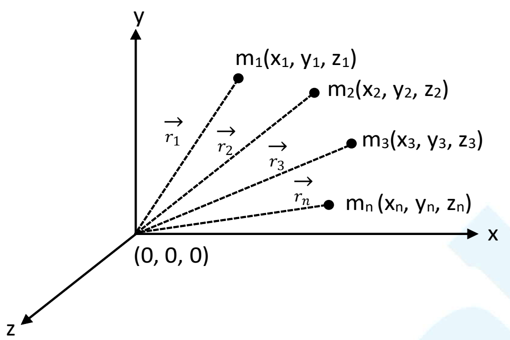
CENTRE OF MASS

• Centre of mass :

For a system of particles, centre of mass is that point at which its total mass is supposed to be concentrated.

• Centre of mass of system of discrete particles :



Total mass of the body : $M = m_1 + m_2 + \ldots\ldots. + m_n$ then

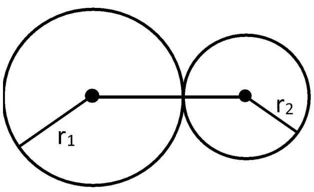
$$\vec{R}_{CM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \ldots + m_n\vec{r}_n}{m_1 + m_2 + m_3 + \ldots + m_n} = \frac{1}{M}\sum_{i=1}^{i=n} m_i\vec{r}_i$$

co-ordinates of centre of mass :

$$x_{cm} = \frac{1}{M}\sum_{i=1}^{i=n} m_i x_i,\ y_{cm} = \frac{1}{M}\sum_{i=1}^{i=n} m_i y_i \text{ and } z_{cm} = \frac{1}{M}\sum_{i=1}^{i=n} m_i z_i$$

• For a two particle system, distances of particles from centre of mass are in the reverse ratio of the masses i.e. $m_1 r_1 = m_2 r_2 \Rightarrow \frac{r_1}{r_2} = \frac{m_2}{m_1}$.

• Two circular discs/sphere of the same material are kept in contact as shown, then distance of centre of mass from the centre of the first disc is $\frac{r_2^2}{(r_1^2 + r_2^2)}(r_1 + r_2)$. Similarly distance of centre of mass from the centre of the second disc is $\frac{r_1^2}{(r_1^2 + r_2^2)}(r_1 + r_2)$.



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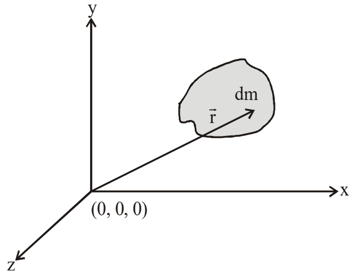
• Centre of mass of continuous distribution of particles

$$\vec{R}_{CM} = \frac{1}{M}\int \vec{r}\,dm$$

$$X_{com} = \frac{\int x\,dm}{\int dm} = \frac{1}{M}\int x\,dm$$

$$Y_{com} = \frac{\int y\,dm}{\int dm} = \frac{1}{M}\int y\,dm$$

$$Z_{com} = \frac{\int z\,dm}{\int dm} = \frac{1}{M}\int z\,dm$$



x, y, z are the co-ordinate of the COM of the dm mass.

• The centre of mass after removal of a part of a body

Original mass (M) – mass of the removed part (m)
= {original mass (M)} + {– mass of the removed part (m)}

When a part is removed from a rigid body. then the position of COM of the remaining portion will be :

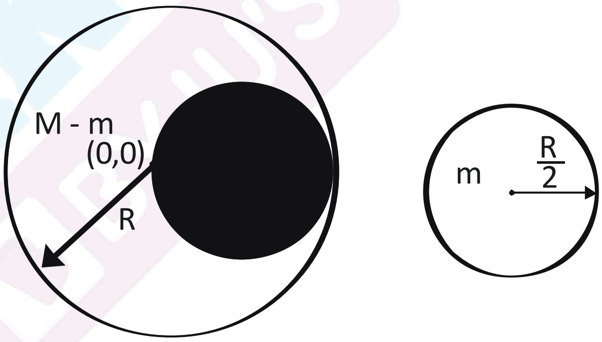
$$\vec{r}_{COM} = \frac{M\vec{r}_1 - m\vec{r}_2}{M - m}$$

The co-oradinates of COM is given by

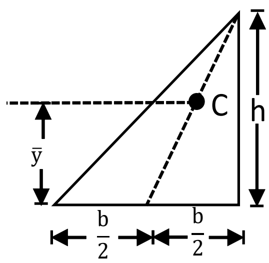
$$x_{COM} = \frac{Mx_1 - mx_2}{M - m}$$

$$y_{COM} = \frac{My_1 - my_2}{M - m}$$

$$z_{COM} = \frac{Mz_1 - mz_2}{M - m}$$



• Centre of mass of some common objects

Shape	Figure	$\bar{x}$	$\bar{y}$
Triangular area			$\frac{h}{3}$

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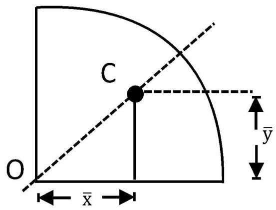
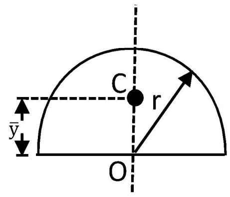
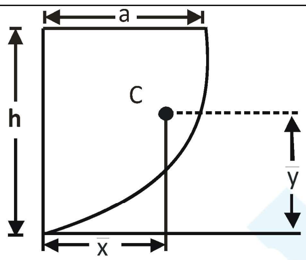
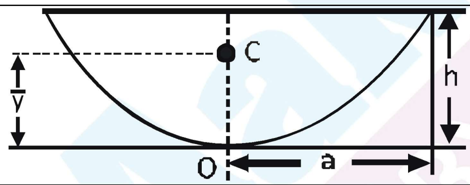
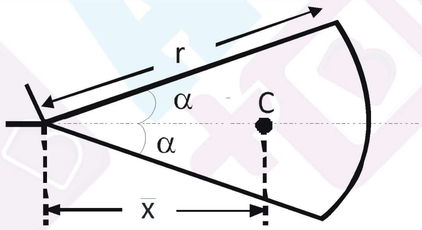
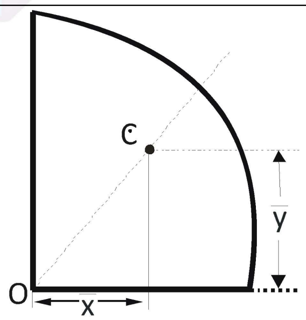
Shape	Figure	$\bar{x}$	$\bar{y}$
Quarter circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$
Semi-circular area		0	$\frac{4r}{3\pi}$
Semi parabolic area		$\frac{3a}{8}$	$\frac{3h}{5}$
Parabolic area		0	$\frac{3h}{5}$
Circular Sector		$\frac{2r\sin\alpha}{3\alpha}$	0
Quarter Circular arc		$\frac{2r}{\pi}$	$\frac{2r}{\pi}$

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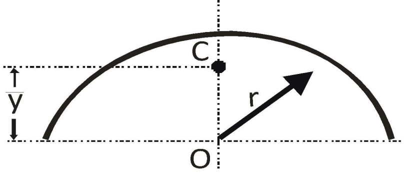
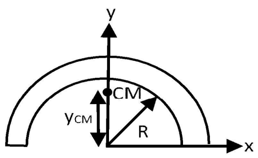
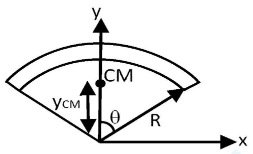
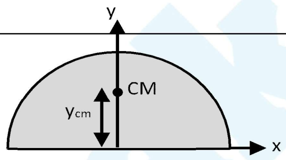
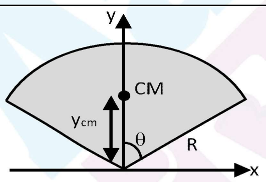
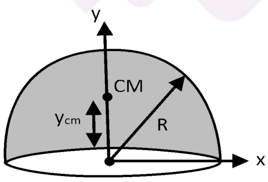
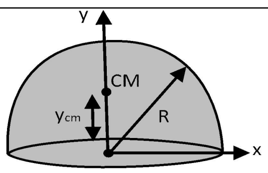
Shape	Figure	$\bar{x}$	$\bar{y}$
Semi Circular arc		0	$\frac{2r}{\pi}$
Half Ring		0	$y_{cm} = \frac{2R}{\pi}$
Segment of a ring		0	$y_{cm} = \frac{R\sin\theta}{\theta}$
Half disc (plate)		0	$y_{cm} = \frac{R\sin\theta}{\theta}$
Segment of a ring		0	$y_{cm} = \frac{2R\sin\theta}{3\theta}$
Hollow hemisphere		0	$y_{cm} = \frac{R}{2}$
Solid hemisphere		0	$y_{cm} = \frac{3R}{8}$

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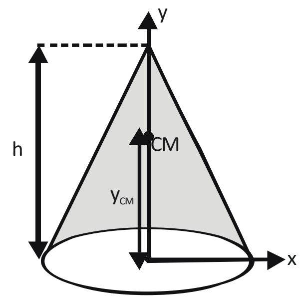
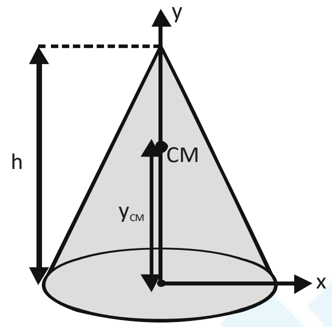
Shape	Figure	$\overline{x}$	$\overline{y}$
Hollow Cone		0	$y_{CM} = \frac{h}{3}$
Solid Cone		0	$y_{CM} = \frac{h}{4}$

Motion of centre of mass

For a system of particles,

velocity of centre of mass $\vec{v}_{CM} = \frac{d\vec{R}_{CM}}{dt} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + \ldots + m_n\vec{v}_n}{m_1 + m_2 + \ldots + m_n}$

Similarly acceleration $\vec{a}_{CM} = \frac{d}{dt}\left(\vec{v}_{CM}\right) = \frac{m_1\vec{a}_1 + m_2\vec{a}_2 + \ldots + m_n\vec{a}_n}{m_1 + m_2 + \ldots + m_n}$

Analysis of Dynamics of COM

$$\vec{F}_{net} = \left(\sum \vec{F}_{ext}\right)_{sys} = \frac{d\vec{P}_{sys}}{dt} = M\vec{a}_{COM}$$

- COM is rest $\boxed{\begin{array}{l} \vec{F}_{net} = 0 \\ \vec{V}_{COM} = 0 \end{array}}$
- COM moves with constant velocity $\boxed{\begin{array}{l} \vec{F}_{net} \neq 0 \\ \vec{V}_{COM} \neq 0 \end{array}}$

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- COM moves with acceleration $\boxed{\vec{F}_{net} \neq 0}$

- If a system is at rest intially and there is no net external force acting on it then there will be no shift in position of the COM of the system.

$$\boxed{\vec{F}_{net} = 0} + \boxed{\vec{V}_{COM} = 0} \Rightarrow \boxed{\Delta\vec{r}_{COM} = 0}$$

• Law of conservation of linear momentum

Linear momentum of a system of particles is equal to the product of mass of the system with velocity of its centre of mass.

From Newton's second law $\vec{F}_{ext.} = \dfrac{d(M\vec{v}_{CM})}{dt}$

If $\vec{F}_{ext.} = \vec{0}$, then $M\vec{v}_{cm}$ = constant

If no external force acts on a system the velocity of its centre of mass remains constant, i.e., velocity of centre of mass is unaffected by internal forces.

• Impulse-Momentum theorem

Impulse of a force is equal to the change of momentum.

Force-time graph area gives change in momentum. $\boxed{\int_{t_1}^{t_2} \vec{F}dt = \Delta\vec{P}}$

• Reduced Mass For Two Body System

1. A two body system can be made equivalent to a single body system by introducing the concept of reduced mass.

2. Let m_1 and m_2 be the masses of two particles with position vectors $\vec{r}_1$ and $\vec{r}_2$ and $\vec{F}_{12}$ be the forces exerted by second body on first body and $\vec{F}_{21}$ by first body on second body respectively.

$$\vec{F}_{12} = m_1\frac{d^2\vec{r}_1}{dt^2} \text{ and } \vec{F}_{21} = m_2\frac{d^2\vec{r}_2}{dt^2}$$

As no external force acts on the system, $\vec{F}_{12} = -\vec{F}_{21} = \vec{F}$

$$\Rightarrow \frac{d^2\vec{r}_1}{dt^2} - \frac{d^2\vec{r}_2}{dt^2} = \vec{F}\left(\frac{1}{m_1} + \frac{1}{m_2}\right) = \vec{F}\left(\frac{m_1 + m_2}{m_1 m_2}\right)$$

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Let $\vec{r}_1 - \vec{r}_2 = \vec{r}$

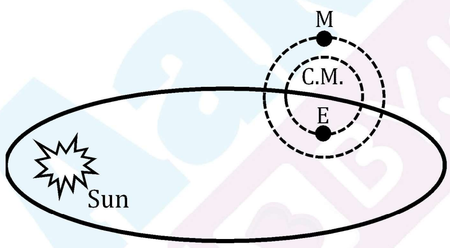
$$\Rightarrow \frac{d^2\vec{r}}{dt^2} = \vec{F}\left(\frac{m_1 + m_2}{m_1 m_2}\right) \text{or} \left(\frac{m_1 + m_2}{m_1 m_2}\right)\frac{d^2\vec{r}}{dt^2} = \vec{F}$$

$$\text{or } \mu\frac{d^2\vec{r}}{dt^2} = \vec{F}$$

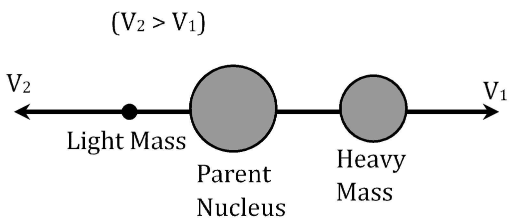
Here $\mu = \left(\frac{m_1 m_2}{m_1 + m_2}\right)$ is called reduced mass.

• Classical Example of Application of COM :-

(1) The earth revolves around the sun in an elliptical orbit whereas the moon revolves round the earth in circular orbit. Both the earth and the moon move in circles about a common centre of mass. The internal force which act on the earth moon system are the gravitational force of attraction on each other. The earth and the moon are always on opposite sides of the centre of mass. Since the earth is heavier than moon, So the centre of mass of the system is very close to the earth. It is this centre of mass which revolves around the sun in an elliptical orbit.



(2) In radioactive decay, the process is caused by the internal forces of the system. Therefore, initial and final momenta are zero. Hence, the decay products fly off in the opposite directions. The centre of mass of the system remains at rest. The heavy mass move with less speed than that of the light mass.



(3) Explosion of a projectile (e.g. fire cracker) in mid air. Let us consider a projectile which explodes in air. Before explosion, the projectile move along a parabolic path. After explosion,

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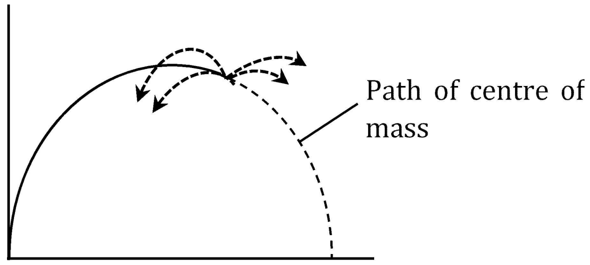
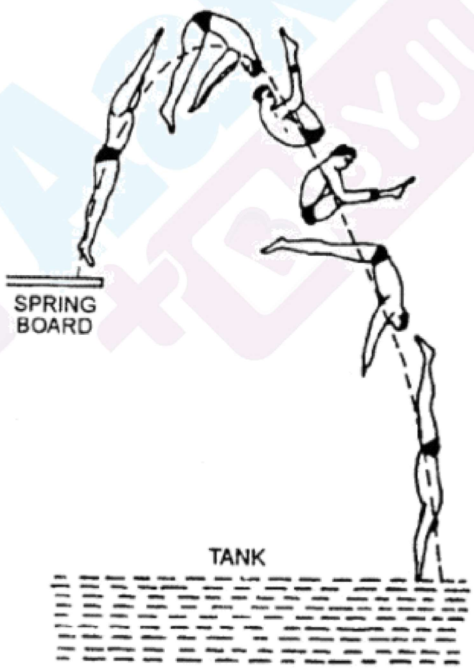
each fragment move along its own parabolic path but the centre of mass of the projectile continues to move in the same parabolic path.



Explanation. The projectile follows a parabolic path under the action of gravity (i.e. earth's gravitational force) Explosion of the projectile occurs due to the internal forces i.e, without any external force. These internal forces cannot change the total momentum of the system al though they may change the momenta of the individual fragments. Thus the centre of mass will remain unaffected after the explosion and hence follow the same parabolic path.

(4) When a diver jumps into water form a height, then body can moves in any path but centre of mass of his body traverses in parabolic path.

So centre of mass follow laws of motion.



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COLLISIONS

1. Collision of bodies :

The event or the process, in which two bodies either coming in contact with each other or due to mutual interaction at distance apart, affect each others motion (velocity, momentum, energy or direction of motion) is defined as a collision.

2. In collision :

- The particles come closer before collision and after collision they either stick together or move away from each other.
- The particles need not come in contact with each other for a collision.
- The law of conservation of linear momentum is necessarily applicable in a collision, whereas the law of conservation of mechanical energy is not.

Types of Collisions

- On the basis of direction
  - One dimensional collision or Head on collision → The collision in which the particles move along the same straight line before and after the collision, is defined as one dimensional collision
  - Two dimensional collision or Oblique collision → The collision, in which the particles move in the same plane at different angles before and after collision, is defined as oblique collision
- On the basis of kinetic energy
  - Elastic collision → A collision is said to be elastic, if the total kinetic energy before and after collision remains the same
  - Inelastic collision → A collision is said to be inelastic, if the total kinetic energy does not remains constant
  - Perfectly inelastic collision → The collision, in which particles get sticked together after the collision. It is called perfectly inelastic collision. In this type of inelastic collision, loss of energy is maximum.

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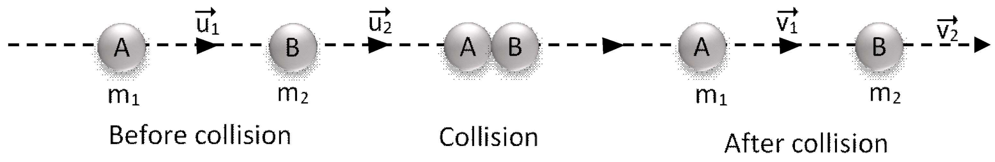
2.1 Head on collision :



2.2 Elastic Collision :

1. **For one dimensional collision between two bodies**
   Total momentum before collision = Total momentum after collision.

2. m_1 and m_2 are masses of two bodies moving with velocities u_1 and u_2 in the same direction ($u_2 < u_1$). After collision their velocities are v_1 and v_2. Then

   - **For one dimensional collision between two bodies,**
     $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$ (conservation of momentum)
   - **If the second body is at rest before collision,** $u_2 = 0$
   - **If they approach each other before collision,** $u_2 = -u_1$
   - **If they move together with velocity v after collision,**
     $m_1 u_1 + m_2 u_2 = (m_1 + m_2) v$

     $$\Rightarrow \quad v = \frac{m_1u_1 + m_2u_2}{(m_1 + m_2)} \text{ (perfect inelastic collision)}$$

3. Coefficient of restitution e = 1
   $\Rightarrow v_2 - v_1 = u_1 - u_2$

   Or relative velocity of separation after collision is equal to relative velocity of approach before collision.

4. **For perfect elastic collision between two bodies which is head on,**

   $$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \text{ and } \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

   $(v_2 - v_1) = (u_1 - u_2)$

   Here
   (i) If $m_1 = m_2 = m$, then $v_2 = u_1$ and $v_1 = 0$
   (ii) If $m_2 >> m_1$, then $v_1 = -u_1$ and $v_2 = 0$
   (iii) If $m_1 >> m_2$ then $v_1 = u_1$ and $v_2 = 2u_1$

5. When a lighter body collides with a stationary heavy body elastically, the second body starts moving with the velocity of the first body while the first body stops.

6. When a heavy body collides elastically with a stationary lighter body, then heavy body continues to move with the same velocity but the lighter body starts moving with double the velocity of heavy body.

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7. When a lighter body collides with a heavy body at rest, then it returns with the same velocity but heavy body remains at rest.

8. When perfect elastic collision takes place between two bodies of same mass moving along a direction, the two bodies interchange their velocities after collision.

9. For perfect elastic collision between a moving body m_1 and stationary body m_2
   Fraction of K.E transferred to the second body = Fraction of K.E retained by the first body.

10. A ball is dropped from certain height. If the collision is perfectly elastic, it rebounds to the same height.

11. When two bodies of equal mass moving towards each other collide elastically with same velocity in magnitude, after collision, they move away with the same velocity in magnitude.

12. A body makes an oblique elastic collision with another body of same mass at rest. After collision, they will move in mutually perpendicular directions.

13. Collisions between atomic, nuclear and fundamental particles are examples of elastic collisions.

2.3 Perfect Inelastic Collision :

1. **For one dimensional collision between two bodies**
   Total momentum before collision = Total momentum after collision

2. After perfect inelastic collision the two bodies stick together and move with same velocity

   $$\Rightarrow v = \frac{m_1u_1 + m_2u_2}{m_1 + m_2}$$

3. The collision between a bullet and a target is perfectly inelastic if the bullet remains embedded in the target.

4. Coefficient of restitution e = 0

   $\Rightarrow v_1 = v_2 = v$

5. Only momentum is conserved and kinetic energy is not conserved.

6. If a body of mass m_1 collides with a body of mass m_2 at rest and the collision is perfectly inelastic, $\frac{\text{Final K.E}}{\text{Initial K.E}} = \frac{m_1}{(m_1+m_2)}$ (for the system)

   If K is initial K.E of m_1, then loss in K.E $= K\left(\frac{m_2}{m_1 + m_2}\right)$

   Fractional loss in K.E is $\left(\frac{m_2}{m_1 + m_2}\right)$

7. Loss in kinetic energy during perfect inelastic collision $= \frac{1}{2}\frac{m_1m_2}{m_1 + m_2}(u_1 - u_2)^2$

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8. If the two bodies approach each other before collision, common velocity after collision is

$$v = \frac{m_1u_1 - m_2u_2}{m_1 + m_2}$$ Loss in kinetic energy in the case is $\frac{1}{2}\left(\frac{m_1m_2}{m_1+m_2}\right)(u_1+u_2)^2$

Coefficent of Restitution :

1. The ratio between relative velocity of separation after collision and relative velocity of approach before collision is know as coefficient of restitution $\mathbf{e} = \frac{v_2 - v_1}{u_1 - u_2}$

2. The value of '**e**' is given by $0 \le \mathbf{e} \le 1$
   If **e** = 0, the collision is perfectly inelastic
   If **e** = 1 the collision is perfectly elastic
   If 0 < **e** < 1 the collision is semi elastic

3. **e** is dimensionless and has no units.

4. The value of **e** is independent of masses and the velocities of the colliding bodies

5. **e** depends on the nature of material of the colliding bodies

6. If a body is dropped from a height 'h' and after first rebound it rises to a height h_1 the coefficient of restitution $\mathbf{e} = \sqrt{\frac{h_1}{h}} \Rightarrow h_1 = \mathbf{e}^2 h$ After n^{th} rebound $h_n = \mathbf{e}^{2n}h$

   If the body strikes the ground with velocity v and rebounds with velocity v_1 then

$\mathbf{e} = \frac{v_1}{v} \Rightarrow v_1 = \mathbf{e}v$ After nth rebound $v_n = \mathbf{e}^n v$

7. A ball dropped from a height h. It strikes the ground and rebounds. Here '**e**' is coefficient of restitution and this collisions took place repeatedly. The total distance travelled by the ball before coming to rest is $d = h\left(\frac{1+\mathbf{e}^2}{1-\mathbf{e}^2}\right)$ Here total time taken by the ball to come to rest is $t = \frac{\sqrt{2h}}{8}\left(\frac{1+\mathbf{e}}{1-\mathbf{e}}\right)$

8. A ball of mass m is dropped from height h and after hitting the ground it rises to height less than 'h'. If '**e**' is coefficient of restitution, the change in momentum of the ball in magnitude is $m\sqrt{2gh}(1+\mathbf{e})$

9. For one dimensional collision between two bodies of masses m_1 and m_2 moving with initial velocities u_1 and u_2 respectively, final velocities after collision are given by

$$v_1 = \left(\frac{m_1 - \mathbf{e}m_2}{m_1 + m_2}\right)u_1 + \frac{m_2(1+\mathbf{e})}{m_1 + m_2}u_2$$

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$$v_2 = \left(\frac{m_1 - \mathbf{e}m_1}{m_1 + m_2}\right)u_2 + \frac{m_2(1+\mathbf{e})}{m_1 + m_2}u_1$$

Here loss in kinetic energy during collision is $\frac{1}{2}\left(\frac{m_1 m_2}{m_1 + m_2}\right)(u_1 - u_2)^2(1-\mathbf{e})^2$

2.4 Head on inelastic collision of two particles :

Let the coefficient of restitution for collision is e

(i) Momentum is conserved $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$ .....(i)

(ii) Kinetic energy is not conserved.

(iii) According to Newton's law $e = \frac{v_2 - v_1}{u_1 - u_2}$ ...(ii)

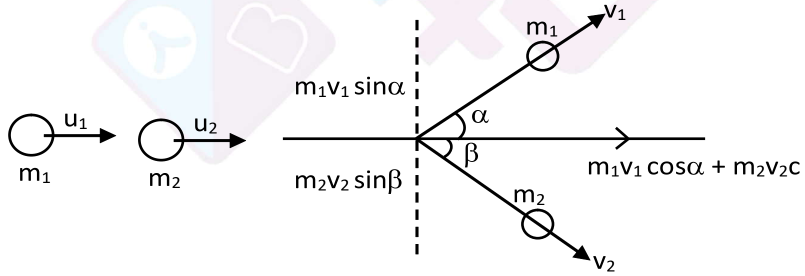
By solving eq. (i) and (ii)

$$v_1 = \left(\frac{m_1 - em_2}{m_1 + m_2}\right)u_1 + \left(\frac{(1+e)m_2}{m_1 + m_2}\right)u_2 = \frac{m_1u_1 + m_2u_2 - m_2e(u_1 - u_2)}{m_1 + m_2}$$

$$v_2 = \left(\frac{m_2 - em_1}{m_1 + m_2}\right)u_2 + \left(\frac{(1+e)m_1}{m_1 + m_2}\right)u_1 = \frac{m_1u_1 + m_2u_2 - m_1e(u_2 - u_1)}{m_1 + m_2}$$

2.5 Two Dimensional Collision :

1. Consider two bodies of masses m_1 and m_2 moving with velocities u_1 and u_2 along the same straight line. They collide and after collision they move in directions making angles α and β with the initial direction of motion. Let v_1 and v_2 be their final velocities. Then



From conservation of momentum

$m_1 u_1 + m_2 u_2 = m_1 v_1 \cos\alpha + m_2 v_2 \cos\beta$

$0 = m_1 v_1 \sin\alpha - m_2 v_2 \sin\beta$

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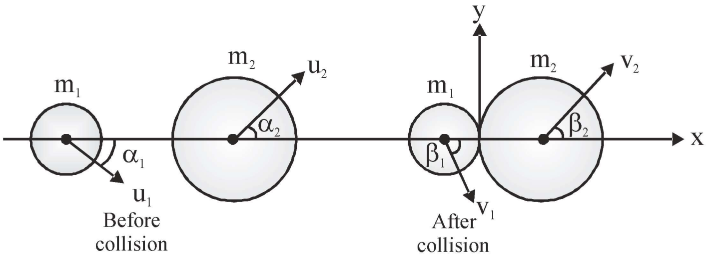
2.6 Oblique Collision :

Conserving the momentum of system in directions along normal (x-axis in our case) and tangential (y-axis in our case)

$m_1u_1\cos\alpha_1 + m_2u_2\cos\alpha_2 = m_1v_1\cos\beta_1 + m_2v_2\cos\beta_2$ and

$m_2u_2\sin\alpha_2 - m_1u_1\sin\alpha_1 = m_2v_2\sin\beta_2 - m_1v_1\sin\beta_1$



Since no force is acting on m_1 and m_2, along the tangent (i.e. y-axis) the individual momentum of m_1 and m_2 remains conserved.

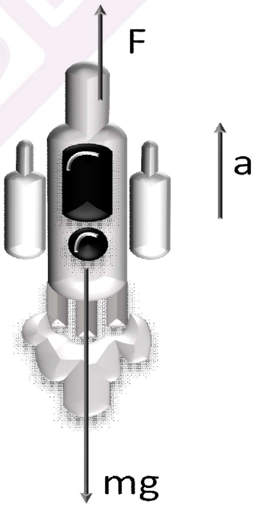
$m_1u_1\sin\alpha_1 = m_1v_1\sin\beta_1$ & $m_2u_2\sin\alpha_2 = m_2v_2\sin\beta_2$

By using Newton's experimental law along the line of impact

$$e = \frac{v_2\cos\beta_2 - v_1\cos\beta_1}{u_1\cos\alpha_1 - u_2\cos\alpha_2}$$

3. Rocket propulsion :

Thrust force on the rocket, $F_{gas} = -v\frac{dm}{dt}$



From Newton's Second law,

$-F_{gas} = F_{rocket}$

Therefore,

$$F_{rocket} = v\frac{dm}{dt}$$

From free body diagram,

$$v\frac{dm}{dt} - mg = ma$$

Where,

v is the relative velocity of gases w.r.t. rocket

m is the mass of the rocket

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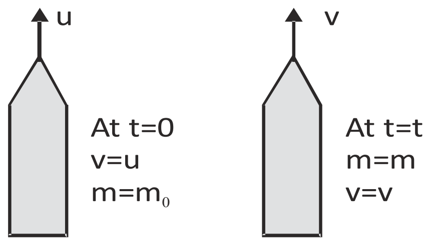
g is the acceleration due to gravity

a is the initial acceleration of the rocket

$\frac{dm}{dt}$ is the rate of consumption of fuel

Velocity of rocket at any instant

$$v = u - gt + v_r \ell n\left(\frac{m_0}{m}\right)$$



exhaust velocity = v_r

4. Ballistic Pendulum :

1. It is used to find velocity of bullet. This arrangement consists of a wooden block suspended using a rope or wire. A bullet fired horizontally into the block, it gets embedded and both move together.

2. Let m be mass of the bullet which strikes the wooden block of mass M with velocity u and gets embedded into it. After this the combined system moves with a velocity v and the system rises to a height *h* above the previous level. Then

$$mu = (M+m)v$$

$$\Rightarrow v = \frac{mu}{(M+m)}$$

As $v = \sqrt{2gh}, \sqrt{2gh} = \frac{mu}{(M+m)}$

$$\Rightarrow u = \left(\frac{M+m}{m}\right)\sqrt{2gh}$$

$$\Rightarrow h = \left(\frac{mu}{M+m}\right)^2 / 2g$$

3. In the previous case if the bullet emerges out from the block with velocity u_1 and the block rises to a height h,

$$m(u - u_1) = Mv$$

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$$\Rightarrow v = \frac{m(u - u_1)}{M} \text{ or } \sqrt{2gh} = \frac{m(u - u_1)}{M}$$

4. Ballistic pendulum is an example for perfect inelastic collision (if the bullet stops in the block).

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ROTATIONAL MOTION

- **Angular velocity**

$$\vec{\omega} = \frac{d\vec{\theta}}{dt}$$

- **Angular acceleration**

$$\vec{\alpha} = \frac{d\vec{\omega}}{dt} = \frac{d^2\vec{\theta}}{dt^2}$$

- **Angular momentum**

$$\vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}$$

- **Torque**

$$\vec{\tau} = \vec{r} \times \vec{F} = \frac{d\vec{L}}{dt}$$

- **Rotational Kinetic energy**

$$K = \frac{1}{2}I\omega^2 = \frac{L^2}{2I}$$

- **Rotational Power :**

$$P = \vec{\tau}.\vec{\omega}$$

- **For constant angular acceleration**

→ $\omega = \omega_0 + \alpha t$

→ $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

→ $\omega^2 = \omega_0^2 + 2\alpha\theta$

→ $\theta_n = \omega_0 + \frac{\alpha}{2}(2n - 1)$

- **Moment of Inertia**

→ A tensor but for fixed axis it is a scalar

→ For discrete distribution of mass $I = m_1 r_1^{\,2} + m_2 r_2^{\,2} + ...+ m_n r_n^2 = \sum_{i=1}^{i=n} m_i r_i^2$

→ For continuous distribution of mass $I = \int dI = \int dmr^2$

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- **Radius of gyration**

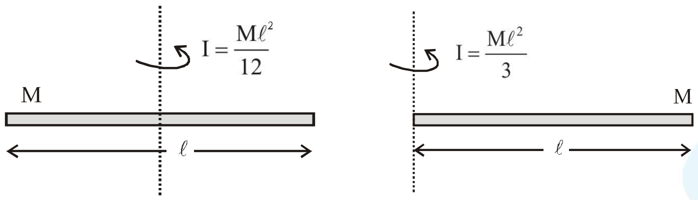
$$k = \sqrt{\frac{I}{M}}$$

- **Theorems regarding moment of inertia**

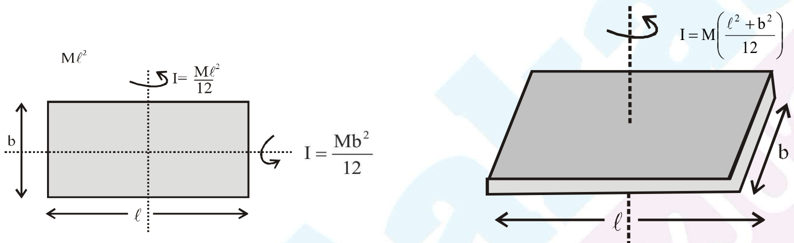
→ Theorem of parallel axes $I_{axis} = I_{cm} + md^2$
where d is the perpendicular distance between parallel axes.

→ Theorem of perpendicular axes $I_z = I_x + I_y$

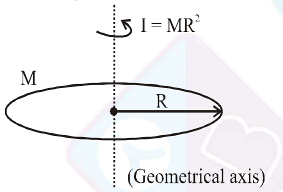
- **Rod**



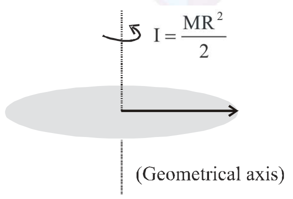
- **Rectangular Lamina**



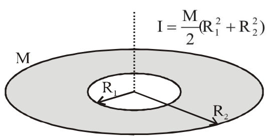
- **Ring :**



- **Disc :**



- **Circular Hollow Disk**



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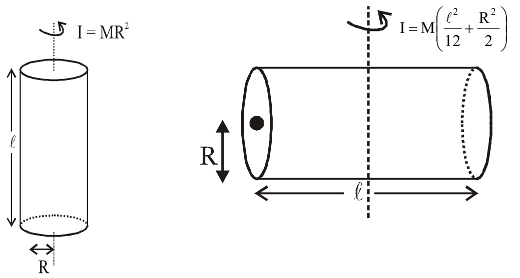
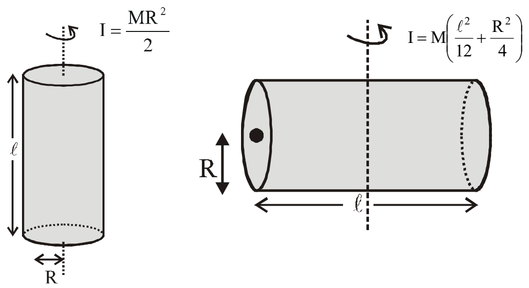
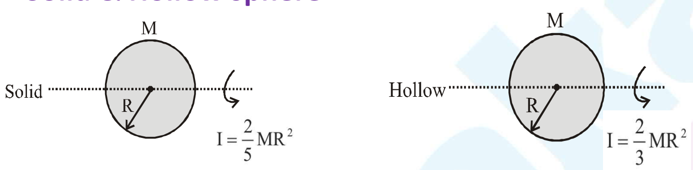
- **Hollow cylinder**



- **Solid cylinder**

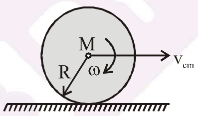


- **Solid & Hollow sphere**



- **Rolling motion**

→ Total kinetic energy $=\frac{1}{2}Mv_{CM}^2+\frac{1}{2}I_{CM}\omega^2$



→ Total angular momentum $=Mv_{CM}R+I\omega_{CM}$

- **Pure rolling (or rolling without slipping) on stationary surface**

→ Condition : $v_{cm} = R\omega$

In accelerated motion $a_{cm} = R\alpha$

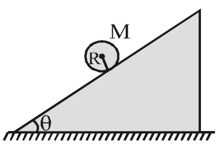
→ If $v_{cm} > R\omega$ then rolling with forward slipping.

→ If $v_{cm} < R\omega$ then rolling with backward slipping.

→ Total kinetic energy in pure rolling

$$K_{total}=\frac{1}{2}Mv_{cm}^2+\frac{1}{2}(Mk^2)\left(\frac{v_{cm}^2}{R^2}\right)=\frac{1}{2}Mv_{cm}^2\left(1+\frac{k^2}{R^2}\right)$$

- **Pure rolling motion on an inclined plane**



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→ Acceleration $a = \frac{g\sin\theta}{1 + k^2 / R^2}$

→ Minimum frictional coefficient $\mu_{min} = \frac{\tan\theta}{1 + R^2 / k^2}$

- **Torque** $\vec{\tau} = I\vec{\alpha} = I\frac{d\vec{\omega}}{dt} = \frac{d(I\vec{\omega})}{dt} = \frac{d\vec{L}}{dt}$ or $\frac{d\vec{J}}{dt}$
- **Change in angular momentum** $\Delta\vec{L} = \vec{\tau}\Delta t$
- **Work done by a torque** $W = \int \vec{\tau} \cdot d\vec{\theta}$

KEY POINTS

→ A ladder is more apt to slip, when you are high up on it than when you just begin to climb because at the high up on a ladder the torque is large and on climbing up, the torque is small.

→ When a sphere rolls on a horizontal table, it slows down and eventually stops because when the sphere rolls on the table, both the sphere and the surface deform near the contact. As a result the normal force does not pass through the centre and provide an angular deceleration.

→ The spokes near the top of a rolling bicycle wheel are more blurred than those near the bottom of the wheel because the spokes near the top of wheel are moving faster than those near the bottom of the wheel.

→ Instantaneous angular velocity is a vector quantity because infinitesimal angular displacement is a vector.

→ The relative angular velocity between any two points of a rigid body is zero at any instant.

→ All particles of a rigid body, which do not lie on an axis of rotation move on circular paths with centres at an axis of rotation.

→ Instantaneous axis of rotation is stationary w.r.t. ground

→ Many greater rivers flow toward the equator. The sediment that they carry increases the time of rotation of the earth about its own axis because the momentum of the angular earth about its rotation axis is conserved.

→ The hard boiled egg and raw egg can be distinguished on the basis of spinning of both.

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