Term II - Part Test

Date: 14/04/2022

Class: X

Subject: Mathematics

Time: 02:00 hrs

Instructions:

General Instructions:
1. The question paper consists of 14 questions divided into 3 sections A, B, C.
2. All questions are compulsory.
3. Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in
two questions.
4. Section B comprises of 4questions of 3 marks each. Internal choice has been provided in
one question.
5. Section C comprises of 4 questions of 4 marks each. An internal choice has been provided
in one question. It contains two case study based questions.

1. Find the values of k for the following quadratic equation, so that they have two equal roots.
   kx (x - 2) + 6 = 0

   OR

   Solve the following quadratic equation:
   $48x^2 - 13x - 1 = 0$

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$kx(x-2)+6=0$
$or\ kx^2-2kx+6=0$
Comparing this equation with $ax^2+bx+c=0$, we get
a = k, b = - 2k and c = 6
$Discriminant = b^2-4ac$
$=(-2k)^2-4(k)\ (6)$
$=4k^2-24k$
For equal roots,
$b^2-4ac=0$
$4k^2-24k=0$
$4k(k-6)=0$
$Either\ 4k=0\ or\ k=6$
However, if k = 0, then the equation will not have the terms 'x^2' and 'x'.
Therefore, for this equation to has two equal roots, k should be 6.

OR

$48x^2-13x-1=0$

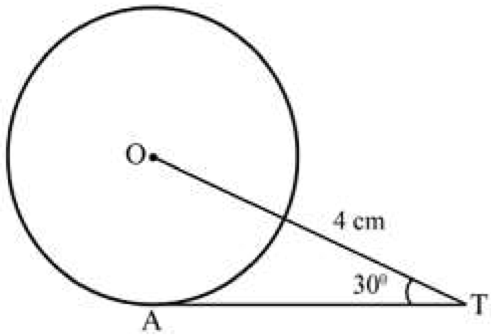
$=48x^2-16x+3x-1=0$

$=16x(3x-1)+1(3x-1)=0$

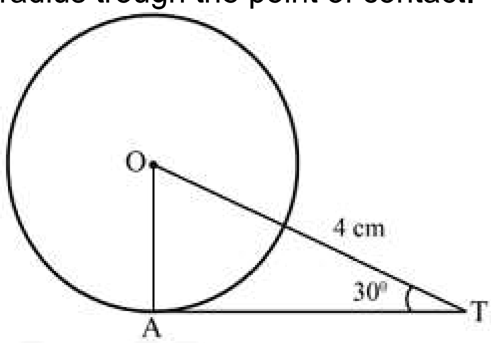
$=(3x-1)(16x+1)=0$

$3x-1=0$
$x=1/3$
and
$16x+1=0$
$x=-1/16$

2. In figure, AT is a tangent to the circle with centre O such that OT = 4 cm and $\angle OTA = 30^\circ$. Then , AT is equal to



Join OA.
We know that, the tangent at any point of a circle is perpendicular to the radius trough the point of contact.



$\therefore \angle OAT = 90^\circ$

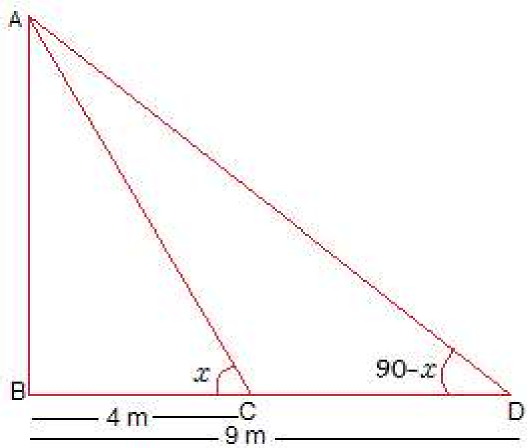
$In\ \Delta OAT, Cos\ 30^\circ = \frac{AT}{OT}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{AT}{4}$

$\Rightarrow AT = 2\sqrt{3}\ cm$

3. The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m from the base of the tower and in the same straight line with it are complementary. Prove that the height of the tower is 6 m.

Let AB be the height of tower and x be the one of the complementry angle.
So, (90-x) be the another complementary angle.
C and D be the two points with distance 4 m and 9 m from the base respectively.



As per question,

In right ΔABC,

tan $x = \frac{AB}{BC}$

$\Rightarrow\ tan\ x = \frac{AB}{4}$

$\Rightarrow\ AB = 4\ tan\ x \ldots (i)$

Also,

In right ΔABD,

tan $(90^\circ - x) = \frac{AB}{BD}$

$\Rightarrow\ cot\ x = \frac{AB}{9}$

$\Rightarrow\ AB =\ 9\ cot\ x \ldots (ii)$

Multiplying equation (i) and (ii)

$AB^2 = 9\ cot\ x \times 4\ tan\ x$

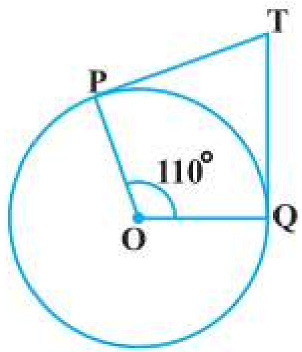
$\Rightarrow AB^2 = 36$

$\Rightarrow AB = \pm 6$

Height cannot be negative.

Therefore, the height of the tower is 6m.

4. In the given figure, if TP and TQ are the two tangents to a circle with centre O so that $\angle \mathrm{POQ} = 110^\circ$, $\angle \mathrm{PTQ}$ is equal to



OP and OQ are radii of the circle to the tangents TP and TQ respectively.
Since, the line drawn from the centre of the circle to the tangent is perpendicular to the tangent.
$\therefore$ OP $\perp$ TP and OQ $\perp$ TQ.
$\angle OPT = \angle OQT = 90^\circ$
In quadrilateral POQT,
Sum of all interior angles = 360°
$\angle PTQ + \angle OPT + \angle POQ + \angle OQT = 360^\circ$
$\Rightarrow \angle PTQ + 90^\circ + 110^\circ + 90^\circ = 360^\circ$
$\Rightarrow \angle PTQ = 70^\circ$
$\angle PTQ$ is equal to 70°.

5. Identify whether the given equation is a quadratic equation.
$x^2 + 2x + 1 = (4 - x)^2 + 3$

Given that, $x^2 + 2x + 1 = (4 - x)^2 + 3$
$\Rightarrow x^2 + 2x + 1 = 16 + x^2 - 8x + 3$
$\Rightarrow 10x - 18 = 0$
Which is not of the form $ax^2 + bx + c$, a $\neq$ 0. Thus the equation is not a quadratic equation.

6. Check whether -150 is a term of the A.P. 11, 8, 5, 2, …

OR

Show that $a_1, a_2 \ldots\ldots, a_n, \ldots\ldots$ form an AP where a_n is defined as below:
$a_n = 3 + 4n$
Also find the sum of the first 15 terms.

$For\ this\ A.P,$
$a = 11$
$d = a_2 - a_1 = 8 - 11 = -3$
$Let\ -150\ be\ the\ n^{th}\ term\ of\ this\ A.P.$
$We\ know\ that,\ a_n = a + (n-1)d$
$-150 = 11 + (n-1)(-3)$
$-164 = -3n$
$n = \frac{164}{3}$
Clearly, n is not an integer.
Therefore, - 150 is not a term of this A.P.

OR

$Given,\ a_n = 3 + 4n$
$a_1 = 3 + 4(1) = 7$
$a_2 = 3 + 4(2) = 3 + 8 = 11$
$a_3 = 3 + 4(3) = 3 + 12 = 15$
$a_4 = 3 + 4(4) = 3 + 16 = 19$
$It\ can\ be\ observed\ that;$
$a_2 - a_1 = 11 - 7 = 4$
$a_3 - a_2 = 15 - 11 = 4$
$a_4 - a_3 = 19 - 15 = 4$
$i.e., a_{k+1} - a_k\ is\ same\ everytime.\ Therefore,\ this\ is\ an\ AP$
$with\ common\ difference\ as\ 4\ and\ first\ term\ as\ 7.$
$S_n = \frac{n}{2}[2a + (n-1)d]$
$S_{15} = \frac{15}{2}[2(7) + (15-1) \times 4]$
$= \frac{15}{2}[(14) + 56]$
$= \frac{15}{2}(70)$
$= 15 \times 35$
$= 525$

7. The ratio of the sums of m and n terms of an A.P. is $m^2 : n^2$. Show that the ratio of the m^{th} and n^{th} term is $(2m–1) : (2n–1)$.

Let a be the first term and d the common difference of the given A.P. Then, the sums of m and n terms are given by

S_m = $(\frac{m}{2})$ [2a + (m – 1) d], and

S_n = $(\frac{n}{2})$ [2a + (n – 1) d]

$\frac{S_m}{S_n} = \frac{m^2}{n^2}$

$\frac{\frac{m}{2}[2a+(m-1)d]}{\frac{n}{2}[2a+(n-1)d]} = \frac{m^2}{n^2}$

$\Rightarrow \frac{2a+(m-1)d}{2a+(n-1)d} = \frac{m}{n}$

$\Rightarrow [2a + (m - 1)d]n = [2a + (n - 1)d]m$

$\Rightarrow 2a(n - m) = d((n - 1)m - (m - 1)n)$

$\Rightarrow 2a(n - m) = d(n - m)$

$\Rightarrow d = 2a$

$\frac{T_m}{T_n} = \frac{a+(m-1)\times 2a}{a+(n-1)\times 2a} = \frac{a+(m-1)\times 2a}{a+(n-1)\times 2a} = \frac{2m-1}{2n-1}$
$(\because d = 2a)$

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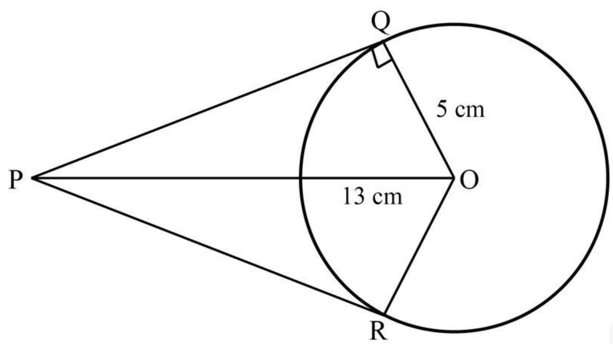
8. From a point P which is at a distance of 13 cm from the centre O of a circle of radius 5 cm, the pair of tangents PQ and PR to the circle is drawn. Then, the area of the quadrilateral PQOR is:

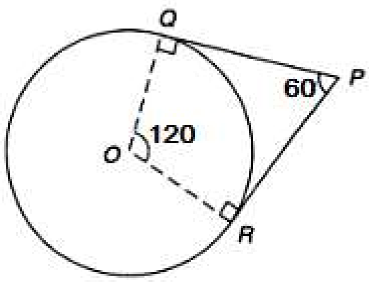
Firstly, draw a circle of radius 5 cm having centre O . P is a point at a distance of 13 cm from O. A pair of tangents PQ and PR are drawn.



Thus, Quadrilateral PQOR is formed.
[Since , QP is a tangent line]
$\therefore OQ \perp QP$
In right angled ΔPQO,
$OP^2 = OQ^2 + QP^2$
$\Rightarrow 13^2 = 5^2 + QP^2$
$\Rightarrow QP^2 = 169 - 25 = 144$
$QP = 12cm$
$Now,\ area\ of\ \Delta OQP = \frac{1}{2} \times QP \times QO$
$= \frac{1}{2} \times 12 \times 5 = 30cm^2$
$\therefore Area\ of\ quadrilateral\ QORP = 2 \times\ Area\ of\ \Delta OQP$
$= 2 \times 30 = 60cm^2$

9. Draw a pair of tangents to a circle of radius 5 cm which are inclined to each other at an angle of 60°.

We know that radius of the circle is perpendicular to the tangents.
Sum of all the 4 angles of quadrilateral $= 360^{\circ}$
$\therefore$ Angle between the radius $(\angle O) = 360^{\circ} - (90^{\circ} + 90^{\circ} + 60^{\circ}) = 120^{\circ}$
Steps of Construction:
Step I: A point Q is taken on the circumference of the circle and OQ is joined. OQ is radius of the circle.
Step II: Draw another radius OR making an angle equal to 120° with the previous one.



Step III: A point P is taken outside the circle. QP and PR are joined which are perpendicular to OQ and OR respectively.
Thus, QP and PR are the required tangents inclined to each other at an angle of 60°.
Justification:
Sum of all angles in the quadrilateral PQOR = 360°
$\angle QOR + \angle ORP + \angle OQR + \angle RPQ = 360^{\circ}$
$\Rightarrow 120^{\circ} + 90^{\circ} + 90^{\circ} + \angle RPQ = 360^{\circ}$
$\Rightarrow \angle RPQ = 360^{\circ} - 300^{\circ}$
$\Rightarrow \angle RPQ = 60^{\circ}$
Hence, QP and PR are tangents inclined to each other at an angle of 60°.

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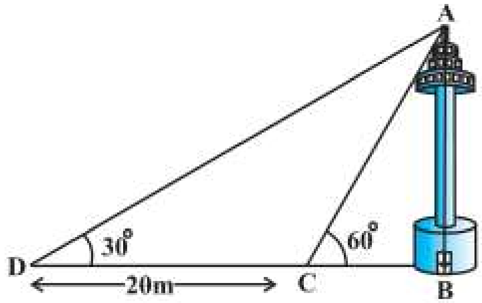
10. A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60°. From another point 20 m away from this point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is 30°. Find the height of the tower and the width of the canal.



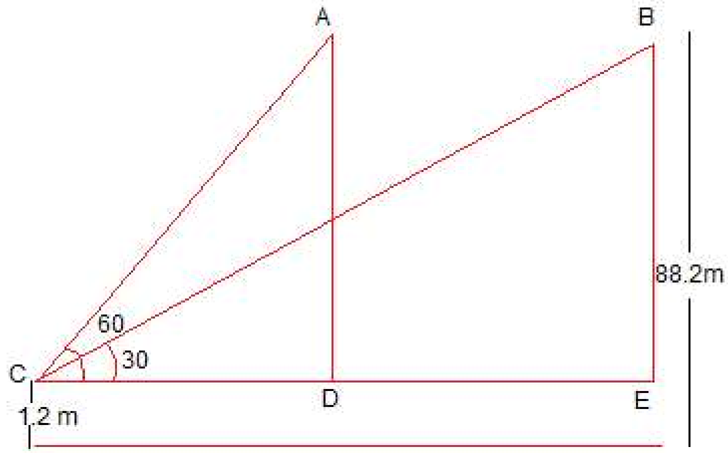
OR

A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60°. After some time, the angle of elevation reduces to 30°. Find the distance travelled by the balloon during the interval.

Here, AB is the height of the tower and BC is the width of canal.
CD = 20 m
As per question,
In right ΔABD,
tan $30^\circ = \frac{AB}{BD}$
$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{(20+BC)}$
$\Rightarrow AB = \frac{(20+BC)}{\sqrt{3}} \ldots (i)$
Also,
In right ΔABC,
tan $60^\circ = \frac{AB}{BC}$
$\Rightarrow \sqrt{3} = \frac{AB}{BC}$
$\Rightarrow AB = \sqrt{3}BC \ldots (ii)$
From equation (i) and (ii)
$AB = \sqrt{3}BC = \frac{(20+BC)}{\sqrt{3}}$
$\Rightarrow 3BC = 20 + BC$
$\Rightarrow 2BC = 20 \Rightarrow BC = 10m$
Putting the value of BC in equation (ii)
$AB = 10\sqrt{3}m$
Thus, the height of the tower is $10\sqrt{3}$ m and the width of the canal is 10 m.

OR

Let the initial position of the balloon be A and final position be B.
Height of balloon above the girl height $= 88.2m - 1.2m = 87m$
Distance travelled by the balloon = $DE = CE - CD$



As per question, In right

ΔBEC,

tan $30^\circ = \frac{BE}{CE}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{87}{CE}$

$\Rightarrow CE = 87\sqrt{3}m$

Also,

In right ΔADC,

tan $60^\circ = \frac{AD}{CD}$

$\Rightarrow \sqrt{3} = \frac{87}{CD}$

$\Rightarrow CD = \frac{87}{\sqrt{3}}m = 29\sqrt{3}m$

Distance travelled by the balloon

$= DE = CE - CD = (87\sqrt{3} - 29\sqrt{3})m = 29\sqrt{3}(3-1)m$ =100.45 m

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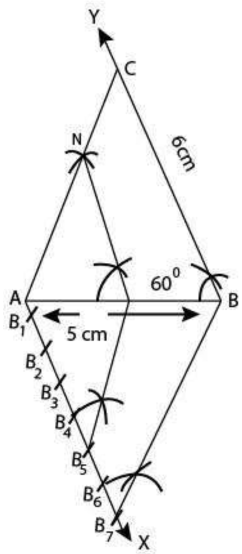
11. Draw a ΔABC in which AB = 5 cm , BC = 6 cm and $\angle ABC = 60^\circ$. Construct a triangle similar to ABC with scale factor $\frac{5}{7}$. Justify the construction.

Steps of construction



1. Draw a line segment AB= 5 cm
2. From Point B, draw $\angle ABY = 60^\circ$ on which take BC = 6cm
3. Join AC, ΔABC is the required triangle.
4. From A, draw any ray AX downwards marking an acute angle.
5. Mark 7 points $B_1, B_2, B_3, B_4, B_5, B_6$ and B_7 on AX. Such that
$AB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5 = B_5B_6 = B_6B_7$
6. Join B_7B and from B_5 draw $B_5M \parallel B_7B$ intersecting AB at M
7. From point M draw $MN \parallel BC$ intersecting AC at N. then ΔAMN is the required triangle whose sides are equal to $\frac{5}{7}$ of the corresponding sides of the ΔABC

Justification

Here $B_5M \parallel B_7B$ (by construction)

$\therefore \frac{AM}{BM} = \frac{5}{2}$

Now, $\frac{AB}{AM} = \frac{AM+MB}{AM}$

$= 1 + \frac{MB}{AM} = 1 + \frac{2}{5} = \frac{7}{5}$

Also, $MN \parallel BC$

$\therefore \Delta AMN \sim \Delta ABC$

Therefore $\frac{AM}{AB} = \frac{AN}{AC} = \frac{NM}{BC} = \frac{5}{7}$

12. Find the roots of the given equation, if they exist, by applying the quadratic formula:
$3a^2x^2 + 8abx + 4b^2 = 0, a \neq 0$

OR

Factorize $\sqrt{3}\,x^2$ + 5x + 2$\sqrt{3}$ = 0 into (ax+b)(cx+d)=0. What is the value of a + b + c - d ?

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$$3a^2x^2 + 8abx + 4b^2 = 0$$

$$a = 3a^2$$
$$b = 8ab$$
$$c = 4b^2$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-8ab \pm \sqrt{(8ab)^2 - 4 \times 3a^2 \times 4b^2}}{2 \times 3a^2}$$

$$= \frac{-8ab \pm \sqrt{64a^2b^2 - 48a^2b^2}}{6a^2}$$

$$= \frac{-8ab \pm \sqrt{16a^2b^2}}{6a^2}$$

$$= \frac{-8ab \pm 4ab}{6a^2}$$

$$= \frac{-8ab + 4ab}{6a^2} \quad or \quad \frac{-8ab - 4ab}{6a^2}$$

$$= \frac{-4ab}{6a^2} \quad or \quad \frac{-12ab}{6a^2}$$

$$x = \frac{-2b}{3a} \quad or \quad \frac{-2b}{a}$$

OR

Multiply $\sqrt{3}$ with $2\sqrt{3}$ and we get 6

Pairs of 6 are

$1 \times 6 = 6$ sum $1 + 6 = 7$

$2 \times 3 = 6$ sum $2 + 3 = 5$

So our required pair is 2 and 3

$\Rightarrow \sqrt{3}\, x^2 + 5x + 2\sqrt{3} = 0$

$\Rightarrow \sqrt{3}\, x^2 + 3x + 2x + 2\sqrt{3} = 0$

$\Rightarrow \sqrt{3}x(x + \sqrt{3}) + 2(x + \sqrt{3}) = 0$

$\Rightarrow (\sqrt{3}x + 2)(x + \sqrt{3}) = 0$

a= $\sqrt{3}$, b = 2, c = 1, d = - $\sqrt{3}$

a + b + c - d = 3

13. A group of students of class X visited India Gate on an education trip. The teacher and students had interest in history as well. The teacher narrated that India Gate, official name Delhi Memorial, originally called All-India War Memorial, monumental sandstone arch in New Delhi, dedicated to the troops of British India who died in wars fought between 1914 and 1919. The teacher also said that India Gate, which is located at the eastern end of the Rajpath (formerly called the Kingsway), is about 138 feet (42 metres) in height.

1. What is the angle of elevation if they are standing at a distance of 42m away from the monument?
2. They want to see the tower at an angle of 60 . So, they want to know the distance where they should stand and hence find the distance.
3. If the altitude of the Sun is at 60 , then the height of the vertical tower that will cast a shadow of length 20 m is
4. The ratio of the length of a rod and its shadow is 1:1 .The angle of elevation of the Sun is

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1. To find the angle of elevation, $\tan\ \theta = \frac{height\ of\ the\ tower}{dis\tan ce\ from\ the\ tower} = \frac{42}{42} = 1$

$$\sqrt{3}\ = \frac{42}{dis\tan ce}$$

$$dis\tan ce = \frac{42}{\sqrt{3}}$$

$$dis\tan ce\ =\ 24.64\ m$$

$\theta\ =\ \tan^{-1}\ (1)\ =\ 45^{\circ}$

2. To find the distance, $\tan\ 60^{\circ}\ = \frac{height\ of\ the\ tower}{dis\tan ce} = \frac{42}{dis\tan ce}$

$$\sqrt{3} = \frac{42}{dis\tan ce}$$

$$dis\tan ce\ = \frac{42}{\sqrt{3}}$$

$$dis\tan ce\ =\ 24.64\ m$$

3. To find the height of the verticle tower,$\tan\ 60^{\circ}\ = \frac{height\ of\ the\ tower}{dis\tan ce}$

$$\sqrt{3} = \frac{height\ of\ the\ tower}{20}$$

$$height\ of\ the\ tower\ = 20\sqrt{3}$$

4. To find the angle of elevation of the sun,

$$\tan\ \theta\ = \frac{height\ of\ the\ tower}{dis\tan ce\ from\ the\ tower}$$

$$= \frac{1}{1}\left(\sin ce\ the\ ratios\ are\ in\ 1:1\right)$$

$$\theta\ =\ \tan^{-1}\left(1\right)\ =\ 45^{\circ}$$

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14. Firozabad is a city near Agra in Firozabad district in the state of Uttar Pradesh in India. It is the centre of India's glassmaking industry and is known for the quality of the bangles and also glasswares produced there.

    Rita lives in Firozabad and aspires to establish her own small scale bangle manufacturing company. In the first month her company manufactured 5000 bangles and she plans to increase the bangle production by 500 each month.

    i) Find the number of bangles her company will be producing in the 12^{th} month.

    ii) Find the total number of bangles produced by her company in 12 months.

Given: Rita's company can produce 5000 bangles in the first month.
so, $a = 5000$
now, we know that the bangle production increase y 500 each month.
so, $d = 500$

i) bangle produce in the 12^{th} month = a_{12} = ?
$a_n = a + (n - 1)d$
so, $a_{12} = 5000 + (12 - 1)500$
$= 5000 + 11 \times 500$
= 10,500
so, 10500 bangles will be produced in the 12^{th} month.

ii) total number of bangles produced in 12 months = $S_{12} = ?$
$S_n = \frac{n(a+l)}{2}$
$a = 5000, n = 12, l = a_{12} = 10500$
so, $S_{12} = \frac{12(5000+10500)}{2}$
$\Rightarrow S_{12} = 6(15500) = 93000$
so, a total of 93000 bangles is produced by the the company in 12 months.