

# Uttar Pradesh Madhyamik Shiksha Parishad Class 12 Model Question Papers for Maths

## Model Question Paper 2021-22

### Mathematics

TIME – 3 Hrs 15 Min

Class-12

Maximum Marks - 100

*Note:* First 15 minutes are allotted for the candidates to read the question paper.

#### **Instructions :**

- (i) There are in all *nine* questions in this question paper.
- (ii) *All* questions are compulsory.
- (iii) In the beginning of each question, the number of parts to be attempted has been clearly mentioned.
- (iv) Marks allotted to the questions are indicated against them.
- (v) Start solving from the first question and proceed to solve till the last one.
- (vi) Do not waste your time over a question you cannot solve.

#### **1. Choose the correct option and write down in your answer sheet.**

- (a) Suppose that the function defined as  $f(x) = 3x$  is  $f: \mathbb{R} \rightarrow \mathbb{R}$ , select the correct option. 01
- (i)  $f$  is one-one onto                      (ii)  $f$  is many-one onto  
(iii)  $f$  is one-one but not onto      (iv)  $f$  is neither one-one nor onto
- (b) If  $R$  is a relation on the set  $N$ , defined as  $R = \{(a, b) : a = b - 2, b > 6\}$ , select the correct option from the following. 01
- (i)  $(2, 4) \in R$                                       (ii)  $(3, 8) \in R$   
(iii)  $(6, 8) \in R$                                       (iv)  $(8, 7) \in R$
- (c) Find the value of integral  $\int xe^x dx$  01
- (i)  $e^x$               (ii)  $(x+1)e^x$               (iii)  $(x-1)e^x$               (iv)  $\frac{x^2}{2}e^x$

- (d) Order of the differential equation  $2x^2 \frac{d^2y}{dx^2} - 3 \frac{dy}{dx} + y = 0$  is - 01
- (i) 2      (ii) 1      (iii) 0      (iv) not defined
- (e) If the vector's  $2\hat{i} + \hat{j} + \hat{k}$  and  $\hat{i} - 4\hat{j} + \lambda\hat{k}$  are mutually perpendicular, then find the value of  $\lambda$  - 01
- (i) 3      (ii) 2      (iii) 4      (iv) 0

**2. Attempt all the parts:**

- (a) Find the principal value of  $\text{Cot}^{-1}\left(\frac{-1}{\sqrt{3}}\right)$ . 01
- (b) Show that the function  $f(x) = |x|$ , is continuous at  $x = 0$ . 01
- (c) Find the order and power of the differential equation  $xy \frac{d^2y}{dx^2} + x \left(\frac{dy}{dx}\right)^2 - y \frac{dy}{dx} = 0$ . 01
- (d) Find the maximum value of  $z = 3x + 4y$  subject to the following constraints  $x + y \leq 4, x \geq 0, y \geq 0$ . 01
- (e) If  $P(A) = \frac{7}{13}$ ,  $P(B) = \frac{9}{13}$  and  $P(A \cap B) = \frac{4}{13}$  then find the value of  $P(A/B)$ . 01

**3. Attempts all the parts:**

- (a) If  $A = \{1,2\}$  and  $B = \{3,4\}$  then find the number of relations between A and B. 02
- (b) If  $y = A \sin x + B \cos x$  then prove that  $\frac{d^2y}{dx^2} + y = 0$ . 02
- (c) Find the angle between the vectors  $\hat{i} - 2\hat{j} + 3\hat{k}$  and  $3\hat{i} - 2\hat{j} + \hat{k}$ . 02
- (d) A problem of mathematics is given to three students. Probabilities of solving the problem by them are  $\frac{1}{2}$ ,  $\frac{1}{3}$  and  $\frac{1}{4}$ . If all the three students try their best, then find the probability that problem is solved. 02

**4. Attempt all the parts.**

- (a) Show that the function defined on  $\mathbb{R}$  as  $f(x) = 7x - 3$  is an increasing function. 02
- (b) Find the unit vector perpendicular to each of vectors  $(\bar{a} + \bar{b})$  and  $(\bar{a} - \bar{b})$  where  $\bar{a} = \hat{i} + \hat{j} + \hat{k}$ ,  $\bar{b} = \hat{i} + 2\hat{j} + 3\hat{k}$ . 02
- (c) Find the area of parallelogram whose adjacent sides are given by vectors  $\bar{a} = 3\hat{i} + \hat{j} + 4\hat{k}$  and  $\bar{b} = \hat{i} - \hat{j} + \hat{k}$ . 02
- (d) A and B are two given events where  $P(A) = \frac{1}{2}$ ,  $P(A \cup B) = \frac{3}{5}$  and  $P(B) = P$ . Find the value of P if events are mutually exclusive. 02

**5. Attempt all the parts.**

- (a) Prove that the relation R on the set of integers  $\mathbb{Z}$  is defined as  $R = \{(a, b) : (a-b) \text{ is divisible by number } 2\}$  is an equivalence relation. 05
- (b) Prove that 
$$\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc.$$
 05
- (c) Differentiate the function  $(\sin x)^{\cos x}$  with respect to x. 05
- (d) Find the 
$$\int_{-\pi/4}^{\pi/4} \sin^2 x \, dx.$$
 05
- (e) Find the shortest distance between the lines  $\bar{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$  and  $\bar{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(2\hat{i} + 3\hat{j} + 6\hat{k})$ . 05

**6. Attempt all the parts:**

- (a) Show that the function  $f(x) = \begin{cases} \frac{|x|}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$  is discontinuous at  $x = 0$ . 05
- (b) Find the area bounded by the parabolas  $y = x^2$  and  $y^2 = x$ . 05
- (c) Find the equation of the plane passing through the intersection of the planes  $\bar{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 6$  and  $\bar{r} \cdot (2\hat{i} + 3\hat{j} + 4\hat{k}) = -5$  and the point  $(1, 1, 1)$ . 05

- (d) Minimize  $z = 3x + 2y$  subject to the constraints; 05  
 $x + y \geq 8$     $3x + 5y \leq 15$     $x \geq 0$ ,    $y \geq 0$
- (e) In a hostel 60% students read Hindi newspaper, 40% students read English newspaper and 20% read both newspapers -
- (i) Find the probability of the students who read neither Hindi newspaper nor English newspaper.  $2\frac{1}{2}$
- (ii) If she reads Hindi newspaper then what is the probability that she also reads English newspaper.  $2\frac{1}{2}$

**7. Attempt any one of the following:**

- (a) If  $A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$  and  $B = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$  08  
then find out the value of  $(AB)^{-1}$ .

- (b) Solve the following system of linear equations by the matrix method:

$$\begin{aligned} 3x - 2y + 3z &= 8 \\ 2x + y - z &= 1 \\ 4x - 3y + 2z &= 4 \end{aligned} \quad \text{08}$$

**8. Attempt any one of the following:**

- (a) Find the area bounded by the parabola  $y^2 = 4ax$  and its latus rectum. 08
- (b) Find the general solution of the differential equation  $\frac{dy}{dx} - y = \text{Cos}x$ . 08

**9. Attempt any one of the following:**

- (a) Find the value of the integral  $\int_0^{\frac{\pi}{2}} \log \sin x \, dx$ . 08
- (b) Evaluate  $\int_0^{\pi} \frac{x \, dx}{a^2 \text{Cos}^2 x + b^2 \text{Sin}^2 x}$ . 08

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