Mock Board Exam

MATHEMATICS

SOLUTIONS

Question 1

(i) The order and degree respectively of differential equation $\left(\frac{dy}{dx}\right)^2 + 3y\left(\frac{d^2y}{dx^2}\right) = 4$ are **[1]**

A. $2 \ and \ 1$
B. $1 \ and \ 2$
C. $2 \ and \ 2$
D. $3 \ and \ 2$

Answer: $(A) \ 2 \ and \ 1$
Solution:
$\left(\frac{dy}{dx}\right)^2 + 3y\left(\frac{d^2y}{dx^2}\right) = 4$
Order of a differential equation is defined to be that of the highest order derivative it contains.
Degree of a differential equation is defined as the power to which the highest order derivative is raised.
Thus, order of the differential equation is 2 and degree is 1.

(ii) If $\int_{-2}^{2} \ f(x)dx = 0$, then $f(x)$ may be
[1]

A. x
B. x
C. $(x + x^3)^3$
D. $ln \ ln \ (x^2)$

Answer: $(C) \ (x + x^3)^3$
Solution:
We know that $\int_{-a}^{a} \ f(x)dx = 0$, if $f(x)$ is an odd function.
And $(x + x^3)^3$ is an odd function.
Because for $f(x) = (x + x^3)^3$, we get
$f(-x) = (-x + (-x)^3)^3 = -(x + x^3)^3 = -f(x)$

(iii) The value of $\int \ \left(\frac{12x^{11} + 8^x ln ln \ 8}{x^{12} + 8^x}\right) dx$ is
[1]
(where c is the constant of integration)

A. $ln \ ln \ |x^{12} + 8^x| \ + c$
B. $\frac{x^{13}}{13} + 8^x \ ln \ ln \ 8 \ + c$
C. $ln \ ln \ |x^{13} + 8^x \ ln \ ln \ 8 \ | \ + c$
D. $\frac{(x^{12} + 8^x)^2}{2} + c$

Answer: $(A)\ \ln \ln |x^{12}+8^x| + c$
Solution:
Let $I=\int \left(\frac{12x^{11}+8^x \ln \ln 8}{x^{12}+8^x}\right) dx$
Put $x^{12}+8^x=t$
$\Rightarrow (12x^{11}+8^x \ln \ln 8)\,dx=dt$
$I=\int \frac{dt}{t} = \ln \ln |t| + c$
$\therefore I = \ln \ln |x^{12}+8^x| + c$

(iv) The general solution of the differential equation $\frac{dy}{dx}=e^{x+y}$ is **[1]**

A. $e^x+e^{-y}=C$
B. $e^x+e^y=C$
C. $e^{-x}+e^y=C$
D. $e^{-x}+e^{-y}=C$

Answer: $(A)\ e^x+e^{-y}=C$
Solution:
Given differential equation, $\frac{dy}{dx}=e^x \cdot e^y$
Separating variables and integrating,
$\int e^{-y}dy=\int e^x\,dx$
$\Rightarrow -e^{-y}=e^x-C$, where $-C$ is constant of integration.
$\therefore e^x+e^{-y}=C$

(v) Let A and B be the two events such that $P(A)=0.3$ and $P(A\cup B)=0.8$. If A and B are independent events, then $P(B)$ is **[1]**

A. $\frac{5}{7}$
B. $\frac{3}{7}$
C. $\frac{1}{2}$
D. $\frac{1}{7}$

Answer: $(A)\ \frac{5}{7}$
Solution:
For independent events, $P(A\cap B)=P(A)\cdot P(B)$
We know that $P(A\cup B)=P(A)+P(B)-P(A\cap B)$
$\Rightarrow 0.8=0.3+P(B)-0.3P(B)$

$$\therefore P(B)=\frac{5}{7}$$

(vi) Two persons A and B speak the truth with the probabilities 0.7 and 0.8 respectively. The probability that they will say the same thing while describing a single event is **[1]**

A. 0.56

B. 0.62
C. 0.5
D. 0.42

Answer: $(B)0.62$
Solution:
Let A_1 be the event that A speaks the truth
and B_1 be the event that B speaks the truth.
$\Rightarrow P(A_1) = 0.7$ and $P(B_1) = 0.8$
Probability that they will say same thing while describing a single event
= Both narrate same event truthfully or Both lie about event
$= P(A_1) \cdot P(B_1) + \left(1 - P(A_1)\right) \cdot \left(1 - P(B_1)\right)$
$= 0.7 \times 0.8 + (1 - 0.7)(1 - 0.8)$
$= 0.56 + \ 0.06$
$= 0.62$

Question 2 **[2]**
(a) The value of $\int (2x \ + 4x\ )\ dx$ is

Answer: $3x + \frac{sinsin\ 2x}{2} + c$
Solution:

$\int \quad (2x \ + 4x\ )dx$

$= \int \quad [2(x \ + x\ ) + 2x\ ]dx$

$= \int \quad [2 + (1 + cos\ cos\ 2x\ )]\ dx \qquad (\because x \ + x \ = 1\ and\ cos\ cos\ 2x \ = 2x \ - 1)$

$= \int \quad (3 + cos\ cos\ 2x \ )dx$

$= 3x + \frac{sinsin\ 2x}{2} + c$

OR

(b) The value of $\int \frac{1}{x^2+13x-2}\ dx$

Answer: $\frac{1}{\sqrt{177}} log\ log\ \left|\frac{2x+13-\sqrt{177}}{2x+13+\sqrt{177}}\right| + c$
Solution:
Let $I = \int \frac{1}{x^2+13x-2}\ dx$

Consider $x^2 + 13x - 2 = x^2 + 13x + \frac{169}{4} - \frac{169}{4} - 2 = \left(x + \frac{13}{2}\right)^2 - \frac{177}{4}$

$I = \int \frac{1}{\left(x+\frac{13}{2}\right)^2 - \left(\frac{\sqrt{177}}{2}\right)^2}\ dx$

$= \frac{1}{2\left(\frac{\sqrt{177}}{2}\right)} log\ log\ \left|\frac{x+\frac{13}{2}-\frac{\sqrt{177}}{2}}{x+\frac{13}{2}+\frac{\sqrt{177}}{2}}\right| + c$

$\therefore I = \frac{1}{\sqrt{177}} log\ log\ \left|\frac{2x+13-\sqrt{177}}{2x+13+\sqrt{177}}\right| + c$

Question 3 **[2]**

(a) The general solution of the differential equation $x\frac{dy}{dx} + 2y = x^2\ (x \neq 0)$ is

Answer: $4x^2y = x^4 + C$

Solution:

Given differential equation, $\frac{dy}{dx} + \frac{2y}{x} = x$

Comparing it with linear form, $\frac{dy}{dx} + Py = Q,$ where $P\ and\ Q$ are the functions of x only.

We have, $P = \frac{2}{x}\ and\ Q = x$

$I.F. = e^{\int Pdx} = e^{\int \frac{2}{x}dx} = e^{2ln ln\, x} = x^2$

Solution of the differential equation is given by

$y(I.F.) = \int Q(I.F.) + K$

$\Rightarrow yx^2 = \int (x \cdot x^2)\, dx + K$

$\Rightarrow yx^2 = \frac{x^4}{4} + K$

$\therefore 4x^2y = x^4 + C\ \ (C = 4K)$

OR

(b) Find the differential equation which has $y = c_1e^x + c_2e^{-x}$ as the general solution.

Answer: $\frac{d^2y}{dx^2} - y = 0$

Solution:

$y = c_1e^x + c_2e^{-x}\ \cdots (i)$

Differentiating on both sides, we get

$\frac{dy}{dx} = c_1e^x - c_2e^{-x}$

Differentiating on both sides again, we get

$\frac{d^2y}{dx^2} = c_1e^x + c_2e^{-x}$

$\Rightarrow \frac{d^2y}{dx^2} = y$ [From (i)]

$\therefore \frac{d^2y}{dx^2} - y = 0$ is the required differential equation.

Question 4 **[2]**

Evaluate $\int_{\pi/6}^{\pi/3} \frac{1}{1+\sqrt{cot cot\, x}}\, dx$

Answer: $\frac{\pi}{12}$

Solution:

Let $I = \int_{\pi/6}^{\pi/3} \frac{1}{1+\sqrt{cot cot\, x}}\, dx$

$\Rightarrow I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{sinsin\ x}}{\sqrt{sinsin\ x}+\sqrt{coscos\ x}}\ dx\ \ \cdots(i)$

We know that $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

$\therefore I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{sinsin\left(\frac{\pi}{2}-x\right)}}{\sqrt{sinsin\left(\frac{\pi}{2}-x\right)}+\sqrt{coscos\left(\frac{\pi}{2}-x\right)}}\ dx$

$= \int_{\pi/6}^{\pi/3} \frac{\sqrt{coscos\ x}}{\sqrt{sinsin\ x}+\sqrt{coscos\ x}}\ dx\ \ \cdots(ii)$

Adding (i) and (ii), we get

$2I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{sinsin\ x}+\sqrt{coscos\ x}}{\sqrt{sinsin\ x}+\sqrt{coscos\ x}}\ dx$

$\Rightarrow\ 2I = \int_{\pi/6}^{\pi/3} 1\ dx$

$\Rightarrow 2I = \frac{\pi}{3} - \frac{\pi}{6}$

$\therefore I = \frac{\pi}{12}$

Question 5 **[4]**

(a) The probability distribution of X is as follows:

x	0	1	2	3	4
$P(X = x)$	0.1	k	$2k$	$2k$	k

Then the value of $P(X \geq 3)$ is

Answer: $\frac{9}{20}$

Solution:

We know that $\sum P(X = x) = 1$

$\Rightarrow\ P(x = 0) + P(x = 1) + P(x = 2) + P(x = 3) + P(x = 4) = 1$

$\Rightarrow 0.1 + +k + 2k + 2k + k = 1$

$\Rightarrow k = \frac{3}{20}$

$P(X \geq 3) = P(x = 3) + P(x = 4)$

$\Rightarrow P(X \geq 3) = 2\left(\frac{3}{20}\right) + \frac{3}{20}$

$\therefore P(X \geq 3) = \frac{9}{20}$

OR

(b) The probability of getting 3 exactly twice in 5 throws of a fair die is

Answer: $\frac{1}{3}\left(\frac{5}{6}\right)^4$

Solution:

The repeated tossing of a die forms a binomial distribution.

Let X represent the number of times of getting 3 in 5 throws of the die.

Probability of getting 3 in a single throw of a die, $p = \frac{1}{6}$

Probability of not getting 3, $q = 1 - \frac{1}{6} = \frac{5}{6}$

We have to find probability of getting 3 exactly twice in 5 throws of a die.

Using Binomial distribution,

$P(X = 2) = 5C_2 \cdot p^2 \cdot q^{5-2}$

$$= 5C_2 \cdot \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^3$$
$$= \frac{5\times4}{2\times1} \times \left(\frac{1}{6}\right)^2 \times \left(\frac{5}{6}\right)^3$$
$$\therefore P(X = 2) = \frac{1}{3}\left(\frac{5}{6}\right)^4$$

Question 6 **[4]**

The value of $\int \frac{1}{(2x+3)(x-4)} dx$

Answer: $ln\, ln\, \left|\frac{x-4}{2x+3}\right|^{\frac{1}{11}} + C$

Solution:

Let $I = \int \frac{1}{(2x+3)(x-4)} dx$

Consider $\frac{1}{(2x+3)(x-4)} = \frac{A}{2x+3} + \frac{B}{x-4}$

$\Rightarrow 1 = A(x-4) + B(2x+3)$

For $x = 4,\ 1 = B(2(4)+3) \Rightarrow B = \frac{1}{11}$

For $x = -\frac{3}{2},\ 1 = A\left(-\frac{3}{2} - 4\right) \Rightarrow A = -\frac{2}{11}$

$\therefore I = \frac{1}{11}\int \left(\frac{-2}{2x+3} + \frac{1}{x-4}\right) dx$

$= \frac{1}{11}[ln\, ln\, |x-4|\ - ln\, ln\, |2x+3|\ ] + C$

$= \frac{1}{11} ln\, ln\, \left|\frac{x-4}{2x+3}\right| + C,$ where C is the constant of integration.

Question 7 **[6]**

Every morning, Mr. Ramesh either reads a book or watches T.V. The probability that he watches T.V. is $\frac{4}{5}$. If he watches T.V., there is a probability of $\frac{3}{4}$ that he falls asleep. If he reads a book, the probability that he falls asleep is $\frac{1}{4}$. On one evening, Mr. Ramesh is found to be asleep. The probability that he watched T.V., is

Answer: $\frac{12}{13}$

Solution:

Probability that Ramesh watches T.V., $P(A) = \frac{4}{5}$.

Probability that Ramesh reads book, $P(B) = 1 - \frac{4}{5} = \frac{1}{5}$

Let S be the event that Ramesh is asleep.

Probability that Ramesh falls asleep while watching T.V. is $P(S|A) = \frac{3}{4}$

Probability that Ramesh falls asleep while reading book is $P(S|B) = \frac{1}{4}$

By Bayes' theorem,

$$P(A|S) = \frac{P(A)\cdot P(S|A)}{P(A)\cdot P(S|A)+P(B)\cdot P(S|B)} = \frac{\frac{4}{5}\times\frac{3}{4}}{\frac{4}{5}\times\frac{3}{4}+\frac{1}{5}\times\frac{1}{4}} = \frac{12}{13}$$

Question 8 **[6]**

(a) $Evaluate \int \frac{(x^2+1)e^x}{(x+1)^2} dx$

Answer: $e^x\left(\frac{x-1}{x+1}\right) + C$

Solution:

$$Let\ I = \int \frac{(x^2+1)e^x}{(x+1)^2} dx$$

$$\Rightarrow I = \int \frac{e^x(x^2-1+1+1)}{(x+1)^2} dx$$

$$\Rightarrow I = \int e^x\left[\frac{x^2-1}{(x+1)^2} + \frac{2}{(x+1)^2}\right] dx$$

$$\Rightarrow I = \int e^x\left[\frac{x-1}{x+1} + \frac{2}{(x+1)^2}\right] dx$$

Consider $f(x) = \frac{x-1}{x+1}$, then $f'(x) = \frac{2}{(x+1)^2}$

Thus, the given integral is of the form $e^x[f(x) + f'(x)]$.

Therefore, $I = \int \frac{(x^2+1)e^x}{(x+1)^2} dx = e^x\left(\frac{x-1}{x+1}\right) + C$

OR

(b) Evaluate $\int_0^{\pi/2} log\ log\ x\ )\ dx$

Answer: $-\frac{\pi}{2} log\ log\ 2$

Solution:

Let $I = \int_0^{\frac{\pi}{2}} log\ log\ x\ )\ dx \quad \cdots (1)$

We know that $\int_a^b f(x) = \int_a^b f(a+b-x)$

$$\therefore I = \int_0^{\frac{\pi}{2}} log\ log\ \left(\frac{\pi}{2} - x\right)\ dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} log\ log\ (cos\ cos\ x\ )\ dx \quad \cdots (2)$$

Adding the equations (1) and (2), we get

$$2I = \int_0^{\frac{\pi}{2}} [log\ log\ (sin\ sin\ x\ )\ + log\ log\ (cos\ cos\ x\ )\ ]dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} [log\ log\ (sin\ sin\ x\ \ cos\ cos\ x\ )\ + log\ log\ 2\ -$$
$$log\ log\ 2\ ]\ dx\ (By\ adding\ and\ subtracting\ log\ log\ 2\ )$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} [log\ log\ (sin\ sin\ 2x\ )\ - log\ log\ 2\ ]\ dx \quad (\because sin\ sin\ 2x\ = 2\ sin\ sin\ x\ \ cos\ cos\ x\ )$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} log\ log\ \ sin\ sin\ 2x\ \ dx - \int_0^{\frac{\pi}{2}} log\ log\ 2\ dx$$

Put $2x = t$ in the first integral. Then, $2\ dx = dt$

Lower limit: when $x = 0, t = 0$ and Upper limit: when $x = \frac{\pi}{2}, t = \pi$

$$\therefore 2I = \frac{1}{2}\int_0^{\pi} \log\log \sin\sin t \ dt - \int_0^{\frac{\pi}{2}} \log\log 2 \, dx$$

$$\Rightarrow 2I = \frac{2}{2}\int_0^{\frac{\pi}{2}} \log\log \sin\sin t \ dt - \frac{\pi}{2}\log\log 2 \quad (\because \sin\sin(\pi - x) = \sin\sin x)$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} \log\log \sin\sin x \ dx - \frac{\pi}{2}\log\log 2 \quad (By\ changing\ variable\ t\ to\ x)$$

$$\Rightarrow 2I = I - \frac{\pi}{2}\log\log 2$$

$$\therefore I = -\frac{\pi}{2}\log\log 2$$

SECTION B – 8 MARKS

Question 9 **[2]**

(i) The equation of plane passing through $A(1, 2, 3)$ and having $3, 2, 5$ as the direction ratios of the normal to the plane, is

A. $3x + 2y - 6z = 9$

B. $3x + 2y + 5z = 22$

C. $3x + 4y - 5z = 18$

D. $3x - 2y + 5z = 22$

Answer: $(B)\ 3x + 2y + 5z = 22$

Solution:

Equation of plane passing through (x_1, y_1, z_1) and having direction ratios of normal a, b, c is given by $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$.

$\therefore$ Required equation of plane is $3(x - 1) + 2(y - 2) + 5(z - 3) = 0$

or, $3x + 2y + 5z = 22$

(ii) Find the acute angle between the planes $\vec{r} \cdot (\hat{\imath} + \hat{\jmath} - 2\hat{k}) = 8$ and $\vec{r} \cdot (-2\hat{\imath} + \hat{\jmath} + \hat{k}) = 3$.

A. $\frac{\pi}{6}$

B. 0

C. $\frac{\pi}{2}$

D. $\frac{\pi}{3}$

Answer: $(D)\frac{\pi}{3}$

Solution:

Normal vector of given planes are $\vec{n_1} = \hat{\imath} + \hat{\jmath} - 2\hat{k}$ and $\vec{n_2} = -2\hat{\imath} + \hat{\jmath} + \hat{k}$.

Angle between the planes is, $\cos\cos\theta = \left|\frac{\vec{n_1}\cdot\vec{n_2}}{|\vec{n_1}||\vec{n_2}|}\right|$

$\Rightarrow \cos\cos\theta = \left|\frac{1(-2)+1(1)+(-2)(1)}{\sqrt{1^2+1^2+(-2)^2}\sqrt{(-2)^2+1^2+1^2}}\right|$

$\Rightarrow \cos\cos\theta = \left|-\frac{3}{6}\right| = \frac{1}{2}$

$\therefore \theta = \frac{\pi}{3}$

Question 10 **[2]**

If lines OA, OB are drawn from O with direction cosines proportional to $(1, -2, -1)$ and $(3, -2, 3)$, then the direction cosines of the normal to the plane AOB is

Answer: $\left(-\frac{4}{\sqrt{29}}, -\frac{3}{\sqrt{29}}, \frac{2}{\sqrt{29}}\right)$

Solution:

$\overrightarrow{OA} = \hat{\imath} - 2\hat{\jmath} - \hat{k}$

$\overrightarrow{OB} = 3\hat{\imath} - 2\hat{\jmath} + 3\hat{k}$

DRs of normal to the plane AOB is.

$\overrightarrow{OA} \times \overrightarrow{OB} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -2 & -1 \\ 3 & -2 & 3 \end{vmatrix} = \hat{\imath}(-6-2) - \hat{\jmath}(3+3) + \hat{k}(-2+6)$

$= -8\hat{\imath} - 6\hat{\jmath} + 4\hat{k}$

So, d.r.'s of normal is $(-4, -3, 2)$

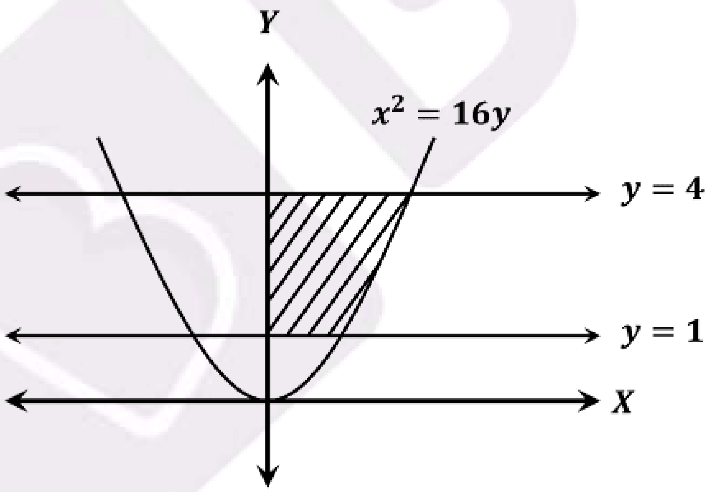
Hence, d.c.'s of normal is $\left(-\frac{4}{\sqrt{29}}, -\frac{3}{\sqrt{29}}, \frac{2}{\sqrt{29}}\right)$

Question 11 **[2]**

The area of the region bounded by $x^2 = 16y, y = 1, y = 4$ and $x = 0$ in the first quadrant is

Answer: $\frac{56}{3}$ sq. units

Solution:



Area $= \int_1^4 4\sqrt{y}\, dy$

$= 4\left[\frac{(y^{3/2})}{3/2}\right]_1^4$

$= \frac{8}{3}[8-1]$

$\therefore Area = \frac{56}{3}$ sq. units

SECTION C – 8 MARKS

Question 12 **[2]**

(i) The two lines of regression are $x + 2y - 5 = 0$ and $2x + 3y - 8 = 0$ and variance of x is 12. Find the variance of y.

A. 2
B. 1
C. 2.5
D. 1.5

Answer: $(A)\ 2$

Solution:

If possible, suppose that regression line, x on y is $2x + 3y - 8 = 0$ and y on x, is $x + 2y - 5 = 0$

$\therefore b_{xy} = -\frac{3}{2}$ and $b_{yx} = -\frac{1}{2}$

Since $r^2 = b_{xy} \cdot b_{yx} = -\frac{3}{2} \times -\frac{1}{2} = \frac{3}{4} < 1$

$\therefore$ Our assumption is right.

$\therefore$ Coefficient of correlation, $r = -\frac{\sqrt{3}}{2}$

We know that $b_{yx} = r \cdot \frac{\sigma_y}{\sigma_x} = -\frac{\sqrt{3}}{2} \times \frac{\sigma_y}{\sqrt{12}}$

$\Rightarrow -\frac{1}{2} = -\frac{\sqrt{3}}{2} \times \frac{\sigma_y}{\sqrt{12}}$

$\therefore \sigma_y = 2$

(ii) Let the regression line of x on y be, $mx - y + 10 = 0$ and y on x be, $-2x + 5y + 14 = 0$. If the coefficient of correlation between x and y is $\frac{1}{\sqrt{10}}$, then the value of m is

A. 6
B. 4
C. 2
D. 8

Answer: $(B)\ 4$

Solution:

Given, regression line of x on y: $mx - y + 10 = 0$

$\Rightarrow mx = y - 10$

$\Rightarrow x = \frac{y}{m} - \frac{10}{m}$

Comparing with $x = (b_{xy})y + c$, we get

$b_{xy} = \frac{1}{m}$

Similarly, $b_{yx} = \frac{2}{5}$

We know that $r^2 = b_{xy} \times b_{yx} = \frac{1}{10}$

$\Rightarrow \frac{2}{5m} = \frac{1}{10}$

$\therefore m = 4$

Question 13 **[2]**

Find the value of $b_{yx} - b_{xy}$ for the following observations: $(3,6), (4,5), (5,4), (6,3), (7,2)$

Answer: 0

Solution:

x	y	x^2	y^2	xy
3	6	9	36	18
4	5	16	25	20
5	4	25	16	20
6	3	36	9	18
7	2	49	4	14
$\sum x = 25$	$\sum y = 20$	$\sum x^2 = 135$	$\sum y^2 = 90$	$\sum xy = 90$

We know that $b_{xy} = \dfrac{\sum xy - \frac{\sum x \cdot \sum y}{n}}{\sum y^2 - \frac{\left(\sum y\right)^2}{n}}$

$\Rightarrow b_{xy} = \dfrac{90 - \frac{25\times20}{5}}{90 - \frac{400}{5}} = \dfrac{90-100}{90-80} = -1$

$\therefore b_{xy} = -1$

Also, $b_{yx} = \dfrac{\sum xy - \frac{\sum x \cdot \sum y}{n}}{\sum x^2 - \frac{\left(\sum x\right)^2}{n}}$

$\Rightarrow b_{yx} = \dfrac{90 - \frac{25\times20}{5}}{135 - \frac{625}{5}} = \dfrac{90-100}{135-125} = -1$

$\therefore b_{yx} = -1$

Hence, the value of $b_{yx} - b_{xy}$ is 0.

Question 14 **[4]**

A company produces two types of items P and Q. Manufacturing of both items requires the metals gold and copper. Each unit of item P requires 3 grams of gold and 1 gram of copper while that of item Q requires 1 gram of gold and 2 grams of copper. The company has 9 grams of gold and 8 grams of copper in its store. If each unit of item P makes a profit of ₹50 and each unit of item Q makes a profit of ₹60 then determine the number of units of each item that the company should produce to maximise profit. What is the maximum profit?

Answer: 280

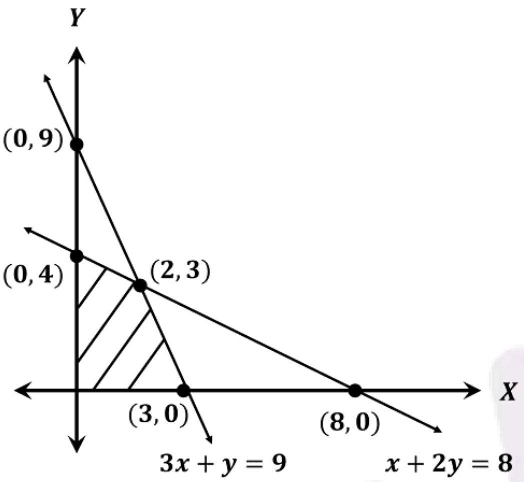
Solution:

Let the number of units item A to be produced be x and that of B to be produced be y. Let z be the total profit in rupees.

The given data can be put in tabular form as:

Items		Gold	Copper	Profit per unit(₹)
P	x	3	1	50
Q	y	1	2	60
Availability		9	8	

Thus, mathematical formulation of given LPP is
Maximize $z = 50x + 60y$
Subject to constraints $3x + y \leq 9$
$x + 2y \leq 8$
$x, y \geq 0$



The shaded region $OABC$ is the feasible region.
The corner points are $O(0,0), A(3,0), B(2,3), C(0,4)$.
At $O(0,0), z = 0$
At $A(3,0), z = 50 \times 3 + 60 \times 0 = 150$
At $B(2,3), z = 50 \times 2 + 60 \times 3 = 280$
At $C(0,4), z = 50 \times 0 + 60 \times 4 = 240$
$\therefore$ z is maximum at $x = 2$ and $y = 3$.
$\therefore$ For maximum profit, company should produce 2 units of item P and 3 units of item Q.
Hence, maximum profit is ₹280.