Full Syllabus Test 1

Subject: Mathematics

1. The values of a for which the number 6 lies in between the roots of the equation $x^2 + 2(a-3)x + 9 = 0$, belong to

   A. $\left(\frac{3}{4}, \infty\right)$

   B. $\left(-\infty, -\frac{3}{4}\right)$

   C. $(-\infty, 0) \cup (6, \infty)$

   D. $(-\infty, 0) \cup (3, \infty)$

2. The line $4x - 3y + 2 = 0$ is rotated through an angle of $\frac{\pi}{4}$ in clockwise direction about the point $(1, 2)$. The equation of the line in its new position is

   A. $x - 7y + 13 = 0$

   B. $y - 7x + 5 = 0$

   C. $x + 7y - 15 = 0$

   D. $y + 7x - 15 = 0$

3. The circle passing through $(1, -2)$ and touching the x-axis at $(3, 0)$ also passes through the point

   A. $(-2, -2)$

   B. $(2, -5)$

   C. $(5, -2)$

   D. $(-2, 5)$

4. Let the coefficients of powers of x in the second, third and fourth terms in the binomial expansion of $(1+x)^n$, where n is a positive integer, be in arithmetic progression. The sum of the coefficients of odd powers of x in the expansion is

A. 32

B. 64

C. 128

D. 256

5. If the imaginary part of $\dfrac{2z+1}{iz+1}$ is -2, then the locus of z is

A. a circle

B. a straight line

C. an ellipse

D. a parabola

6. A geometric progression with common ratio r, consists of an even number of terms. If the sum of all terms is 5 times the sum of the terms occupying the odd places, then $\sum_{i=1}^{4}(ir)^2$ is

A. 1456

B. 120

C. 1172

D. 480

7. If a, b, c, x are positive integers, then $\begin{vmatrix} a^2+x & ab & ac \\ ab & b^2+x & bc \\ ac & bc & c^2+x \end{vmatrix}$ is divisible by

A. x^2

B. x^3

C. x^4

D. $a^2+b^2+c^2$

8. An open cylindrical can has to be made with 100 square units of tin. If its volume is maximum, then the ratio of its base radius and the height is

A. $2:1$

B. $1:1$

C. $1:2$

D. $\sqrt{2}:1$

9. If $(\tan^{-1}x)^2+(\cot^{-1}x)^2=\dfrac{5\pi^2}{8}$, then the sum of the solutions in x is

A. 1

B. -1

C. 0

D. not finite

10. If in a $\triangle ABC$, $\sin C + \cos C + \sin(2B + C) - \cos(2B + C) = 2\sqrt{2}$, then $\triangle ABC$ is

A. isosceles

B. equilateral

C. right-angled isosceles

D. right-angled but not isosceles

11. If the chords of the hyperbola $x^2 - y^2 = a^2$ touch the parabola $y^2 = 4ax$, then the locus of the midpoints of the chords is the curve

A. $y^2(x + a) = x^3$

B. $y^2(x - a) = x^3$

C. $y^2(x + 2a) = 3x^3$

D. $y^2(x - 2a) = 2x^3$

12. Let $f : \mathbb{R} \to \mathbb{R}$ be a function defined by $f(x) = x^3 + x^2 + 3x + \sin x$. Then f is

A. injective and surjective

B. injective but not surjective

C. surjective but not injective

D. neither injective nor surjective

Full Syllabus Test 1

13. If a variable tangent of the circle $x^2 + y^2 = 1$ intersects the ellipse $x^2 + 2y^2 = 4$ at points P and Q, then the locus of the point of intersection of tangent at P and Q is

A. a circle of radius 2 units

B. a parabola with focus at $(2, 3)$

C. an ellipse with latus rectum 2 units

D. a hyperbola with eccentricity $\frac{3}{2}$

14. The two vectors $\hat{i} + \hat{j} + \hat{k}$ and $\hat{i} + 3\hat{j} + 5\hat{k}$ represent the two sides $\overrightarrow{AB}$ and $\overrightarrow{AC}$ respectively of a triangle ABC. The length of the median through A is

A. 7

B. 14

C. $\sqrt{14}$

D. $\frac{\sqrt{14}}{2}$

15. The logical statement $(p \to q) \to \big((\sim p \to q) \to q\big)$ is

A. a tautology

B. equivalent to $\sim p \to q$

C. equivalent to $p \to \sim q$

D. a fallacy

Full Syllabus Test 1

16. $\lim_{x \to 0} \left(\frac{\tan x}{x} \right)^{1/x^2}$ is equal to

   A. $e^{1/3}$

   B. 1

   C. e

   D. 0

17. Let $f : \mathbb{R} \to \mathbb{R}$ be a function defined by $f(x) = \max\{x,\ x^3\}$. Then the set of all points where f is not differentiable, is

   A. $\{-1, 0\}$

   B. $\{-1, 0, 1\}$

   C. $\{0, 1\}$

   D. $\{-1, 1\}$

18. Let $f(x) = \begin{cases} x^2 \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$. Then

   A. f' does not exist at $x = 0$

   B. f' exists and is continuous at $x = 0$

   C. f' exists but not continuous at $x = 0$

   D. f' does not exist at any point

Full Syllabus Test 1

19. The area bounded by $y = x^2$, $y = [x+1]$, $x \le 1$ and the y-axis, where $[.]$ represents the greatest integer function, is

A. $\frac{2}{3}$

B. $\frac{1}{3}$

C. $\frac{7}{3}$

D. 1

20. The solution of the differential equation $ydx - xdy = y^2 \tan\left(\frac{x}{y}\right)dx$ is (C is constant of integration)

A. $\frac{x}{y} = Ce^x$

B. $\sin\left(\frac{x}{y}\right) = Ce^x$

C. $\cos\left(\frac{x}{y}\right) = Ce^x$

D. $x = Cy$

21. The total number of numbers greater than $4,00,000$ that can be formed by using the digits $0,2,2,4,4,5$ is

22. Three persons A, B, C are to speak at a function along with 5 other persons. If the persons speak in random order, the probability that A speaks before B and B speaks before C is $\frac{p}{q}$, where p, q are co-prime. Then $p+q$ is

23. If $x = \log_{24} 12$, $y = \log_{36} 24$ and $z = \log_{48} 36$, then $(1 + xyz)$ equals k times yz. The value of k is

24. The plane $P_1 : 4x + 7y + 4z + 81 = 0$ is rotated through a right angle about its line of intersection with the plane $P_2 : 5x + 3y + 10z = 25$. If the plane in its new position be denoted by P and the distance of plane P from the origin is d units, then the value of $[d/2]$, where $[.]$ represents the greatest integer function, is

25. If $\int\limits_{\ln 2}^{k} \frac{1}{\sqrt{e^x - 1}} dx = \frac{\pi}{6}$, then $k = \ln p$. The value of $p + 1$ is

26. If the mean and variance of eight numbers $3, 7, 9, 12, 13, 20, x$ and y be 10 and 25 respectively, then xy is equal to

27. The distance from the origin to the normal of the curve $x = 2\cos t + 2t \sin t$, $y = 2\sin t - 2t\cos t$ at $t = \frac{\pi}{4}$ is

28. If $y = \cfrac{\sin x}{1 + \cfrac{\cos x}{1 + \cfrac{\sin x}{1 + \cfrac{\cos x}{1 + \cdots \infty}}}}$, then $\frac{dx}{dy}$ at $x = \frac{\pi}{2}$ is

29. If $f(x) = \int \frac{2x^5 + 5x^4}{(4 + 2x + 3x^5)^2} dx, (x \geq 0)$ and $f(0) = 0$, then the value of $72 \cdot f(1)$ is

30. In triangle ABC, $\angle B = \frac{\pi}{3}$ and $\angle C = \frac{\pi}{4}$. Let D divide BC internally in the ratio $1 : 3$ and the angles $\angle BAD = \theta$, $\angle CAD = \phi$. The value of $\frac{\sin^2 \phi}{\sin^2 \theta} =$