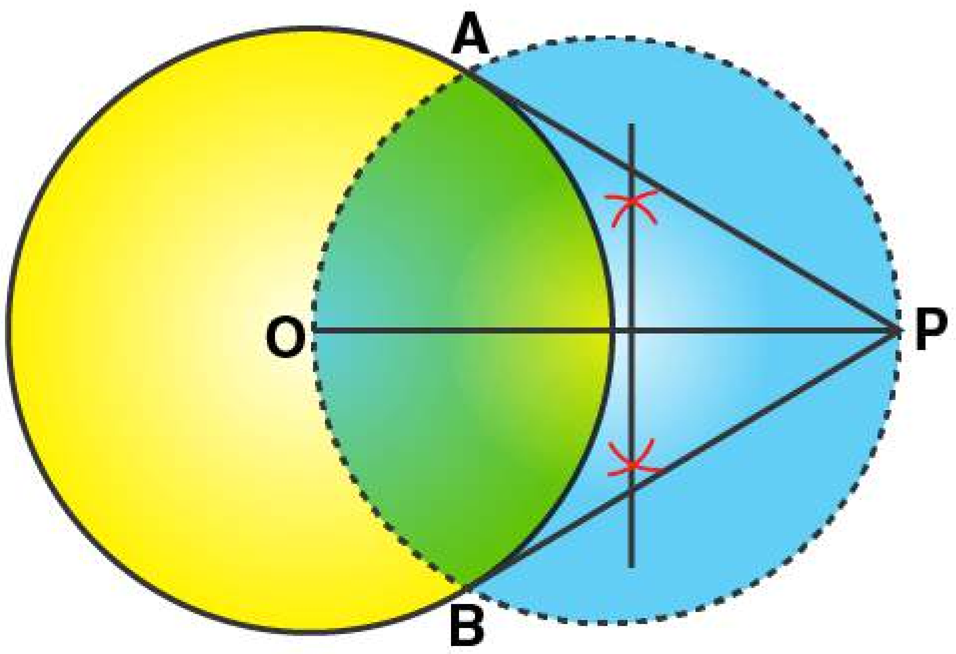
$$= \frac{Original\ CSA - new\ CSA}{original\ CSA} \times 100\ \%$$

$$= \frac{2\pi rh - (0.88)2\pi rh}{2\pi rh} = 0.12 \times 100\ \% = 12\%$$ [1]

(iv) Draw a circle of diameter of 9 cm. Mark a point at a distance of 7.5 cm from the centre of the circle. Draw tangents to the given circle from this exterior point. [3]

{Ch:Constructions (Circles), Topic:- Drawing tangents to a circle, Bloom: Analysis}

Solution:



[1]

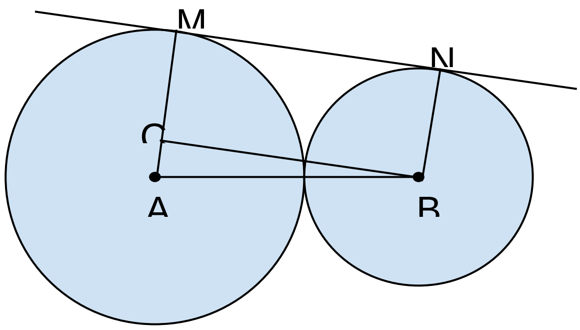
Steps of construction:
i) Taking O as the centre, draw a circle of diameter 9 cm (radius = 4.5 cm).
ii) Mark a point P outside the circle, such that PO = 7.5 cm.
iii) Taking OP as the diameter, draw a circle such that it cuts the circle with centre O at A and B. [1]
iv) Now, join PA and PB.
Thus, PA and PB are required tangents.
On measuring, we get PA = PB = 6 cm. [1]

Question 5

(i) The centres of two circles of radii 3 cm and 2 cm are 8 cm apart. Find the length of the common tangent. [2]

Solution:

In the figure, MN is the common tangent. A and B are the centres of the two circles with radii 3 cm and 2 cm respectively.



AB = 8 cm. AM = 3 cm and BN = 2 cm. Draw BC perpendicular to AM extended forward.

Since BCMN is a rectangle, CM = BN = 2 cm.

Now, AC = AM + CM = 3 cm + 2 cm = 5 cm. [1]

In ABC, $AB^2 = BC^2 + AC^2$

$BC^2 = AB^2 - AC^2$

$BC^2 = (8\ cm)^2 - (5\ cm)^2 = 39\ cm^2$

$BC = \sqrt{39}$ cm [1]

(ii) If 3sinθ + 4cosθ = 5, then the value of find the value of sinθ. [2]

Solution:

Given, 3 sin**θ** +4 cos**θ** = 5.......(1)

Squaring both sides, we have:

$(3 \sin\theta + 4 \cos\theta)^2 = 5^2$

$\Rightarrow 9 \sin^2\theta + 16 \cos^2\theta + 24 \sin\theta\cos\theta = 25$

$\Rightarrow 9(1 - \cos^2\theta) + 16(1 - \sin^2\theta) + 24 \sin\theta \cos\theta = 25$

$\Rightarrow 9 - 9 \cos^2\theta + 16 - 16 \sin^2\theta + 24 \sin\theta \cos\theta = 25$

$\Rightarrow 16\sin^2\theta + 9\cos^2\theta - 24 \sin\theta \cos\theta = 0$ [1]

$\Rightarrow (4 \sin\theta - 3 \cos\theta)^2 = 0$

$\Rightarrow 4 \sin\theta - 3 \cos\theta = 0$

$\Rightarrow 4 \sin\theta = 3 \cos\theta$

$\Rightarrow \tan\theta = \frac{3}{4}$

$\Rightarrow \sin\theta = \frac{3}{5}$ [1]

(iii) The probability of selecting a white ball at random from a container that contains only white, yellow and red balls is **1**/**7** . The probability of selecting a yellow ball at random from the same container is **1**/**5** . If the container contains 23 red balls, find the total number of balls in the container. [3]

Solution:

Let the total number of balls in the container be *x*.

Since, the total number of red balls in the container is 23.

Then, the probability of selecting a red ball from the container is,

$$P(Red\ Ball) = \frac{total\ number\ of\ red\ balls\ in\ the\ container}{total\ number\ of\ balls\ in\ the\ container} = \frac{23}{x} \quad \ldots(1)$$

[1]

It's given that,

$P(White\ Ball) = \frac{1}{7}$, $P(Yellow\ Ball) = \frac{1}{5}$

Since, the sum of the probabilities of all events of an experiment is always one. Then,

$P(White\ Ball) + P(Yellow\ Ball) + P(Red\ Ball) = 1$

[1]

$$\Rightarrow \frac{1}{7} + \frac{1}{5} + P(Red\ Ball) = 1$$

$$\Rightarrow P(Red\ Ball) = 1 - \frac{1}{7} - \frac{1}{5}$$

$\Rightarrow P(Red\ Ball) = \frac{23}{35}$ ...(2)

From (1) and (2),

$\frac{23}{x} = \frac{23}{35} \Rightarrow\ x = 35$

Hence, the total number of balls in the container is 35. [1]

(iv) The marks obtained by the students of a class, in an exam which was out of 50 marks,is given below.

Marks	No of Students
5	1
7	2
11	2
16	3
21	5
24	6
28	3
32	5
34	8
39	6
43	4
45	2
49	3

Represent the same as grouped data, with class intervals of width = 10 and find the mode for the grouped data. [3]

Solution:

Grouped data:

Class intervals	Frequency
0 -10	3
10 - 20	5
20 - 30	14
30 - 40	19
40 - 50	9

Modal Class is 30-40. [1]

Lower limit, l = 30

$f_1 = 19$, $f_0 = 14$ and $f_2 = 9$

Mode = l + $\left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)$ x h [1]

=30 + $\left(\frac{19 - 14}{2\times 19 - 14 - 9}\right)$ x 10

= 30 + $\left(\frac{5}{15}\right)$ x 10

= 33.33 [1]

Question 6

(i) Given a dartboard of radius 50 cm, find the probability of hitting the bull's eye of radius 5 cm.

[2]

Solution:

Area of board = π x $(50\ cm)^2$ = 2500π cm^2

Area of the bull's eye = π x $(5\ cm)^2$ = 25π cm^2 [1]

The probability of hitting the bull's eye

$= \frac{Area\ of\ required\ region}{Total\ area} = \frac{25\pi}{2500\pi} = \frac{1}{100} = 0.01$ [1]

(ii) A farmer connects a pipe of internal diameter 20 cm from a canal into a cylindrical tank in his field that is 10 m in diameter and 2 m deep. If water flows through the pipe at the rate of 3 km/hr, in how much time will the tank be filled? [2]

Solution:

Let r be the radius of the circular end of the pipe, R be the radius of the circular end of cylindrical tank and h be the depth of the cylindrical tank.

Then,

$$r = \frac{20}{2}\ cm = \frac{20}{2 \times 100}\ m = 0.1\ m$$

$$R = \frac{10}{2}\ m = 5\ m$$

$$h = 2\ m$$

Area of cross section= $\pi r^2 = \pi(0.1)^2 = 0.01\pi\ m^2$ [1]

Speed of water $= 3km/h = \frac{3\times1000}{60}\ m/min = 50\ m/min$

Volume of water flowing in one minute from pipe= $Speed\ of\ water\ \times\ Area\ of\ cross\ section$

$= 50\ \times 0.01\pi = 0.5\pi\ m^3$

Volume of water flowing in t minute from pipe $= t \times 0.5\pi\ m^3$

Volume of water flowing in t minute from pipe = Volume of cylindrical water tank

$$\Rightarrow t \times 0.5\pi = \pi R^2 h$$

$$\Rightarrow t \times 0.5\pi = \pi(5)^2 \times 2$$

$$\Rightarrow t = \frac{\pi(5)^2\times2}{0.5\,\pi}$$

$$\Rightarrow t = 100\ minutes$$

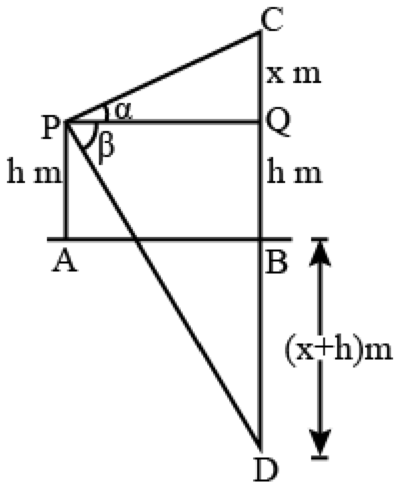
Hence, the required time to fill the water tank is $100\ minutes$. [1]

(iii) The angle of elevation of a cloud from a point h metres above a lake is α and the angle of depression of its reflection in the lake is β. Then find the height of the cloud in terms of trigonometric ratios . [3]

Solution:

Let AB be the surface of the lake and let P be a point vertically above A such that AP = h metres.

Let C be the position of the cloud and let D be its reflection in the lake.



[1]

Draw $(PQ \perp CD)$. Then,

$\angle$ QPC = α , $\angle$ QPD = β,

(BQ = AP = h metres)

Let (CQ = x metres).

Then,(BD = BC = (x + h) metres

From right triangle PQC, we have

$\frac{PQ}{CQ}$ = cotα $\Rightarrow \frac{PQ}{x\ m}$ = cotα

$\Rightarrow$ PQ = x cotα metres) .... (i)

From right (triangle PQD), we have

$\frac{PQ}{QD}$ = cotβ $\Rightarrow \frac{PQ}{(x+2h)\,m}$ = cotβ

$\Rightarrow$ PQ = (x + 2h)cotβ metres .... (ii) [1]

From (i) and (ii), we get

xcotα = (x + 2h) cotβ

$\Rightarrow$ x(cotα - cotβ) = 2h cotβ

$\Rightarrow x\left(\frac{1}{tan\alpha} - \frac{1}{tan\beta}\right) = \frac{2h}{tan\beta}$

$\Rightarrow x\left(\frac{tan\beta - tan\alpha}{tan\alpha tan\beta}\right) = \frac{2h}{tan\beta}$

$\Rightarrow x = \frac{2h\,tan\alpha}{tan\beta - tan\alpha}$

$\Rightarrow$ height of the cloud from the surface of the lake

$= (x + h) = \frac{2h\,tan\alpha}{tan\beta - tan\alpha} + h$

$= \frac{h\,(tan\alpha + tan\beta)}{tan\beta - tan\alpha}$ metres [1]

(iv) Using the frequency distribution table given below, draw 'less than ogive'. Then from the ogive, find the interquartile range.

[3]

Class Interval	10 – 20	20 – 30	30 – 40	40 – 50	50 - 60
Frequency	2	4	6	8	10

Solution:

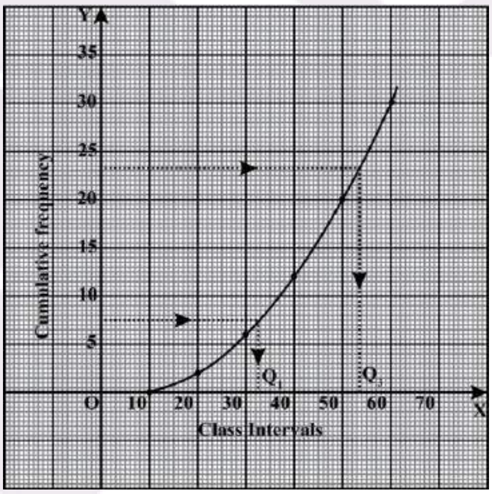
We prepare the cumulative frequency table as given below:

Class interval	Frequency	Cumulative frequency
10 – 20	2	2
20 – 30	4	6
30 – 40	6	12
40 – 50	8	20
50 - 60	10	30

[1]

We need to plot the points (10, 0), (20, 2), (30, 6), (40, 12), (50, 20) and (60, 30). Plotting these points on a graph paper and joining by a freehand curve, we get the cumulative frequency curve (a 'less than ogive') as given below:

Finding the lower quartile and upper quartile to find the interquartile range



To find the lower quartile Q_1, we locate $\frac{n+1}{4}$, i.e., $\frac{31}{4}$ = 7.5 on the y-axis and proceed horizontally to meet the ogive. From this point, draw a perpendicular to meet the x-axis.

⇒ From figure Q_1 = 32.5 [1]

To find the upper quartile Q_3, we proceed similarly as in (i) taking

$\frac{3(n+1)}{4} = \frac{(3)\,(7.75)}{4}$ =23.25 on y-axis

⇒ From figure $Q_3 = 53.5$

Interquartile range = $Q_3 - Q_1 = 53.5 - 32.5 = 21$ [1]