COM, Collision and Rotational dynamics

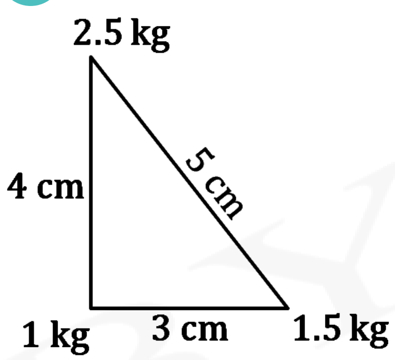
1. Three point particles of mass 1 kg, 1.5 kg and 2.5 kg are placed at three corners of a right triangle of sides 4.0 cm, 3.0 cm and 5.0 cm as shown in the figure. The centre of mass of the system is at the point:

A. 0.9 cm right and 2.0 cm above 1 kg mass

B. 2.0 cm right and 0.9 cm above 1 kg mass

C. 1.5 cm right and 1.2 cm above 1 kg mass

D. 0.6 cm right and 2.0 cm above 1 kg mass

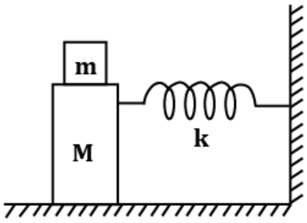


Taking 1 kg as the origin

$$x_{com} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$
$$x_{com} = \frac{1 \times 0 + 1.5 \times 3 + 2.5 \times 0}{5}$$
$$x_{com} = 0.9$$
$$y_{com} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3}$$
$$y_{com} = \frac{1 \times 0 + 1.5 \times 0 + 2.5 \times 4}{5}$$
$$y_{com} = 2$$

Centre of mass is at $(0.9, 2)$

2. In the given figure, a mass M is attached to a horizontal spring, which is fixed on one side to a rigid support. The spring constant of the spring is k. The mass oscillates on a frictionless surface with time period T and amplitude A. When the mass is in equilibrium position, as shown in the figure, another mass m is gently fixed upon it then the new amplitude of oscillation will be :



A. $A\sqrt{\dfrac{M}{M+m}}$

B. $A\sqrt{\dfrac{M}{M-m}}$

C. $A\sqrt{\dfrac{M-m}{M}}$

D. $A\sqrt{\dfrac{M+m}{M}}$

Before placing the mass m, angular frequency,

$$\omega_i = \sqrt{\frac{k}{M}}$$

After placing the mass m, angular frequency,

$$\omega_f = \sqrt{\frac{k}{M+m}}$$

As there is no impulsive force, so, linear momentum will remain conserved.

$$\therefore p_i = p_f$$
$$\Rightarrow M\omega_i A_i = (M+m)\omega_f A_f$$

$$\Rightarrow M \times \sqrt{\frac{k}{M}} \times A = (M+m) \times \sqrt{\frac{k}{M+m}} \times A_f$$

$$\Rightarrow A_f = A\sqrt{\frac{M}{M+m}}$$

COM, Collision and Rotational dynamics

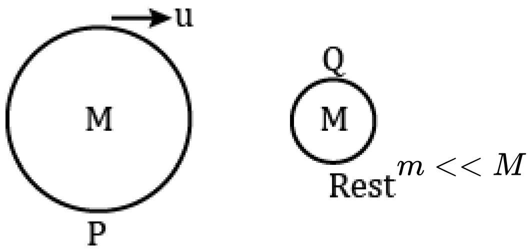
3. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A: Body P having mass M moving with speed u has head-on collision elastically with another body Q having mass m initially at rest. If $m << M$, body Q will have a maximum speed equal to $2u$ after collision.

Reason R : During elastic collision, the momentum and kinetic energy are both conserved.

In the light of the above statements, choose the most appropriate answer from the options given below:

- ✗ **A.** A is correct but R is not correct.
- ✗ **B.** Both A and R are correct but R is NOT the correct explanation of A.
- ✗ **C.** A is not correct but R is correct.
- ✓ **D.** Both A and R are correct and R is the correct explanation of A.



$$e=\frac{v_2-v_1}{u_1-u_2}$$

[Newton's Law of Restitution]

For elastatic collision, $e=1$

$$1=\frac{v_2-u}{u-0}$$

$$u=v_2-u$$

$$v_2=2u$$

In elastic collision kinetic energy & momentum are conserved.

Hence, option (d) is the correct answer.

COM, Collision and Rotational dynamics

4. A rubber ball is released from a height of $5\ \mathrm{m}$ above the floor. It bounces back repeatedly, always rising to $\frac{81}{100}$ of the height through which it falls. Find the average speed of the ball.
(Take $g = 10\ \mathrm{m\ s^{-2}}$)

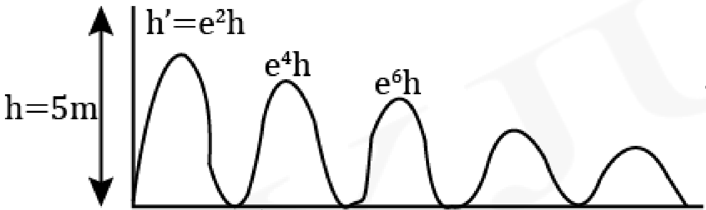
A. $2.50\ \mathrm{m\ s^{-1}}$

B. $3.50\ \mathrm{m\ s^{-1}}$

C. $3.0\ \mathrm{m\ s^{-1}}$

D. $2.0\ \mathrm{m\ s^{-1}}$

Let the situation of the ball be as shown in the below figure.



Total distance:

$$d = h + 2e^2h + 2e^4h + 2e^6h + 2e^8h + \ldots$$

$$d = h + 2e^2h(1 + e^2 + e^4 + e^6 + \ldots)$$

$$d = h + 2e^2h\left(\frac{1}{1-e^2}\right)$$

$$d = \frac{(1-e^2)h + 2e^2h}{1-e^2} = \frac{h(1+e^2)}{1-e^2}$$

Total time: $t = T + 2eT + 2e^2T + 2e^3T + \ldots$ Where $T = \sqrt{\frac{2h}{g}} = 1$

$$t = T + 2eT(1 + e + e^2 + e^3 + \ldots)$$

$$t = T + 2eT\left(\frac{1}{1-e}\right)$$

$$t = \frac{T(1+e)}{1-e}$$

Now, average speed of the ball

$$V_{avg} = \frac{d}{t} = \frac{h\dfrac{(1+e^2)}{(1-e^2)}}{T\left(\dfrac{1+e}{1-e}\right)}$$

$$V_{avg} = \frac{5}{1}\left(\frac{1+e^2}{(1+e)(1-e)}\frac{(1-e)}{(1+e)}\right)$$

$$V_{avg} = \frac{5(1+e^2)}{(1+e)^2}$$

$\because h' = e^2 h$

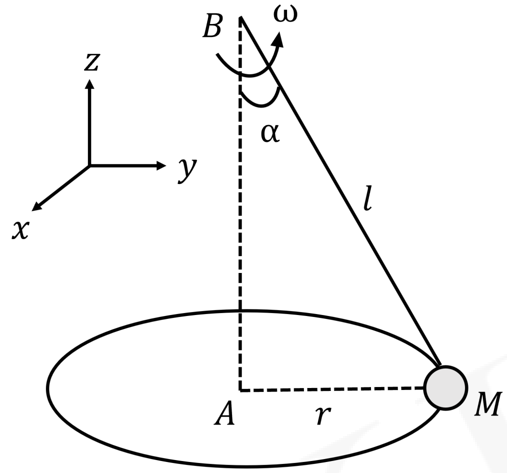
From the question: $\frac{81}{100} = e^2$

$e = \frac{9}{10} = 0.9$

$$V_{avg} = \frac{5\left(1+\frac{81}{100}\right)}{(1+0.9)^2}$$

$V_{avg} = 2.50 \text{ m/s}$

5. A mass M hangs on a massless rod of length l which rotates at a constant angular frequency. The mass M moves with steady speed in a circular path of constant radius. Assume that the system is in steady circular motion with constant angular velocity ω. The angular momentum of M about point A is L_A which lies in the positive $z-$direction and the angular momentum of M about point B is L_B. The correct statement for this system is :



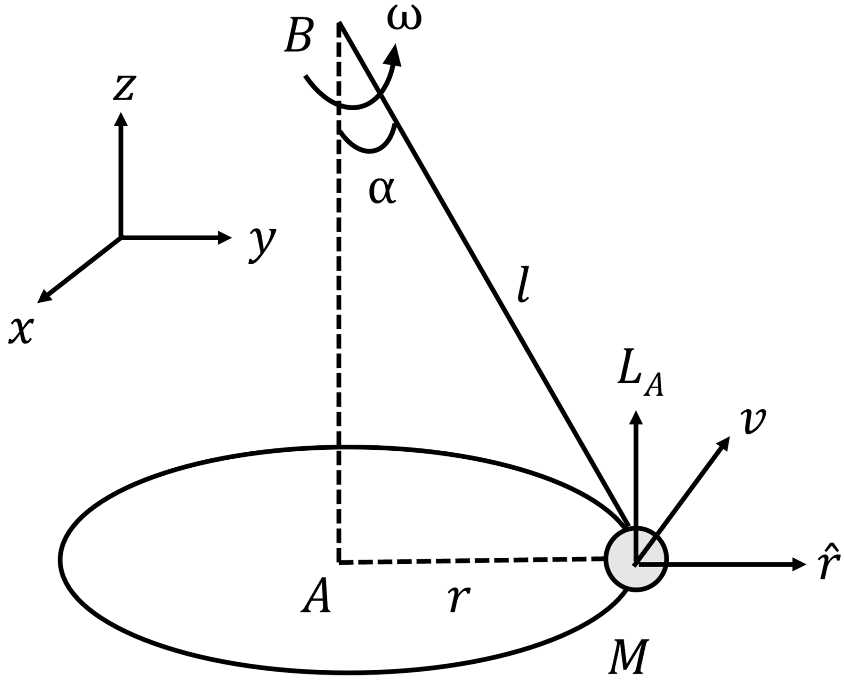
A. L_A and L_B are both constant in magnitude and direction.

B. L_B is constant, both in magnitude and direction.

C. L_A is constant, both in magnitude and direction.

D. L_A is constant in direction with varying magnitude.

About point A :



Angular momentum is given by,

$$\vec{L} = \vec{r} \times \vec{p} = M\left(\vec{r} \times \vec{v}\right)$$

So,
$L_A = Mvr\sin\theta = Mvr\sin 90^\circ = Mvr$

Speed v is constant as ω and r are constant, $\because v = \omega r$.

Also, mass M is constant.

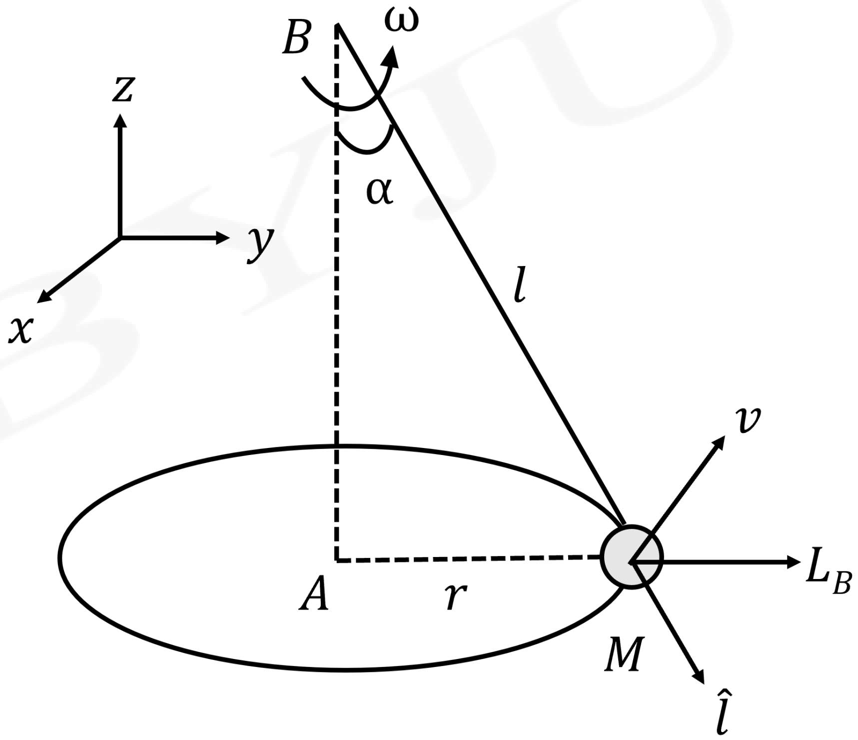
Therefore, L_A is constant in magnitude.

The direction of velocity $\vec{v}$ is always tangential to the path.

$\vec{r} \times \vec{v}$ points in vertically upwards direction.
So, the direction of $\overrightarrow{L_A}$ about A is also constant.

Hence, L_A is constant, both in magnitude and direction.

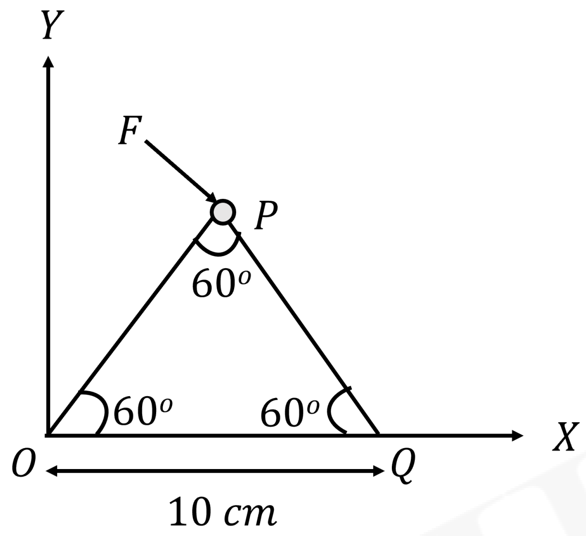
About point B :



Here, $L_B = Mvl\sin\theta$
The angle between $\vec{l}$ and $\vec{v}$ is always 90°, so the magnitude is constant.
However, the direction of $\overrightarrow{L_B}$ changes continuously as the mass rotate.

Hence, option (C) is the correct answer.

6. A triangular plate is shown in the figure. A force $\overrightarrow{F} = 4\hat{i} - 3\hat{j}$ is applied at point P. The torque acting at point P with respect to point O and point Q respectively are :



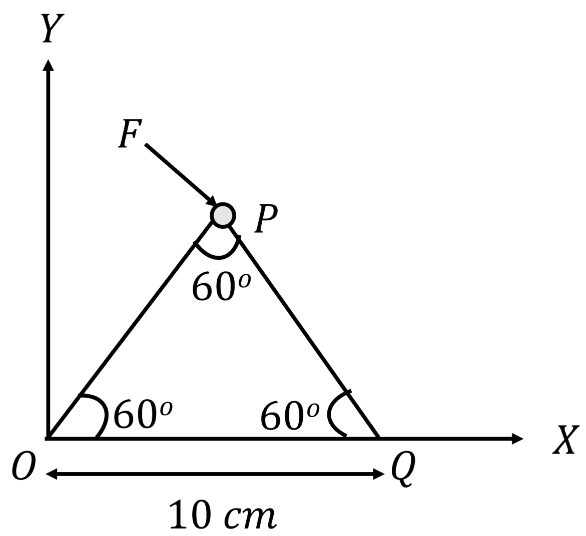
A. $15 - 20\sqrt{3}$; $15 + 20\sqrt{3}$

B. $15 + 20\sqrt{3}$; $15 - 20\sqrt{3}$

C. $-15 + 20\sqrt{3}$; $15 + 20\sqrt{3}$

D. $-15 - 20\sqrt{3}$; $15 - 20\sqrt{3}$

COM, Collision and Rotational dynamics



Given,

$\overrightarrow{F} = 4\hat{i} - 3\hat{j}$

Position vector of $\overrightarrow{F}$ about point O,

$\overrightarrow{r_1} = 10\cos 60^\circ \hat{i} + 10\sin 60^\circ \hat{j}$

$\Rightarrow \overrightarrow{r_1} = 5\ \hat{i} + 5\sqrt{3}\ \hat{j}$

Now, torque,

$\overrightarrow{\tau_1} = \overrightarrow{r_1} \times \overrightarrow{F}$

$\Rightarrow \overrightarrow{\tau_1} = \left(5\hat{i} + 5\sqrt{3}\hat{j}\right) \times \left(4\hat{i} - 3\hat{j}\right)$

$\Rightarrow \overrightarrow{\tau_1} = (-15 - 20\sqrt{3})\hat{k}$

Similarly, position vector of $\overrightarrow{F}$ about point Q,

$\overrightarrow{r_2} = -10\cos 60^\circ \hat{i} + 10\sin 60^\circ \hat{j}$

$\Rightarrow \overrightarrow{r_2} = -5\ \hat{i} + 5\sqrt{3}\ \hat{j}$

Now, torque,

$\overrightarrow{\tau_2} = \overrightarrow{r_2} \times \overrightarrow{F}$

$\Rightarrow \overrightarrow{\tau_2} = \left(-5\ \hat{i} + 5\sqrt{3}\ \hat{j}\right) \times \left(4\ \hat{i} - 3\ \hat{j}\right)$

$\Rightarrow \overrightarrow{\tau_2} = \left(15 - 20\sqrt{3}\right)\hat{k}$

7. A thin circular ring of mass M and radius r is rotating about its axis with an angular speed ω. Two particles having mass m each are now attached at diametrically opposite points. The angular speed of the ring will become:

A. $\omega \frac{M}{M+2m}$

B. $\omega \frac{M}{M+m}$

C. $\omega \frac{M+2m}{M}$

D. $\omega \frac{M-2m}{M+2m}$

External torque is zero on the system. $\tau_{net} = 0$, so angular momentum is conserved.

By angular momentum conservation:
$I_i \omega_i = I_f \omega_f$

$$(MR^2)\omega = (MR^2 + 2mR^2)\omega_f$$

$$\omega_f = \frac{(MR^2)\omega}{MR^2 + 2mR^2} = \frac{M\omega}{M+2m}$$

$$\omega_f = \frac{M\omega}{M+2m}$$

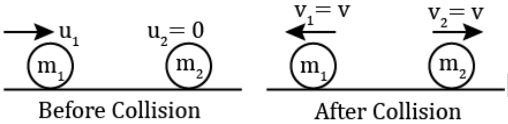
8. An object of mass m_1 collides elastically with another object of mass m_2, which is at rest. After the collision, the objects move with equal speeds in opposite directions. The ratio of the masses, $m_2 : m_1$ is -

A. $2:1$

B. $1:1$

C. $1:2$

D. $3:1$



From conservation of linear momentum;

$p_i = p_f$

$\Rightarrow m_1u_1 + m_2(0) = m_1(-v) + m_2v$

$\Rightarrow m_1u_1 = v(m_2 - m_1) \quad .....(i)$

Also, co-efficient of restitution,

$e = \dfrac{v_{\text{separation}}}{v_{\text{approach}}} = \dfrac{v-(-v)}{u_1} = \dfrac{2v}{u_1} = 1 \quad [\text{For elastic collision}]$

$\Rightarrow u_1 = 2v \quad .....(ii)$

From (i) and (ii),

$m_1(2v) = v(m_2 - m_1)$

$\Rightarrow 2m_1 = m_2 - m_1$

$\Rightarrow 3m_1 = m_2$

$\Rightarrow \dfrac{m_2}{m_1} = \dfrac{3}{1} = 3:1$

9. A uniform sphere of mass 500 g rolls without slipping on a plane horizontal surface with its centre moving at a speed of 5.00 cm/s. Its kinetic energy is

A. 8.75×10^{-4} J

B. 8.75×10^{-3} J

C. 6.25×10^{-4} J

D. 1.13×10^{-3} J

$K.E$ of the sphere = translational $K.E + \text{rotational K.E}$

$$= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

Where, I = moment of inertia,
ω = Angular, velocity of rotation
m = mass of the sphere
v = linear velocity of centre of mass of sphere

$$\because \text{ Moment of intertia of sphere } I = \frac{2}{5}mR^2$$

$$\therefore K.E = \frac{1}{2}mv^2 + \frac{1}{2} \times \frac{2}{5}mR^2 \times \omega^2$$

$$\Rightarrow K.E = \frac{1}{2}mv^2 + \frac{1}{2} \times \frac{2}{5}mR^2 \times \left(\frac{v}{R}\right)^2 \left(\because \omega = \frac{v}{R}\right)$$

$$\Rightarrow KE = \frac{1}{2}\left(\frac{2}{5}mR^2 + mR^2\right)\left(\frac{v}{R}\right)^2$$

$$\Rightarrow KE = \frac{1}{2}mR^2 \times \frac{7}{5} \times \frac{v^2}{R^2} = \frac{7}{10} \times \frac{1}{2} \times \frac{25}{10^4}$$

$$\Rightarrow KE = \frac{35}{4} \times 10^{-4} \text{J}$$

$$\Rightarrow KE = 8.75 \times 10^{-4} \text{ J}$$

10. A particle of mass m is dropped from a height h above the ground. At the same time another particle of the same mass is thrown vertically upwards from the ground with a speed of $\sqrt{2\,gh}$. If they collide head-on completely inelastically, the time taken for the combined mass to reach the ground, in units of $\sqrt{\frac{h}{g}}$ is

A. $\sqrt{\frac{1}{2}}$

B. $\sqrt{\frac{3}{4}}$

C. $\frac{1}{2}$

D. $\sqrt{\frac{3}{2}}$

Let S_1 be distance travelled by particle being dropped from height h before collision
and S_2 be distance travelled by vertically projected particle before collision
Here,

$S_1 = \frac{1}{2} g t^2$

$S_2 = ut - \frac{1}{2} g t^2$

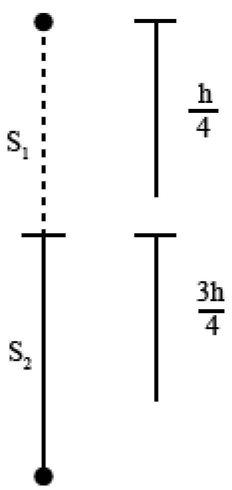
Given that $u = \sqrt{2gh}$

We know that,

$S_1 + S_2 = h$

$\sqrt{2gh}t = h$

$t = \sqrt{\frac{h}{2g}}$



Velocity of dropped particle just before collision is $v_1 = gt = \sqrt{\frac{hg}{2}}$

Velocity of projected particle just before collision is

$$v_2 = u - gt = \sqrt{2gh} - \sqrt{\frac{hg}{2}}$$

For inelastic collission, using principle of conversation of linear momentum
$mv_1 + mv_2 = 2mv_f$

$$\Rightarrow v_f = \frac{m\left(\sqrt{2gh} - \sqrt{\frac{gh}{2}}\right) - m\sqrt{\frac{gh}{2}}}{2m} = 0$$

ie after collision combined mass as zero velocity.
Distance travelled by this combined mass after collision before reaching ground is $S_2 = h - S_1 = h - \frac{h}{4} = \frac{3h}{4}$
After collison, time taken (t_1) for combined mass to reach the ground is

$$\Rightarrow \frac{3h}{4} = \frac{1}{2}gt_1^2$$

$$\Rightarrow t_1 = \sqrt{\frac{3h}{2g}}$$

COM, Collision and Rotational dynamics

11. Mass per unit area of a circular disc of radius $'a'$ depends on the distance r from its centre, as $\sigma(r) = A + Br$. The moment of inertia of the disc about the axis, perpendicular to the plane and passing through its centre, is:

A. $2\pi a^4\left(\frac{A}{4}+\frac{aB}{5}\right)$

B. $2\pi a^4\left(\frac{aA}{4}+\frac{B}{5}\right)$

C. $\pi a^4\left(\frac{A}{4}+\frac{aB}{5}\right)$

D. $2\pi a^4\left(\frac{A}{4}+\frac{B}{5}\right)$

Given,
mass per unit area of circular disc, $\sigma = A + Br$

Consider a small elemental ring of thickness dr at a distance r from the centre,

Area of the element $= 2\ \pi r dr$
Mass of the element, $dm = \sigma 2\pi r dr$

The moment of inertia of the ring about an axis, perpendicular to the plane and passing through its centre, is given by,

$I = \int dm r^2 = \int \sigma 2\pi r dr . r^2$

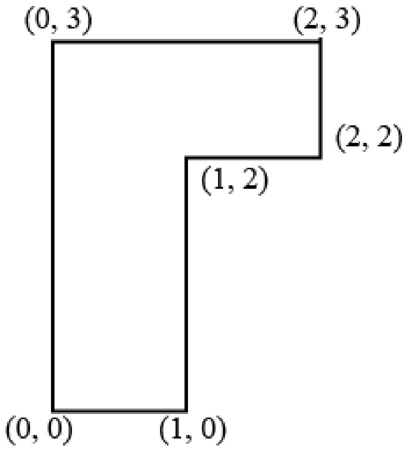
$\Rightarrow I = 2\pi \int_0^a (A + Br) r^3 dr$

$$\Rightarrow I = 2\pi\left[\frac{Aa^4}{4}+\frac{Ba^5}{5}\right]$$

$$\Rightarrow I = 2\pi a^4\left[\frac{A}{4}+\frac{Ba}{5}\right]$$

Hence, option (A) is correct.

12. The coordinates of centre of mass of a uniform flag shaped lamina (thin flat plate) of mass $4\ \mathrm{kg}$. (The coordinates of the same are shown in figure) are:



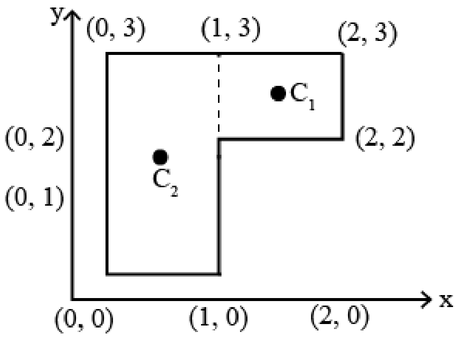
A. $1.25\ \mathrm{m}, 1.50\ \mathrm{m}$

B. $0.75\ \mathrm{m}, 1.75\ \mathrm{m}$

C. $0.75\ \mathrm{m}, 0.75\ \mathrm{m}$

D. $1\ \mathrm{m}, 1.75\ \mathrm{m}$

COM, Collision and Rotational dynamics



The given Lamina can be divided into two parts, as shown.

The mass and the position of centre of mass of these parts are given by,

$m_1 = 1, C_1 = (1.5, 2.5)$

$m_2 = 3, C_2 = (0.5, 1.5)$

$$X_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$

$$\Rightarrow X_{cm} = \frac{1.5 + 1.5}{4} = 0.75$$

$$Y_{cm} = \frac{m_1y_1 + m_2y_2}{m_1 + m_2}$$

$$\Rightarrow Y_{cm} = \frac{2.5 + 4.5}{4} = 1.75$$

$\therefore$ Coordinate of the centre of mass of flag shaped lamina $(0.75, 1.75)$

Hence, option (B) is correct.

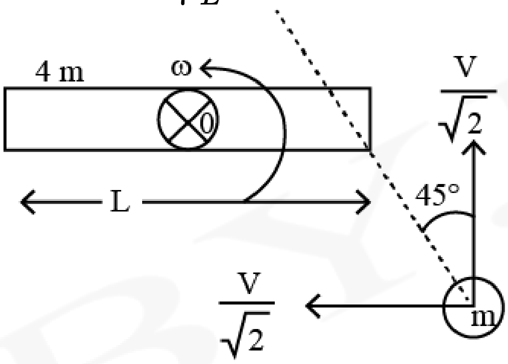
13. Consider a uniform rod of mass $M = 4m$ and length L pivoted about its centre. A mass m moving with a velocity V making an angle $\theta = \frac{\pi}{4}$ to the rod's long axis collides with one end of the rod, and sticks to it. The angular speed of the rod-mass system just after the collision is:

A. $\frac{3}{7\sqrt{2}} \frac{V}{L}$

B. $\frac{3}{7} \frac{V}{L}$

C. $\frac{3\sqrt{2}}{7} \frac{V}{L}$

D. $\frac{4}{7} \frac{V}{L}$



Angular momentum of the rod-mass system about point O is

$$\Rightarrow L = (mg \times 0) + \frac{mV}{\sqrt{2}} \times \frac{L}{2} = \frac{mVL}{2\sqrt{2}}$$

Let, I be the moment of inertia about O of the rod-mass system, and, ω be the angular speed just after collision.

$$I = \frac{4mL^2}{12} + \frac{mL^2}{4} = \frac{7}{12} mL^2$$

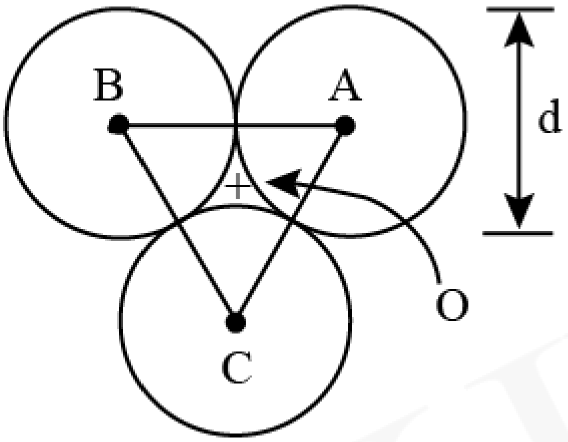
$$L = I\omega$$

$$\Rightarrow \frac{mVL}{2\sqrt{2}} = \frac{7}{12} mL^2 \times \omega$$

$$\therefore \omega = \frac{6V}{7\sqrt{2}L} = \frac{3\sqrt{2}V}{7L}$$

Hence, option (C) is correct.

14. Three solid spheres each of mass m and diameter d are stuck together such that the lines connecting the centres form an equilateral triangle of side of length d. The ratio $\frac{I_0}{I_A}$ of moment of inertia I_0 of the system about an axis passing the centroid and about centre of any of the spheres I_A and perpendicular to the plane of the triangle, is:



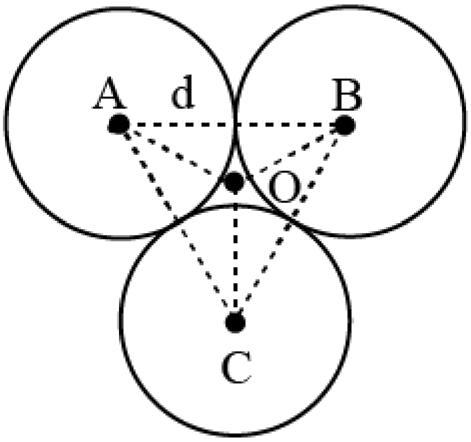
A. $\frac{13}{23}$

B. $\frac{15}{13}$

C. $\frac{23}{13}$

D. $\frac{13}{15}$

COM, Collision and Rotational dynamics



Moment of inertia of a sphere, about an axis passing through O, is,

$$I_1 = \frac{2}{5}m\left(\frac{d}{2}\right)^2 + m(AO)^2$$

and $AO = \dfrac{d}{\sqrt{3}}$

Moment of inertia of the system about O, is

$$I_0 = 3I_1 = 3\left[\frac{2}{5}m\left(\frac{d}{2}\right)^2 + m\left(\frac{d}{\sqrt{3}}\right)^2\right]$$

$$\Rightarrow I_0 = \frac{13}{10}md^2$$

Similarly, Moment of inertia of the system about A, is

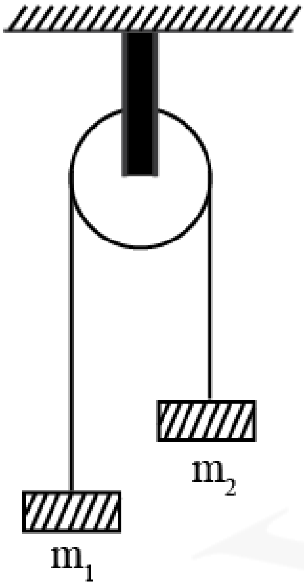
$$I_A = 2\left[\frac{2}{5}m\left(\frac{d}{2}\right)^2 + md^2\right] + \frac{2}{5}m\left(\frac{d}{2}\right)^2$$

$$\Rightarrow I_A = \frac{23}{10}md^2$$

$$\therefore \frac{I_0}{I_A} = \frac{\frac{13}{10}md^2}{\frac{23}{10}md^2} = \frac{13}{23}$$

Hence, (A) is the correct answer.

15. A uniformly thick wheel, with moment of inertia I and radius R, is free to rotate about its centre of mass (see fig.). A massless string is wrapped over its rim and two blocks of masses m_1 and $m_2 > m_2$ are attached to the ends of the string. The system is released from rest. The angular speed of the wheel, when m_1 descents through a distance h, is



A. $\left[\frac{2(m_1 - m_2)gh}{(m_1 + m_2)R^2 + I}\right]^{1/2}$

B. $\left[\frac{2(m_1 + m_2)gh}{(m_1 + m_2)R^2 + I}\right]^{1/2}$

C. $\left[\frac{(m_1 - m_2)}{(m_1 + m_2)R^2 + I}\right]^{1/2} gh$

D. $\left[\frac{(m_1 + m_2)}{(m_1 + m_2)R^2 + I}\right]^{1/2} gh$

Using the principal of conservation of energy,
$(m_1 - m_2)gh = \frac{1}{2}(m_1 + m_2)v^2 + \frac{1}{2}I\omega^2$

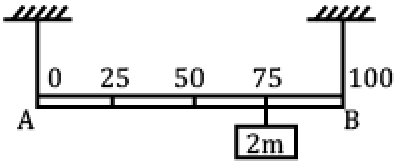
As, $v = \omega R$

$\Rightarrow (m_1 - m_2)gh = \frac{1}{2}(m_1 + m_2)(\omega^2 R^2) + \frac{1}{2}I\omega^2$

$\Rightarrow (m_1 - m_2)gh = \frac{\omega^2}{2}\left[(m_1 + m_2)R^2 + I\right]$

$\Rightarrow \omega = \sqrt{\frac{2(m_1 - m_2)gh}{(m_1 + m_2)R^2 + I}}$

Hence, (A) is the correct answer.

16. 

Shown in the figure is a rigid and uniform one meter long rod AB held in horizontal position by two strings tied to its ends and attached to the ceiling. The rod is of mass m and has another weight of mass $2m$ hung at a distance of 75 cm from A. The tension in the string at A is :

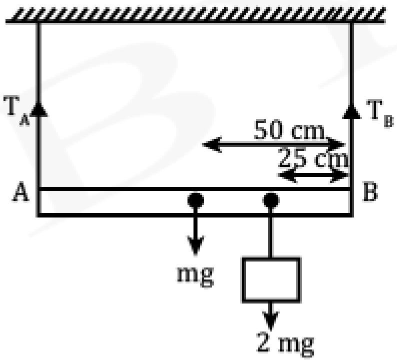
A. $0.5mg$

B. $2mg$

C. $0.75mg$

D. $1mg$

Net torque, τ_{net} about B is zero at equilibrium,



$\Rightarrow (T_A \times 100) - (mg \times 50) - (2mg \times 25) = 0$

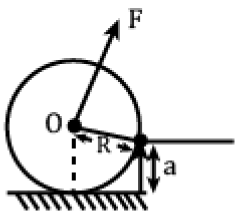
$\Rightarrow T_A \times 100 = 100mg$

Therefore, Tension in the string at A

$T_A = 1mg$

Hence, option (D) is correct.

17. A uniform cylinder of mass M and radius R is to be pulled over a step of height a $(a < R)$ by applying a force F at its centre O perpendicular to the plane through the axes of the cylinder on the edge of the step (see figure). The minimum value of F required is :

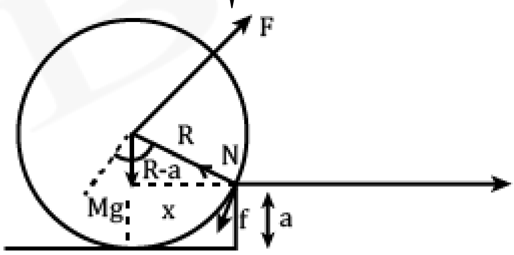


A. $Mg\sqrt{1-\left(\frac{R-a}{R}\right)^2}$

B. $Mg\sqrt{\left(\frac{R}{R-a}\right)^2-1}$

C. $Mg\frac{a}{R}$

D. $Mg\sqrt{1-\frac{a^2}{R^2}}$



With respect to the point of contact on the step,

For step up,

$\tau_F \geq \tau_{mg}$

$\Rightarrow F \times R \geq Mg \times x$

From the figure, $x = \sqrt{R^2 - (R-a)^2}$

$\Rightarrow F_{min} = \frac{Mg}{R} \times \sqrt{R^2 - (R-a)^2}$

$\Rightarrow F_{min} = Mg\sqrt{1-\left(\frac{R-a}{R}\right)^2}$

Hence, option (A) is correct.

COM, Collision and Rotational dynamics

18. Moment of inertia of a cylinder of mass M, length L and radius R about an axis passing through its centre and perpendicular to the axis of the cylinder is $I = M\left(\frac{R^2}{4} + \frac{L^2}{12}\right)$. If such a cylinder to be made for a given mass of a material, the ratio $\frac{L}{R}$ for it to have minimum possible I is-

A. $\frac{2}{3}$

B. $\frac{3}{2}$

C. $\sqrt{\frac{3}{2}}$

D. $\sqrt{\frac{2}{3}}$

Given, $I = \frac{MR^2}{4} + \frac{ML^2}{12}$

$$\because \quad V = \pi R^2 L \quad \Rightarrow \quad R^2 = \frac{V}{\pi L}$$

$$\Rightarrow I = \frac{M}{4} \times \frac{V}{\pi L} + \frac{ML^2}{12} = \frac{MV}{4\pi L} + \frac{ML^2}{12}$$

For I to be minimum, $\frac{dI}{dL} = 0$

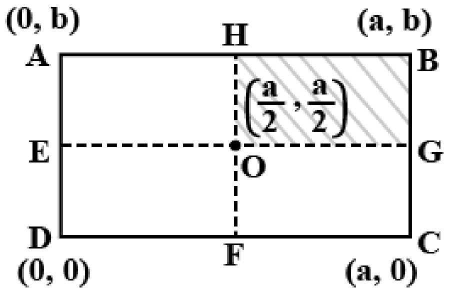
$$\frac{dI}{dL} = -\frac{MV}{4\pi L^2} + \frac{M \times 2L}{12} = 0$$

$$\Rightarrow \left(\frac{R}{L}\right)^2 = \frac{2}{3}$$

$$\therefore \frac{L}{R} = \sqrt{\frac{3}{2}}$$

Hence, (C) is the correct answer.

19. A uniform rectangular thin sheet ABCD of mass M has length a and breadth b, as shown in the figure. If the shaded portion HBGO is cut-off, the coordinates of the centre of mass of the remaining portion will be:



A. $\left(\frac{3a}{4}, \frac{3b}{4}\right)$

B. $\left(\frac{5a}{3}, \frac{5b}{3}\right)$

C. $\left(\frac{2a}{3}, \frac{2b}{3}\right)$

D. $\left(\frac{5a}{12}, \frac{5b}{12}\right)$

With respect to point O, the CM of the cut-off potion will have coordinates, $\left(\frac{a}{4}, \frac{b}{4}\right)$

$\therefore$ With respect to point D, the CM of the cut-off potion will have coordinates, $\left[\left(\frac{a}{2}+\frac{a}{4}\right), \left(\frac{b}{2}+\frac{b}{4}\right)\right]$

So, With respect to point D, the coordinates of CM of the cut-off portion are, $\left(\frac{3a}{4}, \frac{3b}{4}\right)$

Let the mass of the removed portion is m and mass per unit area of the sheet is σ.

$$\Rightarrow \sigma = \frac{M}{ab} = \frac{m}{\frac{a}{2} \times \frac{b}{2}}$$

$$\Rightarrow m = \frac{M}{4}$$

$$\Rightarrow x_{\text{CM}} = \frac{MX - mx}{M - m}$$

$$= \frac{M \times \frac{a}{2} - \frac{M}{4} \times \frac{3a}{4}}{M - \frac{M}{4}} = \frac{5a}{12}$$

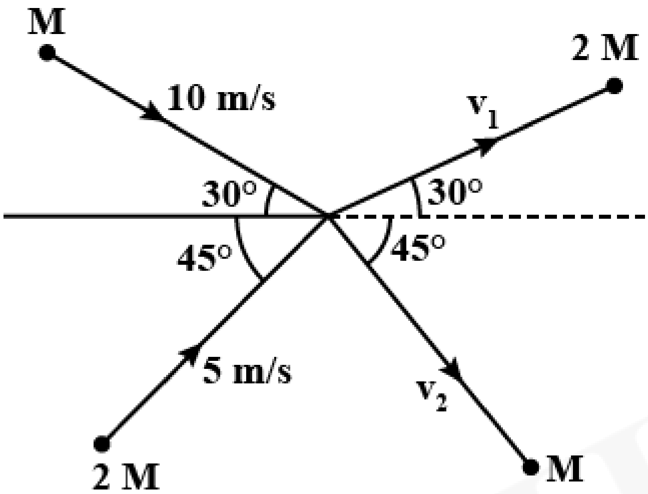
and $y_{\text{CM}} = \frac{MY - my}{M - m}$

$$= \frac{M \times \frac{b}{2} - \frac{M}{4} \times \frac{3b}{4}}{M - \frac{M}{4}} = \frac{5b}{12}$$

So, coordinates of CM of the remaining portion are, $\frac{5a}{12}, \frac{5b}{12}$

Hence, (D) is the correct answer.

20. Two particles, of masses M and $2M$, moving, as shown, with speeds of $10 \mathrm{~m/s}$ and $5 \mathrm{~m/s}$, collide elastically at the origin. After the collision, they move along the indicated directions with speeds v_1 and v_2, respectively. The values of v_1 and v_2 are nearly



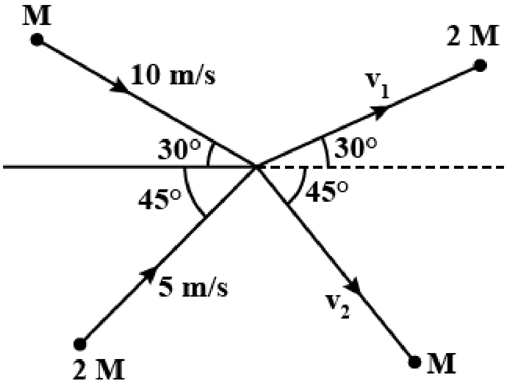
A. $6.5 \mathrm{~m/s}$ and $6.3 \mathrm{~m/s}$

B. $3.2 \mathrm{~m/s}$ and $6.3 \mathrm{~m/s}$

C. $6.5 \mathrm{~m/s}$ and $3.2 \mathrm{~m/s}$

D. $3.2 \mathrm{~m/s}$ and $12.6 \mathrm{~m/s}$

Given:
Speed of mass M particle $= 10\ \mathrm{m/s}$
Speed of mass $2M$ particle $= 5\ \mathrm{m/s}$



Applying conservation of linear momentum in X and Y direction for the system then

$$M(10\cos 30^\circ) + 2M(5\cos 45^\circ) = 2M(v_1\cos 30^\circ) + M(v_2\cos 45^\circ)$$

$$5\sqrt{3} + 5\sqrt{2} = \sqrt{3}v_1 + \frac{v_2}{\sqrt{2}} \quad ...\ (1)$$

Also

$$2M(5\sin 45^\circ) - M(10\sin 30^\circ) = 2Mv_1\sin 30^\circ - Mv_2\sin 45^\circ$$

$$5\sqrt{2} - 5 = v_1 - \frac{v_2}{\sqrt{2}} \quad ...\ (2)$$

Solving equation (1) and (2)

$$(\sqrt{3}+1)v_1 = 5\sqrt{3} + 10\sqrt{2} - 5$$

$$\Rightarrow v_1 = 6.5\ \mathrm{m/s}$$

$$\Rightarrow v_2 = 6.3\ \mathrm{m/s}$$

Hence option (A) is correct.

21. A piece of wood of mass 0.03 kg is dropped from the top of a 100 m height building. At the same time, a bullet of mass 0.02 kg is fired vertically upward, with a velocity 100 ms^{-1}, from the ground. The bullet gets embedded in the wood. Then the maximum height to which the combined system reaches above the top of the building before falling below is: $(g = 10 \text{ ms}^{-2})$

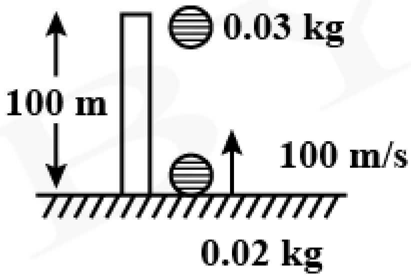
A. 20 m

B. 30 m

C. 40 m

D. 10 m

Given:
Wooden piece of mass $= 0.03 \text{ kg}$
Building height $= 100 \text{ m}$
Bullet mass $= 0.02 \text{ kg}$
Initial velocity of bullet $= 100 \text{ ms}^{-1}$,



Time taken for the particles to collide,

For wooden ball

Let h_1 be the distance which is travelled, then using equation of motion

$$h_1 = 0 + \frac{1}{2}gt^2$$

Similarly, h_2 is the distance covered by the bullet in verticxal direction

$$h_2 = 100.t - \frac{1}{2}gt^2$$

Since, total distance covered by both,

$h_1 + h_2 = 100 \text{ m}$

$100t = 100 \text{ m} \Rightarrow t = 1 \text{ s}$

Speed of wood just before collision

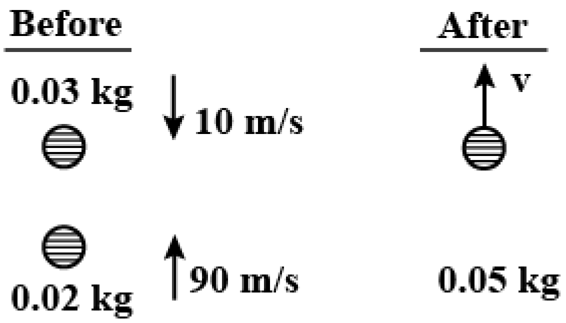
$u_1 = 0 + g.t = 10 \text{ m/s}$

and speed of bullet just before collision

$u_2 = v - gt = 100 - 10 \times 1 = 90 \text{ m/s}$

Distance travelled by the bullet (collision point)

$S = 100 \times 1 - \frac{1}{2} \times 10 \times 1 = 95 \text{ m}$



Now, using conservation of linear momentum just before and after the collision

$m_1 u_1 + m_2 U_2 = (m_1 + m_2) v$

$-(0.03)(10) + (0.02)(90) = (0.05)v$

$\Rightarrow 150 = 5v$

$\therefore v = 30 \text{ m/s}$

Max. height reached by body,

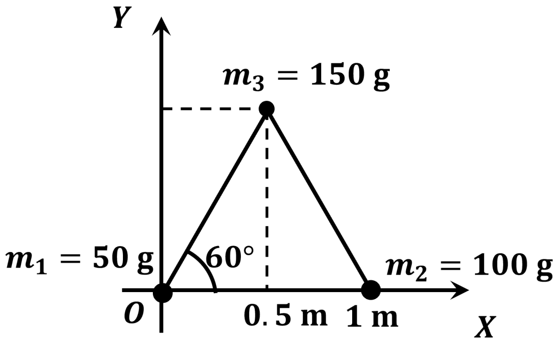
$v^2 = u^2 - 2gh$

$\Rightarrow h = \frac{v^2}{2g} = \frac{30 \times 30}{2 \times 10} = 45 \text{ m}$

$\therefore$ Height above tower $45 - 5 = 40 \text{ m}$

Hence, option (C) is correct.

22. Three particles of masses $50\ \mathrm{g}, 100\ \mathrm{g}$ and $150\ \mathrm{g}$ are placed at the vertices of an equilateral triangle of side $1\ \mathrm{m}$ (asshown in the figure). The (x, y) coordinates of the centre of mass will be :



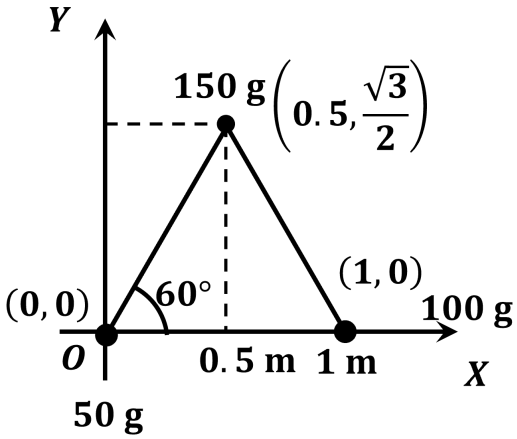
A. $\left(\frac{\sqrt{3}}{4}\mathrm{m}, \frac{5}{12}\mathrm{m}\right)$

B. $\left(\frac{7}{12}\mathrm{m}, \frac{\sqrt{3}}{8}\mathrm{m}\right)$

C. $\left(\frac{7}{12}\mathrm{m}, \frac{\sqrt{3}}{4}\mathrm{m}\right)$

D. $\left(\frac{\sqrt{3}}{8}\mathrm{m}, \frac{7}{12}\mathrm{m}\right)$

Given:
masses $m_1 = 50$ g,
$m_2 = 100$ g
$m_3 = 150$ g



For the center of mass of the given system ,
Using formula

$$x_{cm} = \frac{m_1.x_1 + m_2.x_2 + m_3.x_3}{m_1 + m_2 + m_3}$$

$$= \frac{50 \times 0 + 100 \times 1 + 150 \times 0.5}{50 + 100 + 150}$$

$$= \frac{7}{12}\text{m}$$

Similarly,

$$y_{cm} = \frac{m_1.y_1 + m_2.y_2 + m_3.y_3}{m_1 + m_2 + m_3}$$

$$= \frac{50 \times 0 + 100 \times 0 + 150 \times \frac{\sqrt{3}}{2}}{50 + 100 + 150}$$

$$= \frac{\sqrt{3}}{4}\text{m}$$

Hence the coordinates $(x,\ y)$ are $\left(\frac{7}{12}\text{m},\ \frac{\sqrt{3}}{4}\text{m}\right)$

Hence, option (C) is correct.

23. A uniform thin rod AB of length L has linear mass density $\mu(x) = a + \frac{bx}{L}$, where x is measured from A. If the CM of the rod lies at a distance of $\left(\frac{7}{12}\right) L$ from A, then a and b are related as :

- **A.** $a = 2b$
- **B.** $2a = b$
- **C.** $a = b$
- **D.** $3a = 2b$

Centre of mass of the rod is given by:

$$x_{cm} = \frac{\int_0^L \left(ax + \frac{bx^2}{L}\right) dx}{\int_0^L \left(a + \frac{bx}{L}\right) dx}$$

$$= \frac{\left(\frac{aL^2}{2} + \frac{bL^2}{3}\right)}{\left(aL + \frac{bL}{2}\right)} = L.\frac{\left(\frac{a}{2} + \frac{b}{3}\right)}{\left(a + \frac{b}{2}\right)}$$

$$\Rightarrow \frac{7L}{12} = L.\frac{\left(\frac{a}{2} + \frac{b}{3}\right)}{\left(a + \frac{b}{2}\right)}$$

$$7a + \frac{7}{2}b = 6a + 4b \Rightarrow a = 4b - \frac{7}{2}b$$

$$\Rightarrow b = 2a$$

Hence, option (B) is correct.

24. A boy of mass $20\ \mathrm{kg}$ is standing on a $80\ \mathrm{kg}$ free to move long cart. There is negligible friction between cart and ground. Initially, the boy is standing $25\ \mathrm{m}$ from a wall. If he walks $10\ \mathrm{m}$ on the cart towards the wall, then the final distance of the boy from the wall will be

A. $15\ \mathrm{m}$

B. $12.5\ \mathrm{m}$

C. $15.5\ \mathrm{m}$

D. $17\ \mathrm{m}$

Given:
Mass of boy $m_1 = 20\ \mathrm{kg}$
Mass of cart $m_2 = 80\ \mathrm{kg}$
Initial distance between boy and cart is$25\ \mathrm{m}$
Distance moved by boy towards wall is $10\ \mathrm{m}$

As there is no external force, so displacement of centre of mass of the (cart + boy) system parallel to the surface is zero.

$$\therefore x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = 0$$

Let when the boy moves $10\ \mathrm{m}$ towards the wall, the cart moves away from the wall a distance x
So, displacement of man w.r.t. ground towards the wall is
$x_1 = 10 - x$
And the displacement of cart w.r.t. ground towards the wall is
$x_2 = -x$

$$20 \times (10 - x) + (80 \times (-x)) = 0$$

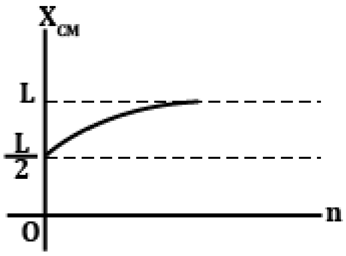
$$\Rightarrow x = 2\ \mathrm{m}$$

i.e. Final distance between boy and wall,
$= 25 - 10 + 2 = 17\ \mathrm{m}$

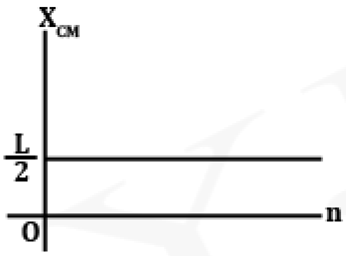
Hence, option (D) is correct.

25. A thin rod of length $'L'$ is lying along the x-axis with its ends at $x=0$ and $x=L$. Its linear density $\left(\frac{\text{mass}}{\text{length}}\right)$ varies with x as $k\left(\frac{x}{L}\right)^n$, where n can be zero or any positive number. If the position x_{cm} of the center of mass of the rod is plotted against $'n'$ which of the following graphs best approximates the dependence of x_{cm} on n?

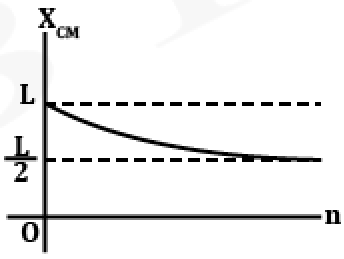
A.



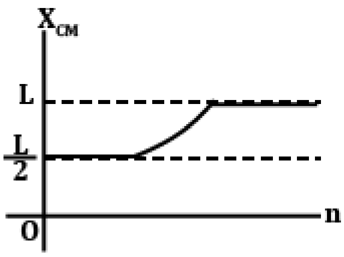
B.



C.



D.



Given:

The linear mass density $\lambda = k\left(\frac{x}{L}\right)^n$

for varying mass,
we can write the equation for the center of mass as,

$$x_{CM} = \frac{\int_0^L xdm}{\int_0^L dm} = \frac{\int_0^L x(\lambda dx)}{\int_0^L \lambda\, dx} = \frac{\int_0^L k\left(\frac{x}{L}\right)^n xdx}{\int_0^L k\left(\frac{x}{L}\right)^n dx}$$

$$= \frac{k\left[\frac{x^{n+2}}{(n+2)L^n}\right]_0^L}{\left[\frac{kx^{n+1}}{(n+1)L^n}\right]_0^L} = \frac{L(n+1)}{n+2}$$

For $n = 0,\ \ x_{CM} = \frac{L}{2};$

$n = 1,\ \ x_{CM} = \frac{2L}{3}$

$n = 2\ \ x_{CM} = \frac{3L}{4}; \dots.$

For $n \to \infty,\ \ x_{cm} = L$

So we can observe that the Graph A satisfies the above conditions.

Hence, option (A) is correct.

26. A circular disc of radius R is removed from a bigger circular disc of radius $2R$ such that the circumferences of the discs coincide. The center of mass of the new disc is $\frac{\alpha}{R}$ from the center of the bigger disc. The value of α is

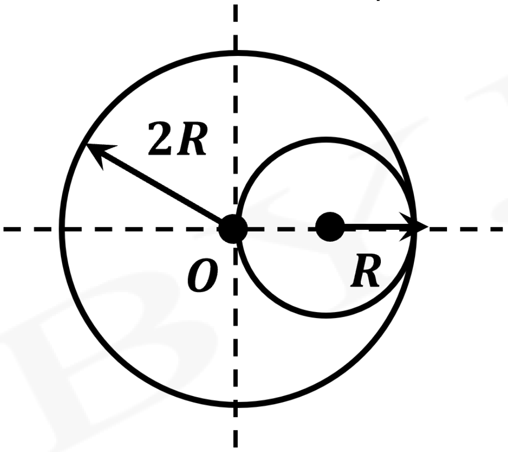
A. $\frac{1}{4}$

B. $\frac{1}{3}$

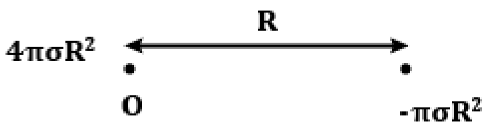
C. $\frac{1}{2}$

D. $\frac{1}{6}$

Let σ be the mass per unit area of the disc.
Then the mass of the complete disc $= \sigma(\pi(2R)^2)$



The mass of the removed disc $= \sigma(\pi R^2) = \pi\sigma R^2$
Let us consider the above situation to be a complete disc of radius $2R$ on which a disc of radius R of negative mass is superimposed.
Let O be the origin.
Then the above figure can be redrawn keeping in mind the concept of centre of mass as :



$$x_{cm} = \frac{(6\pi(2R^2)) \times 0 + (-6(\pi R^2))R}{4\pi\sigma R^2 - \pi\sigma R^2}$$

$$\therefore x_{cm} = \frac{-\pi\sigma R^2 \times R}{3\pi\sigma R^2}$$

$$\therefore x_{cm} = -\frac{R}{3} = \alpha R \Rightarrow \alpha = \frac{1}{3}$$

Hence, option (B) is correct.

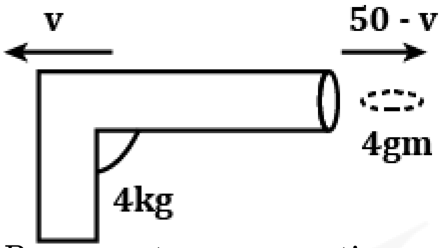
27. A bullet of 4 g mass is fired from a gun of mass 4 kg. If the bullet moves with the muzzle speed of 50 ms^{-1}, the impulse imparted to the gun and velocity of recoil of gun are :

A. $0.4 \text{ kg ms}^{-1}, 0.1 \text{ ms}^{-1}$

B. $0.2 \text{ kg ms}^{-1}, 0.05 \text{ ms}^{-1}$

C. $0.2 \text{ kg ms}^{-1}, 0.1 \text{ ms}^{-1}$

D. $0.4 \text{ kg ms}^{-1}, 0.05 \text{ ms}^{-1}$



By momentum conservation

$4 \times 10^{-3}(50 - v) - 4v = 0$

$$\Rightarrow v = \frac{4 \times 10^{-3} \times 50}{4 + 4 \times 10^{-3}} \approx 0.05 \text{ ms}^{-1}$$

Impulse $J = mv = 4 \times .05 = 0.2 \text{ kg ms}^{-1}$

Hence, (B) is the correct answer.

28. A body rolls down an inclined plane without slipping. The kinetic energy of rotation is 50% of its translational kinetic energy. The body is :

A. Solid sphere

B. Solid cylinder

C. Hollow cylinder

D. Ring

Give:

$(KE)_R = 50\%$ of $(KE)_T = \frac{1}{2} \times (KE)_T$

$$\Rightarrow \frac{1}{2} I\omega^2 = \frac{1}{2} \times \frac{1}{2} mv^2$$

Here, body moves without slipping, $v = R\omega$

$$\therefore I = \frac{1}{2} mR^2$$

And we know that, MOI of disc or solid cylinder is $mR^2/2$.

Hence, option (B) is correct.

29. The moment of inertia of a square plate of side l about the axis passing through one of the corner and perpendicular to the plane of the square plate is given by:

A. $\frac{Ml^2}{12}$

B. $\frac{2}{3}Ml^2$

C. $\frac{Ml^2}{6}$

D. Ml^2

Moment of inertia of the square plate about an axis passing through its center of mass and perpendicular to its plane is :

$I_{com} = \frac{Ml^2}{6}$

According to the parallel axis theorem, moment of inertia of the plate about the axis passing from the corner and perpendicular to its plane is :

$I = I_{com} + Mx^2$

Here, the distance between this axis from com is equal to half the length of the diagonal of the square.

$i.e. x = \frac{l\sqrt{2}}{2} = \frac{l}{\sqrt{2}}$

$\Rightarrow I = \frac{Ml^2}{6} + \frac{Ml^2}{2}$

Or, $I = \frac{2Ml^2}{3}$

30. Two discs have moments of inertia $I_1 \, and \, I_2$ about their respective axes perpendicular to the plane and passing through the center. They are rotating with angular speeds, $\omega_1 \, and \, \omega_2$ respectively and are brought into contact face to face with their axes of rotation coaxial. The loss in kinetic energy of the system in the process is given by :

A. $\frac{I_1 I_2}{2(I_1+I_2)}(\omega_1-\omega_2)^2$

B. $\frac{I_1 I_2}{(I_1+I_2)}(\omega_1-\omega_2)^2$

C. $\frac{(\omega_1-\omega_2)^2}{2(I_1+I_2)}$

D. $\frac{(I_1+I_2)^2\omega_1\omega_2}{2(I_1+I_2)}$

Let their final common angular velocity is ω.
From conservation of angular momentum we have :

$$I_1\omega_1 + I_2\omega_2 = (I_1+I_2)\omega$$

$$\Rightarrow \omega = \frac{(I_1\omega_1 + I_2\omega_2)}{(I_1+I_2)} \ldots (1)$$

Initial kinetic energy of the system :

$$KE_i = \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2$$

Final kinetic energy of the system :

$$KE_f = \frac{1}{2}(I_1+I_2)\omega^2$$

Loss in kinetic energy ;

$$\Delta KE = KE_i - KE_f = \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2 - \frac{1}{2}(I_1+I_2)\omega^2$$

From (1) :

$$\Delta KE = \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2 - \frac{1}{2}\frac{(I_1\omega_1 + I_2\omega_2)^2}{(I_1+I_2)}$$

$$\Rightarrow \Delta KE = \frac{1}{2}\left(\frac{I_1 I_2}{I_1+I_2}\right)(\omega_1-\omega_2)^2$$