Topic : SHM, Wave on string and Sound wave

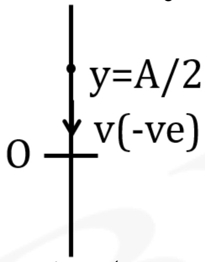
1. $y = A\sin(\omega t + \phi_o)$ is the time – displacement equation of an SHM. At $t = 0$, the displacement of the particle is $Y = \dfrac{A}{2}$ and it is moving along negative $x-$direction. Then, the initial phase angle ϕ_o will be.

A. $\dfrac{\pi}{6}$

B. $\dfrac{\pi}{3}$

C. $\dfrac{2\pi}{3}$

D. $\dfrac{5\pi}{6}$



$y = A\sin(\omega t + \phi_o)$

At $t = 0, x = \dfrac{A}{2}$

$\dfrac{1}{2} = \sin\phi_o$

$\phi_o = \dfrac{\pi}{6}, \dfrac{5\pi}{6}$

$v = \dfrac{dy}{dt} = A\omega\cos(\omega t + \phi_o)$

At $t = 0, v = A\omega\cos(\phi_o)$

For, $\phi_o = \dfrac{\pi}{6},\ v \to +ve,$ and,

For, $\phi_o = \dfrac{5\pi}{6},\ v \to -ve$

$\therefore \phi_o = \dfrac{5\pi}{6}$

2. Which of the following equations represents a travelling wave?

A. $y=Ae^{-x^2}(vt+\theta)$

B. $y=A\sin(15x-2t)$

C. $y=Ae^{x}\cos(\omega t-\theta)$

D. $y=A\sin x\cos\omega t$

For travelling wave equation,

$y=f(x,t)$

For a travelling wave, the argument of the sinusoidal trigonometric function should be a linear function of x and t.

From given options,
$y=A\sin(15x-2t)$ is correct answer.

3. The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{\frac{L}{g}}$. Measured value of $'L'$ is $1.0\ \mathrm{m}$ from meter scale having minimum division of $1\ \mathrm{mm}$ and time of one complete oscillation is $1.95\ \mathrm{s}$ measured from stopwatch of $0.01\ \mathrm{s}$ resolution. The percentage error in the determination of $'g'$ will be:

A. $1.33\ \%$

B. $1.30\ \%$

C. $1.13\ \%$

D. $1.03\ \%$

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$T^2 = 4\pi^2\frac{L}{g}$$

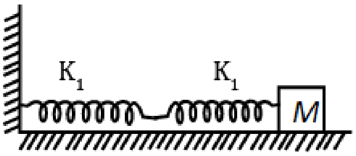
$$g = 4\pi^2\frac{L}{T^2}$$

$$\frac{\Delta g}{g} = \frac{\Delta L}{L} + \frac{2\Delta T}{T}$$

$$\frac{\Delta g}{g} \times 100 = \left[\frac{1\ \mathrm{mm}}{1\ \mathrm{m}} + \frac{2(10 \times 10^{-3})}{1.95}\right] \times 100$$

$$= 1.13\%$$

4. If two similar springs, each of spring constant K_1 are joined in series, the new spring constant and the time period would be changed by a factor of:



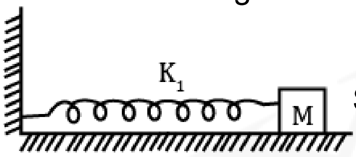
A. $\frac{1}{2}, \sqrt{2}$

B. $\frac{1}{4}, 2\sqrt{2}$

C. $\frac{1}{2}, 2\sqrt{2}$

D. $\frac{1}{4}, \sqrt{2}$

Before connecting in series,



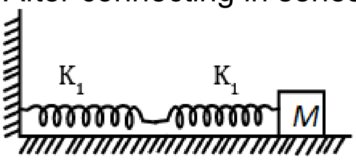
Spring constant,

$$K = K_1$$

Time period,

$$T = 2\pi\sqrt{\frac{M}{K_1}}$$

After connecting in series,



Spring constant, $K' = K_{eq} = \frac{K_1 \times K_1}{K_1 + K_1} = \frac{1}{2}K_1 = \frac{1}{2}K$

Time period,

$$T' = 2\pi\sqrt{\frac{M}{K_{eq}}} = 2\pi\sqrt{\frac{M}{K_1/2}}$$

$$\Rightarrow T' = \sqrt{2} \times 2\pi\sqrt{\frac{M}{K_1}} = \sqrt{2}T$$

Hence, option (A) is the correct answer.

5. Amplitude of a mass spring system, which is executing simple harmonic motion decreases with time. If mass $= 500\ \mathrm{g}$, damping constant $= 20\ \mathrm{g/s}$ then how much time is required for the amplitude of the system to drop to half of its initial value?
$(ln2 = 0.693)$

A. $15.01\ \mathrm{s}$

B. $17.32\ \mathrm{s}$

C. $0.034\ \mathrm{s}$

D. $34.65\ \mathrm{s}$

In damped oscillation, amplitude is given as:

$$A = A_0 e^{-\frac{bt}{2m}}$$

From question, $A = \frac{A_0}{2}$

$$\Rightarrow \frac{1}{2} = e^{-\frac{bt}{2m}}$$

$$\Rightarrow \frac{bt}{2m} = ln2 = 0.693$$

$$\Rightarrow t = \frac{2m}{b} \times 0.693$$

$$\Rightarrow t = 2 \times \frac{500}{20} \times 0.693$$

$$\therefore t = 50 \times 0.693 = 34.6\ \mathrm{s}$$

6. Given below are two statements.

Statement (1) : A second's pendulum has a time period of 1 second.

Statement (2) : It takes precisely one second to move between the two extreme positions.

In the light of the above statements, choose the correct answer from the options given below.

A. Both statement 1 and statement 2 are false.

B. Statement 1 is true, but statement 2 is false.

C. Statement 1 is false, but statement 2 is true.

D. Both statement 1 and statement 2 are true.

Second pendulum has a time period of 2 second, so, statement 1 is false.

But to move from one extreme position to other, it takes 1 second, so, statement 2 is true.

7. A particle performs simple harmonic motion with a period of 2 second. The time taken by the particle to cover a displacement equal to half of its amplitude from the mean position is $\frac{1}{a}\mathrm{s}$. The value of a to the nearest integer is.

Accepted Answers

6 6.0 6.00

Solution:

Time period $(T)=2\ \mathrm{s}$
Assume particle starts from its mean position $(t=0, x=0)$, so its equation, $x=A\sin(\omega t)$

Let particle takes time t_1 to move from mean position to half of its amplitude.

$$\frac{A}{2}=A\sin(\frac{2\pi}{T})t$$

$$\Rightarrow \frac{1}{2}=\sin(\frac{2\pi}{T})t$$

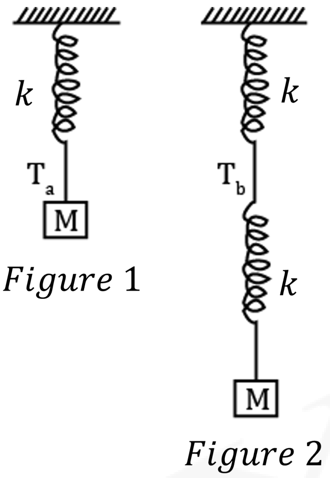
$$\Rightarrow \frac{\pi}{6}=\frac{2\pi}{T}t$$

$$\Rightarrow t=\frac{1}{6}\mathrm{s}=\frac{1}{a}\mathrm{s}$$

$$\Rightarrow a=6$$

8. Consider two identical springs each of spring constant k and negligible mass as compared to the mass M are as shown in the figure. Figure 1 shows one of them connected to mass M and Figure 2 shows their series combination connected to the same mass. The ratio of time period of oscillation of the two SHM is $\frac{T_b}{T_a} = \sqrt{x}$, where value of x is _______.

(Round off to the nearest integer)



Accepted Answers

2 2.0 2.00

Solution:

For Case I,

$$T_a = 2\pi\sqrt{\frac{M}{k}}$$

For case Case II,

$$k_{eq} = \frac{k_1 k_2}{k_1 + k_2} = \frac{k}{2}$$

$$\therefore T_b = 2\pi\sqrt{\frac{M}{k/2}} = 2\pi\sqrt{\frac{2M}{k}}$$

From the above two expressions,

$$T_b = \sqrt{2}\, T_a$$

$$\Rightarrow \frac{T_b}{T_a} = \sqrt{2} = \sqrt{x}$$

$$\therefore x = 2$$

9. A particle is making simple harmonic motion along the $x-\mathrm{axis}$. If at a distance x_1 and x_2 from the mean position the velocities of the particle are v_1 and v_2 respectively. The time period of its oscillation is given as :

A. $T=2\pi\sqrt{\dfrac{x_2^2+x_1^2}{v_1^2-v_2^2}}$

B. $T=2\pi\sqrt{\dfrac{x_2^2+x_1^2}{v_1^2+v_2^2}}$

C. $T=2\pi\sqrt{\dfrac{x_2^2-x_1^2}{v_1^2+v_2^2}}$

D. $T=2\pi\sqrt{\dfrac{x_2^2-x_1^2}{v_1^2-v_2^2}}$

We know that,
$v^2=\omega^2(A^2-x^2)$

$$\Rightarrow A^2=x^2+\frac{v^2}{\omega^2}$$

Given:
At x_1, velocity $=v_1$, and
at x_2, velocity $=v_2$

$$A^2=x_1^2+\frac{v_1^2}{\omega^2}=x_2^2+\frac{v_2^2}{\omega^2}$$

$$\Rightarrow \frac{v_1^2-v_2^2}{\omega^2}=x_2^2-x_1^2$$

$$\Rightarrow \omega^2=\frac{v_1^2-v_2^2}{x_2^2-x_1^2}$$

$$\therefore T=\frac{2\pi}{\omega}=2\pi\sqrt{\frac{x_2^2-x_1^2}{v_1^2-v_2^2}}$$

Hence, option (D) is correct.

10. T_0 is the time period of a simple pendulum at a place. If the length of the pendulum is reduced to $\frac{1}{16}$ times of its initial value, the modified time period is :

A. T_0

B. $8\pi T_0$

C. $4T_0$

D. $\frac{1}{4}T_0$

Let, the initial length of the pendulum is l. So, time period

$$T_0 = 2\pi\sqrt{\frac{l}{g}}$$

When, the length is reduced, the new time period is given by,

$$T = 2\pi\sqrt{\frac{l/16}{g}} = \frac{2\pi}{4}\sqrt{\frac{l}{g}}$$

$$\therefore T = \frac{T_0}{4}$$

Hence, (D) is the correct answer.

11. A transverse wave travels on a taut steel wire with a velocity of v when tension in it is 2.06×10^4 N. When the tension is changed to T the velocity changed to $v/2$. The value of T is close to:

A. 2.50×10^4 N

B. 5.15×10^3 N

C. 30.5×10^4 N

D. 10.2×10^2 N

The velocity of a transverse wave in a stretched wire is given by

$$v = \sqrt{\frac{T}{\mu}}$$

Where,

$T =$ Tension in the wire

$\mu =$ linear density of wise

$(\because\ v \propto T)$

$$\therefore\ \frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}}$$

$$\Rightarrow \frac{v}{v/2} = \sqrt{\frac{2.06 \times 10^4}{T_2}}$$

$$\Rightarrow T_2 = \frac{2.06 \times 10^4}{4} = 0.515 \times 10^4 \text{ N}$$

$$\Rightarrow T_2 = 5.15 \times 10^3 \text{ N}$$

12. Three harmonic waves, having equal frequency f and intensity I_0, have phase angles $0, \frac{\pi}{4}$ and $-\frac{\pi}{4}$ respectively. When they are superimposed, the intensity of the resultant wave is close to:

A. $5.8I_0$

B. $0.2I_0$

C. $3I_0$

D. I_0

Let the amplitude of each wave is $a_1 = a_2 = a_3 = a_0$.

Now, $\phi_1 = 0,\ \phi_2 = \frac{\pi}{4},\ \phi_3 = -\frac{\pi}{4}$

The resultant amplitude due to superposition of second and third wave is,

$$a_r = \sqrt{a_2^2 + a_3^2 + 2a_2a_3\cos(\pi/2)}$$

$$= \sqrt{a_0^2 + a_0^2 + 2a_0^2(0)}$$

$$\therefore a_r = a_0\sqrt{2}$$

Also, $\tan\delta = \dfrac{a_2\sin\phi_2 + a_2\sin\phi_3}{a_2\cos\phi_2 + a_3\cos\phi_3}$

$$= \frac{\left(\frac{a_0}{\sqrt{2}} - \frac{a_0}{\sqrt{2}}\right)}{\left(\frac{a_0}{\sqrt{2}} + \frac{a_0}{\sqrt{2}}\right)} = 0$$

The resultant amplitude of the above wave $(a_r = a_0\sqrt{2},\ \delta = 0)$ and the first wave $(a_1 = a_0,\ \phi_1 = 0)$ is,

$$A = \sqrt{a_1^2 + a_r^2 + 2a_1a_r\cos(0)}$$

$$= \sqrt{a_0^2 + 2a_0^2 + 2a_0(a_0\sqrt{2})} = a_0\sqrt{5.828}$$

Now, the intensity is directly proportional to the square of the amplitude,

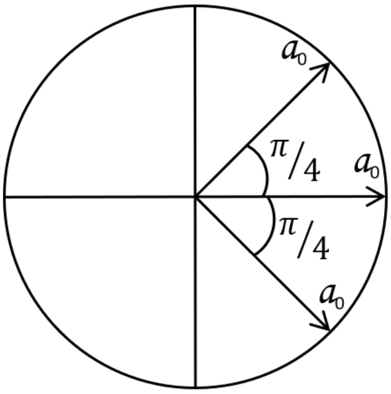
i.e. $I \propto a^2$

$$\Rightarrow \frac{I_0}{I_R} = \frac{a_0^2}{(a_0\sqrt{5.828})^2}$$

$\therefore I_R = 5.828 I_0 \approx 5.8 I_0$

Alternate solution:

Consider the phasor diagram of the given point, at which, the three waves are arriving simultaneously.



By the geometry of the figure, $a_r = a_0 + a_0\sqrt{2}$

As, the intensity is directly proportional to the square of the amplitude,

$$\Rightarrow \frac{I_0}{I_R} = \frac{a_0^2}{(a_0\sqrt{5.828})^2}$$

$\therefore I_R = 5.828 I_0 \approx 5.8 I_0$

Hence, (A) is the correct answer.

13. A wire of length L and mass per unit length $6.0 \times 10^{-3}\ \mathrm{kgm}^{-1}$ is put under tension of $540\ \mathrm{N}$. Two consecutive frequencies that resonates with it are : $420\ \mathrm{Hz}$ and $490\ \mathrm{Hz}$. Then the value of L (in meters) is:

A. $2.1\ \mathrm{m}$

B. $1.1\ \mathrm{m}$

C. $8.1\ \mathrm{m}$

D. $5.1\ \mathrm{m}$

Let the frequency of p^{th} harmonic is $420\ \mathrm{Hz}$ and that of $(p+1)^{\text{th}}$ harmonic is $490\ \mathrm{Hz}$.

Then, $p \cdot f = 420\ \ (f \rightarrow \text{fundamental frequency})$

And, $(p+1) \cdot f = 490$

$\Rightarrow (p+1) \cdot f - p \cdot f = 490 - 420$

$\Rightarrow f = 70\ \mathrm{Hz}$

The fundamental frequency of wire vibrating under tension T is given by,

$$f = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

Here, $\mu =$ mass per unit length of wire
$L =$ length of wire

$$70 = \frac{1}{2L}\sqrt{\frac{540}{6 \times 10^{-3}}}$$

$\Rightarrow L \approx 2.1\ \mathrm{m}$

Hence, (A) is the correct answer.

14. A wire of density 9×10^{-3}kg cm^{-3} is stretched between two clamps 1 m apart. The resulting strain in the wire is 4.9×10^{-4}. The lowest frequency of the transverse vibrations in the wire is (Young's modulus of wire $Y = 9 \times 10^{10} \text{ Nm}^{-2}$), (to the nearest integer),

Accepted Answers

35 35.0 35.00

Solution:

Given, Denisty of wire, $\sigma = 9 \times 10^{-3} \text{ kg cm}^{-3}$

Young's modulus of wire,$Y = 9 \times 10^{10} \text{ Nm}^{-2}$

Strain$= 4.9 \times 10^{-4}$

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{T/A}{\text{Strain}}$$

$$\therefore \frac{T}{A} = Y \times \text{Strain} = 9 \times 10^{10} \times 4.9 \times 10^{-4}$$

Also, mass of wire, $m = Al\sigma$

Mass per unit length, $\mu = \dfrac{m}{J} = A\sigma$

Fundamental frequency in the string

$$f = \frac{1}{2l}\sqrt{\frac{T}{\mu}} = \frac{1}{2l}\sqrt{\frac{T}{\sigma A}} = \frac{1}{1 \times 2}\sqrt{\frac{9 \times 10^{10} \times 4.9 \times 10^{-4}}{9 \times 10^{3}}}$$

$$\frac{1}{2}\sqrt{49 \times 10^{9-4-3}} = \frac{1}{2} \times 70 = 35 \text{ Hz.}$$

15. Two identical strings X and Z made of same material have tension T_X and T_Z in them. If their fundamental frequencies are 450 Hz and 300 Hz, respectively, then the ratio $\frac{T_X}{T_Z}$ is:

A. 2.25

B. 0.44

C. 1.25

D. 1.5

Te fundamental frequency in the string is given by,

$$f = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$$

Where, $T \rightarrow \text{tension}$ and $\mu = \frac{\text{mass}}{\text{length}}$

$$f_X = \frac{1}{2l}\sqrt{\frac{T_X}{\mu}} \text{and } f_Z = \frac{1}{2l}\sqrt{\frac{T_Z}{\mu}}$$

As the strings are identical and made of same material, l and μ are same for both the string,

$$\Rightarrow \frac{f_X}{f_Z} = \frac{450}{300} = \sqrt{\frac{T_X}{T_Z}}$$

$$\Rightarrow \frac{T_X}{T_Z} = \frac{9}{4} = 2.25.$$

Hence, option (A) is correct.

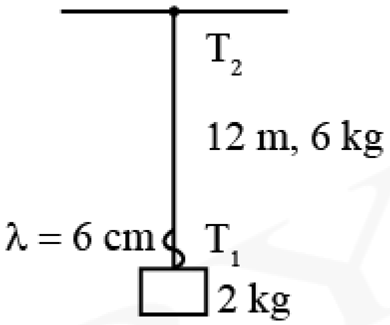
16. A uniform thin rope of length $12 \mathrm{~m}$ and mass $6 \mathrm{~kg}$ hangs vertically from a rigid support and a block of mass $2 \mathrm{~kg}$ is attached to its free end. A transverse short wave-train of wavelength $6 \mathrm{~cm}$ is produced at the lower end of the rope. What is the wavelength of the wave train (in cm) when it reaches the top of the rope?

A. 3

B. 6

C. 12

D. 9



As we know, $v = f\lambda$ and $v = \sqrt{\frac{T}{\mu}}$

$\Rightarrow \quad v \propto \lambda \propto \sqrt{T}$

$\Rightarrow \quad \frac{\lambda_1}{\lambda_2} = \frac{\sqrt{T_1}}{\sqrt{T_2}}$

Here, $T_1 = 2g \;;\; T_2 = 8g$

$\Rightarrow \quad \lambda_2 = \lambda_1 \frac{\sqrt{T_2}}{\sqrt{T_1}} = 6 \times \frac{\sqrt{8g}}{\sqrt{2g}} = 12 \mathrm{~cm}$

Hence, (C) is the correct answer.

17. The amplitude of wave disturbance propagating in the positive x-direction is given by $y=\frac{1}{(1+x)^2}$ at time $t=0$ and $y=\frac{1}{1+(x-2)^2}$ at $t=1\ \mathrm{s}$, where x and y are in meter. The shape of wave does not change during the propagation. The velocity of the wave in $(\mathrm{m/s})$ will be _______.

Accepted Answers

2 2.0 2.00

Solution:

The wave equation, in general, is given by,

$y=f(kx-vt)$

$\therefore$ the equation of the given wave is, $y=\frac{1}{1+(x-vt)^2} \quad \ldots(1)$

Now,

At $t=0,\ y=\frac{1}{1+x^2}$

At $t=1\ \mathrm{s},\ y=\frac{1}{1+(x-2)^2} \quad \ldots(2)$

Comparing equations (1) and (2), we get,

$vt=2$

$\Rightarrow v\times 1=2$

$\Rightarrow v=2\ \mathrm{m/s}$

18. Two travelling waves produces a standing wave represented by equation, $y = 1.0\ \text{mm} \cos[(1.57\ \text{cm}^{-1})x] \sin[(78.5\ \text{s}^{-1})t]$. The node closest to the origin in the region $x > 0\ \text{cm}$ will be at $x =$____cm.

Accepted Answers

1 1.0 1.00

Solution:

Given:

$$y = 1.0\ \text{mm} \cos[(1.57\ \text{cm}^{-1})x] \sin[(78.5\ \text{s}^{-1})t]$$

For nodes,

$$\cos[(1.57\ \text{cm}^{-1})x] = 0$$

$$\Rightarrow (1.57\ \text{cm}^{-1})x = \frac{\pi}{2}$$

$$\Rightarrow x = 1\ \text{cm}$$

19. Two waves are simultaneously passing through a string and their equations are : $y_1 = A_1 \sin\ k(x - vt),\ y_2 = A_2 \sin\ k(x - vt + x_0)$ Given amplitudes $A_1 = 12\ \text{mm}$ and $A_2 = 5\ \text{mm},\ x_0 = 3.5\ \text{cm}$ and wave number $k = 6.28\ \text{cm}^{-1}$. The amplitude of resulting wave will be _____ mm.

Accepted Answers

7 7.0 7.00

Solution:

$y_1 = 12 \sin 6.28(x - vt)$

$y_2 = 5 \sin 6.28(x - vt + 3.5)$

$\Delta\phi = \frac{2\pi}{\lambda}(\Delta x) = k\Delta x$

$\Delta\phi = 7\pi$

$A_{net} = \sqrt{{A_1}^2 + {A_2}^2 + 2A_1A_2 \cos\phi}$

$A_{net} = \sqrt{12^2 + 5^2 + (2)(12)(5) \cos 7\pi}$

$A_{net} = \sqrt{144 + 25 - 120}$

$A_{net} = 7\ \text{mm}$

20. A tuning fork is vibrating at $250 \mathrm{~Hz}$. The length of the shortest closed organ pipe that will resonate with the tuning fork will be _________ cm.

(Take speed of sound in air as $340 \mathrm{~ms}^{-1}$)

Accepted Answers

34 34.0 34.00

Solution:

Length of shortest closed organ pipe for resonance is $l=\frac{\lambda}{4} \Rightarrow \lambda=4 l$

using,
$v=\lambda f$
$\lambda=\frac{v}{f}$
$4 l=\frac{v}{f}$
$l=\frac{v}{4 f}=\frac{340}{4(250)}=0.34 \mathrm{~m}=34 \mathrm{~cm}$

21. Two cars are approaching each other at an equal speed of $7.2\ \mathrm{km/hr}$. When they see each other, both blow horns having frequency of $676\mathrm{Hz}$. The beat frequency heard by each driver will be Hz.

[Velocity of sound in air is $340\mathrm{m/s}$]

Accepted Answers

8 8.0 8.00

Solution:

Speed = 7.2 $\mathrm{km/h}$ = $2\ \mathrm{m/s}$

Frequency as heard by A,

$$f'_{\mathrm{A}} = f_B\left(\frac{v+v_0}{v-v_s}\right)$$

$$f'_{\mathrm{A}} = 676\left(\frac{340+2}{340-2}\right)$$

$$f'_{\mathrm{A}} = 684\ \mathrm{Hz}$$

$$\therefore f_{\mathrm{beat}} = f'_A - f_B = 684 - 676 = 8\ \mathrm{Hz}$$

22. A student is performing the experiment of resonance column. The diameter of the column tube is 6 cm. The frequency of the tuning fork is 504 Hz. The speed of the sound at the given temperature is 336 m/s. The zero of the meter scale coincides with the top end of the resonance column tube. The reading of the water level in the column when the first resonance occurs is -

A. 13 cm

B. 14.8 cm

C. 16.6 cm

D. 18.4 cm

Given:
$d = 6 \text{ cm}$
$f = 504 \text{ Hz}$
$v = 336 \text{ m/s}$
$e = 0.3d$

So, $\lambda = \frac{v}{f} = \frac{336}{504} = 0.6667 \text{ m} = 66.67 \text{ cm}$

Now,

$$l + e = \frac{\lambda}{4}$$

$$\Rightarrow l + 0.3d = \frac{\lambda}{4}$$

$$\Rightarrow l + 0.3 \times 6 = \frac{66.67}{4}$$

$$\Rightarrow l = 14.87 \text{ cm}$$

23. A tuning fork A of unknown frequency produces 5 beats/s with a fork of known frequency 340 Hz. When fork A is filed, the beat frequency decreases to 2 beats/s. What is the frequency of fork A?

- **A.** 342 Hz
- **B.** 335 Hz
- **C.** 338 Hz
- **D.** 345 Hz

Suppose the frequency of fork A is f.

Then, $f \pm 340 = 5$

$\Rightarrow f = 335 \text{ Hz}$ or 345 Hz

Now, after filing, suppose the frequency of fork A is f'.

Then, $f' \pm 340 = 2$

$\Rightarrow f' = 338 \text{ Hz}$ or 342 Hz

After filing, time period decreases slightly and frequency increases slightly.

Therefore, frequency of fork A increases from 335 Hz to 338 Hz.

Hence, original frequency is 335 Hz.

24. The frequency of a car horn encountered a change from $400 \mathrm{~Hz}$ to $500 \mathrm{~Hz}$, when the car approaches a vertical wall. If the speed of sound is $330 \mathrm{~m/s}$. Then the speed of car is _____ $\mathrm{km/h}$.

Accepted Answers

132 132.0 132.00

Solution:

At first, the wall act as the observer.

Frequency received by wall,

$$f_1 = f_0\left(\frac{v+v_0}{v-v_s}\right) = f_0\left(\frac{v}{v-v'}\right) \quad \ldots(1)$$

Now, wall acts as a source.

Frequency received by observer on car,

$$f_2 = f_1\left(\frac{v+v_0}{v-v_s}\right) = f_1\left(\frac{v+v'}{v}\right) \quad \ldots(2)$$

From (1) and (2),

$$f_2 = f_0\left(\frac{v+v'}{v-v'}\right)$$

$$\Rightarrow 500 = 400\left(\frac{330+v'}{330-v'}\right)$$

$$\Rightarrow v' = \frac{110}{3}\mathrm{m/s} = 132 \mathrm{~km/hr}$$

25. A stationary observer receive sound from two identical tuning forks, one of which approaches and the other one recedes with the same speed (much less than the speed of sound). The observer hears 2 beats per second. The oscillation frequency of each tuning fork is $f_0 = 1400 \text{ Hz}$ and the velocity of sound in air is 350 ms^{-1}. The speed of each tuning fork is close to:

A. $\frac{1}{2} \text{ms}^{-1}$

B. 1 ms^{-1}

C. $\frac{1}{4} \text{ms}^{-1}$

D. $\frac{1}{8} \text{ms}^{-1}$

As the observer is stationary,

From Doppler's effect, frequency of sound heard (f_1) by observer when source is approaching will be

$$f_1 = f_0 \frac{v}{v - v_s}$$

Here, $v =$ velocity of sound
$v_s =$ velocity of source

Frequency of sound heard (f_2) by observer when source is receding

$$f_1 = f_0 \frac{v}{v + v_s}$$

Beat frequency $f = |f_1 - f_2|$

$$\Rightarrow 2 = |f_1 - f_2| = f_0 v \left[\frac{1}{v - v_s} - \frac{1}{v + v_s}\right]$$

$$\Rightarrow 2 = f_0 v \frac{2v_s}{v^2 \left[1 - \frac{v_s^2}{v^2}\right]}$$

For $v >> v_s$, $\left(\frac{v_s^2}{v^2}\right)$ can be neglected

$$\Rightarrow v_s = \frac{v}{f_0} = \frac{350}{1400} = \frac{1}{4} \text{ms}^{-1}$$

Hence, option (C) is correct.

26. One meter long (both ends open) organ pipe is kept in a gas that has double the density of air at STP. Assuming the speed of sound of air at STP is $300\ \mathrm{ms}^{-1}$, the frequency difference between the fundamental and second harmonic of the pipe is_____Hz.

Accepted Answers

106

Solution:

Given: $v_{\text{air}} = 300\ \mathrm{ms}^{-1}$, $\rho_{\text{gas}} = 2\rho_{\text{air}}$

The speed of sound in gas is given by,

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

$$\Rightarrow \frac{v_{\text{gas}}}{v_{\text{air}}} = \frac{\sqrt{\frac{\gamma P}{2\rho_{\text{air}}}}}{\sqrt{\frac{\gamma P}{\rho_{\text{air}}}}}$$

$$\Rightarrow v_{gas} = \frac{v_{air}}{\sqrt{2}} = \frac{300}{\sqrt{2}} = 150\sqrt{2}\ \mathrm{ms}^{-1}$$

And n^{th} harmonic in an open pipe is given by,

$$f_n = \frac{nv}{2L}$$

$$\text{fundamental frequency, } f_1 = \frac{v}{2L}$$

$$\text{second harmonic, } f_2 = \frac{2v}{2L}$$

$$\Rightarrow \Delta f = f_2 - f_1 = \frac{v}{2L}$$

$$\text{Given that, } L = 1\ \mathrm{m}$$

$$\Delta f = \frac{150\sqrt{2}}{2} = 106\ \mathrm{Hz}$$

Hence, 106 is the correct answer.

27. Assume that the displacement (s) of air is proportional to the pressure difference (Δp) created by a sound wave. Displacement (s) further depends on the speed of sound (v), density of air (ρ) and the frequency (f). If $\Delta p \sim 10 \text{ Pa}, v \sim 300 \text{ m/s}, \rho \sim 1 \text{ kg/m}^3$ and $f \sim 1000 \text{ Hz}$, then s will be of the order of (take the multiplicative constant to be 1)

A. $\frac{3}{100} \text{mm}$

B. 10 mm

C. $\frac{1}{10} \text{mm}$

D. 1 mm

As we know,
Pressure amplitude,

$$\Delta P_0 = s_0 K B = S_0 \times \frac{\omega}{V} \times \rho V^2$$

$$\left[\because K = \frac{\omega}{V}, V = \sqrt{\frac{B}{\rho}}\right]$$

$$\Rightarrow s_0 = \frac{1}{2\pi} \frac{\Delta P_0}{\rho V f}$$

Taking propotionality constant as 1,

Order of $s_0 \approx \frac{10}{1 \times 300 \times 1000} \text{m} = \frac{1}{30} \text{mm} \approx \frac{3}{100} \text{mm}$

28. In a resonance tube experiment when the tube is filled with water up to a height of $17.0 \mathrm{~cm}$ from bottom, it resonates with a given tuning fork. When the water level is raised the next resonance with the same tuning fork occurs at a height of $24.5 \mathrm{~cm}$. If the velocity of sound in air is $330 \mathrm{~m/s}$, the tuning fork frequency is

A. $2200 \mathrm{~Hz}$

B. $550 \mathrm{~Hz}$

C. $1100 \mathrm{~Hz}$

D. $3300 \mathrm{~Hz}$

Given that
$l_1 = 17 \mathrm{~cm}$ and
$l_2 = 24.5 \mathrm{~cm}$,
$V = 330 \mathrm{~m/s}$,
$f = ?$

$$\lambda = 2(l_2 - l_1) = 2 \times (24.5 - 17) = 15 \mathrm{~cm}$$

Now, from
$\Rightarrow v = f\lambda$
$\Rightarrow 330 = \lambda \times 15 \times 10^{-2}$

$$\therefore \lambda = \frac{330}{15} \times 100 = \frac{1100 \times 100}{5} = 2200 \mathrm{~Hz}$$

29. The driver of a bus, approaching a big wall, notices that the frequency of his bus's horn changes from $420 \text{ Hz to } 490 \text{ Hz}$ when he hears it after it gets reflected from the wall. Find the speed of the bus, if the speed of the sound is 330 ms^{-1}

A. 91 km h^{-1}

B. 81 km h^{-1}

C. 61 km h^{-1}

D. 71 km h^{-1}

From the Doppler's effect of sound, frequency appeared/heard at the wall is,

$$f_w = \frac{330}{330 - v} . f \quad \ldots . . (i)$$

Here,
$v \rightarrow$ speed of bus,
$f \rightarrow$ actual frequency of the source,

The frequency (f') heard by the driver of the bus, after reflection from the wall is,

$$f' = \frac{330 + v}{330} . f_w = \frac{330 + v}{330 - v} . f$$

$$\Rightarrow 490 = \frac{330 + v}{330 - v} \times 420$$

$$\Rightarrow v = \frac{330 \times 7}{91} = 25.38 \text{ m/s}$$

$$= 25.38 \times \frac{18}{5} \approx 91 \text{ km/hr}$$

Hence, (A) is the correct answer.

30. Two cars X and Y are approaching each other with velocities $36 \mathrm{~km/h}$ and $72 \mathrm{~km/h}$ respectively. The frequency of a whistle sound as emitted by a passenger in car X, heard by the passenger in car Y is $1320 \mathrm{~Hz}$. If the velocity of sound in air is $340 \mathrm{~m/s}$, the actual frequency of the whistle sound produced is Hz.

Accepted Answers

1210 1210.0

Solution:

Given :-

$V_s = V_x = 36 \mathrm{~km/h} = 36 \times \frac{5}{18} = 10 \mathrm{~m/s}$

$V_o = V_y = 72 \mathrm{~km/h} = 72 \times \frac{5}{18} = 20 \mathrm{~m/s}$

$V = 340 \mathrm{~m/s}$

$f = 1320 \mathrm{~Hz}$

Using Doppler's law :

$f = f_o \left(\frac{V + V_o}{V - V_s}\right)$

$1320 = f_o \left(\frac{340 + 20}{340 - 10}\right)$

$\Rightarrow f_o = 1210 \mathrm{~Hz}$