Subject: Mathematics

1. For which of the following curves, the line $x+\sqrt{3}y=2\sqrt{3}$ is the tangent at the point $\left(\frac{3\sqrt{3}}{2},\frac{1}{2}\right)$?

   A. $x^2+9y^2=9$

   B. $2x^2-18y^2=9$

   C. $y^2=\frac{1}{6\sqrt{3}}x$

   D. $x^2+y^2=7$

2. Let L be a tangent line to the parabola $y^2=4x-20$ at $(6,\ 2)$. If L is also a tangent to the ellipse $\frac{x^2}{2}+\frac{y^2}{b}=1$ then the value of b is equal to :

   A. 20

   B. 14

   C. 16

   D. 11

3. If a tangent to the ellipse $x^2+4y^2=4$ meets the tangents at the extremities of its major axis at B and C, then the circle with BC as diameter passes through the point

   A. $(-1,1)$

   B. $(\sqrt{3},0)$

   C. $(1,1)$

   D. $(\sqrt{2},0)$

4. A tangent is drawn to the parabola $y^2 = 6x$ which is perpendicular to the line $2x + y = 1$. Which of the following points does $\mathbf{NOT}$ lie on it ?

   A. $(0, 3)$

   B. $(-6, 0)$

   C. $(4, 5)$

   D. $(5, 4)$

5. Let C be the locus of the mirror image of a point on the parabola $y^2 = 4x$ with respect to the line $y = x$. Then the equation of tangent to C at $P(2, 1)$ is :

   A. $2x + y = 5$

   B. $x + 2y = 4$

   C. $x + 3y = 5$

   D. $x - y = 1$

6. If the common tangent to the parabolas, $y^2 = 4x$ and $x^2 = 4y$ also touches the circle, $x^2 + y^2 = c^2$, then c is equal to:

   A. $\frac{1}{2}$

   B. $\frac{1}{4}$

   C. $\frac{1}{\sqrt{2}}$

   D. $\frac{1}{2\sqrt{2}}$

7. Consider a hyperbola $H : x^2 - 2y^2 = 4$. Let the tangent at a point $P\left(4,\ \sqrt{6}\right)$ meet the $x-$axis at Q and latus rectum at $R\left(x_1,\ y_1\right), x_1 > 0$. If F is a focus of H which is nearer to the point P, then the area of ΔQFR is equal to:

A. $\sqrt{6} - 1$

B. $4\sqrt{6} - 1$

C. $4\sqrt{6}$

D. $\frac{7}{\sqrt{6}} - 2$

8. Let a line $L : 2x + y = k,\ k > 0$ be a tangent to the hyperbola $x^2 - y^2 = 3$. If L is also a tangent to the parabola $y^2 = \alpha x$, then α is equal to:

A. -24

B. 24

C. 12

D. -12

9. A line parallel to the straight line $2x - y = 0$ is tangent to the hyperbola $\frac{x^2}{4} - \frac{y^2}{2} = 1$ at the point (x_1, y_1). Then $x_1^2 + 5y_1^2$ is equal to :

A. 6

B. 10

C. 8

D. 5

10. Equation of a common tangent to the parabola $y^2 = 4x$ and the hyperbola $xy = 2$ is:

A. $x - 2y + 4 = 0$

B. $x + y + 1 = 0$

C. $4x + 2y + 1 = 0$

D. $x + 2y + 4 = 0$

11. The locus of the mid-point of the line segment joining the focus of the parabola $y^2 = 4ax$ to a moving point of the parabola, is another parabola whose directrix is:

A. $x = a$

B. $x = 0$

C. $x = -\frac{a}{2}$

D. $x = \frac{a}{2}$

12. A ray of light through $(2, 1)$ is reflected at a point P on the $y-$axis and then passes through the point $(5, 3)$. If this reflected ray is the directrix of an ellipse with eccentricity $\frac{1}{3}$and the distance of the nearer focus from this directrix is $\frac{8}{\sqrt{53}}$, then the equation of the other directrix can be

A. $2x - 7y + 29 = 0$ or $2x - 7y - 7 = 0$

B. $11x + 7y + 8 = 0$ or $11x + 7y - 15 = 0$

C. $2x - 7y - 39 = 0$ or $2x - 7y - 7 = 0$

D. $11x - 7y - 8 = 0$ or $11x + 7y + 15 = 0$

13. The locus of the centroid of the triangle formed by any point P on the hyperbola $16x^2 - 9y^2 + 32x + 36y - 164 = 0$, and its foci is

A. $16x^2 - 9y^2 + 32x + 36y - 36 = 0$

B. $9x^2 - 16y^2 + 36x + 32y - 144 = 0$

C. $9x^2 - 16y^2 + 36x + 32y - 36 = 0$

D. $16x^2 - 9y^2 + 32x + 36y - 144 = 0$

14. If the distance between the foci of an ellipse is 6 and the distance between its directrices is 12, then the length of its latus rectum is

A. $2\sqrt{3}$

B. $\sqrt{3}$

C. $\frac{3}{\sqrt{2}}$

D. $3\sqrt{2}$

15. If e_1 and e_2 are the eccentricities of the ellipse , $\frac{x^2}{18} + \frac{y^2}{4} = 1$ and the hyperbola, $\frac{x^2}{9} - \frac{y^2}{4} = 1$ respectively and (e_1, e_2) is a point on th ellipse , $15x^2 + 3y^2 = k$. Then k is equal to :

A. 14

B. 15

C. 17

D. 16

16. The area (in sq. units) of an equilateral triangle inscribed in the parabola $y^2 = 8x$, with one of its vertices on the vertex of this parabola, is:

A. $128\sqrt{3}$

B. $192\sqrt{3}$

C. $64\sqrt{3}$

D. $256\sqrt{3}$

17. The shortest distance between the point $\left(\frac{3}{2}, 0\right)$ and the curve $y = \sqrt{x}, (x > 0)$, is :

A. $\frac{\sqrt{5}}{2}$

B. $\frac{\sqrt{3}}{2}$

C. $\frac{3}{2}$

D. $\frac{5}{4}$

18. Let the length of the latus rectum of an ellipse with its major-axis along x-axis and centre at the origin, be 8. If the distance between the foci of this ellipse is equal to the length of its minor axis, then which one of the following points lies on it?

A. $(4\sqrt{2}, 2\sqrt{2})$

B. $(4\sqrt{2}, 2\sqrt{3})$

C. $(4\sqrt{3}, 2\sqrt{3})$

D. $(4\sqrt{3}, 2\sqrt{2})$

19. A hyperbola has its centre at the origin, passes through the point $(4, 2)$ and has transverse axis of length 4 along the x-axis. Then the eccentricity of the hyperbola is :

A. $\sqrt{3}$

B. $\frac{2}{\sqrt{3}}$

C. 2

D. $\frac{3}{2}$

20. If the three normals drawn to the parabola, $y^2 = 2x$ pass through the point $(a, 0), a \neq 0$ then $'a'$ must be greater than:

A. 1

B. $\frac{1}{2}$

C. $-\frac{1}{2}$

D. -1

21. Let $x = 4$ be a directrix to an ellipse whose centre is at the origin and its eccentricity is $\frac{1}{2}$. If $P(1, \beta),\ \beta > 0$ is a point on this ellipse, then the equation of the normal to it at P is

A. $8x - 2y = 5$

B. $4x - 2y = 1$

C. $7x - 4y = 1$

D. $4x - 3y = 2$

22. Two sets A and B are as under :
$A = \{(a, b) \in \mathbb{R} \times \mathbb{R} : |a - 5| < 1 \text{ and } |b - 5| < 1\}$;
$B = \{(a, b) \in \mathbb{R} \times \mathbb{R} : 4(a - 6)^2 + 9(b - 5)^2 \leq 36\}$. Then

A. neither $A \subset B$ nor $B \subset A$

B. $B \subset A$

C. $A \subset B$

D. $A \cap B = \phi$ (an empty set)

23. An ellipse is drawn by taking a diameter of the circle $(x - 1)^2 + y^2 = 1$ as its semi-minor axis and a diameter of the circle $x^2 + (y - 2)^2 = 4$ as its semi-major axis. If the centre of the ellipse is at the origin and its axes are the coordinate axes, then the equation of the ellipse is

A. $4x^2 + y^2 = 4$

B. $x^2 + 4y^2 = 8$

C. $x^2 + 4y^2 = 8$

D. $x^2 + 4y^2 = 16$

24. Let a parabola P be such that its vertex and focus lie on the positive x-axis at a distance 2 and 4 units from the origin, respectively. If tangents are drawn from $O(0, 0)$ to the parabola P which meet P at S and R, then the area (in sq. units) of ΔSOR is equal to

A. 16

B. 32

C. $16\sqrt{2}$

D. $8\sqrt{2}$

25. If the line $y = mx + c$ is a common tangent to the hyperbola $\frac{x^2}{100} - \frac{y^2}{64} = 1$ and the circle $x^2 + y^2 = 36$, then which one of the following is true?

A. $4c^2 = 369$

B. $c^2 = 369$

C. $8m + 5 = 0$

D. $5m = 4$

Subject: Mathematics

1. Let L be a common tengent line to the curves $4x^2 + 9y^2 = 36$ and $(2x)^2 + (2y)^2 = 31$. Then the square of the slope of the line L is

2. A line is a common tangent to the circle $(x-3)^2 + y^2 = 9$ and the parabola $y^2 = 4x$. If the two points of contact (a, b) and (c, d) are distinct and lie in the first quadrant, then $2(a + c)$ is equal to

3. Let $A(\sec\theta, 2\tan\theta)$ and $B(\sec\phi, 2\tan\phi)$, where $\theta + \phi = \frac{\pi}{2}$, be two point on the hyperbola $2x^2 - y^2 = 2$. If (α, β) is the point of the intersection of the normals to the hyperbola at A and B, then $(2\beta)^2$ is equal to

4. Let $y = mx + c, m > 0$ be the focal chord of $y^2 = -64x$, which is tangent to $(x + 10)^2 + y^2 = 4$. Then the value of $4\sqrt{2}(m + c)$ is equal to

5. Let E be an ellipse whose axes are parallel to the co-ordinates axes, having its center at $(3, -4)$, one focus at $(4, -4)$ and one vertex at $(5, -4)$. If $mx - y = 4, m > 0$ is a tangent to the ellipse E, then the value of $5m^2$ is equal to