Full Syllabus Test 2

Subject: Mathematics

1. Let $X = \{1, 2, 3, 4, 5\}$. The number of different ordered pairs (Y, Z) that can be formed such that $Y \subseteq X, Z \subseteq X$ and $Y \cap Z$ is empty, is

A. 5^3

B. 5^2

C. 3^5

D. 2^5

2. Let $f(x) = \begin{cases} \sqrt{\{x\}}\ , & x \notin \mathbb{Z} \\ 1\ , & x \in \mathbb{Z} \end{cases}$ and $g(x) = \{x\}^2$, the area bounded by $f(x)$ and $g(x)$ for $x \in [0, n];\ (n \in \mathbb{Z})$ is denoted by $S(n)$, then which of the following is correct?

A. $S(10) = \dfrac{5}{3}$

B. $\dfrac{S(10)}{S(5)} = 4$

C. $\lim\limits_{n \to \infty} \dfrac{S(n)}{n} = \dfrac{1}{3}$

D. $\sum\limits_{n=1}^{9} S(n) = 30$

3. Number of 4 digit numbers that can be formed using the digits $0, 1, 2, 3, 4, 5$ which are divisible by 6 when repetition of digits is not allowed are

A. 24

B. 52

C. 72

D. 48

Full Syllabus Test 2

4. The sum of the series of
$\cot^{-1}\frac{5}{\sqrt{3}}+\cot^{-1}\frac{9}{\sqrt{3}}+\cot^{-1}\frac{15}{\sqrt{3}}+\cot^{-1}\frac{23}{\sqrt{3}}+\cdots\infty$ is equal to

   A. $\frac{\pi}{4}$

   B. $\frac{\pi}{3}$

   C. $\frac{\pi}{6}$

   D. $\frac{\pi}{12}$

5. $px+qy=40$ is a chord of minimum length of the circle $(x-10)^2+(y-20)^2=729$. If the chord passes through $(5,15)$, then $p^{2019}+q^{2019}$ is equal to

   A. 0

   B. 2

   C. 2^{2019}

   D. 2^{2020}

6. $\displaystyle\int\frac{\cos x+\sqrt{3}}{1+4\sin\left(x+\frac{\pi}{3}\right)+4\sin^2\left(x+\frac{\pi}{3}\right)}dx$ is
where c is constant of integration

   A. $\dfrac{\cos x}{1+2\sin\left(x+\frac{\pi}{3}\right)}+c$

   B. $\dfrac{\sec x}{1+2\sin\left(x+\frac{\pi}{3}\right)}+c$

   C. $\dfrac{\sin x}{1+2\sin\left(x+\frac{\pi}{3}\right)}+c$

   D. $\frac{1}{2}\tan^{-1}\left(1+2\sin\left(x+\frac{\pi}{3}\right)\right)+c$

7. If $z_n = \cos \frac{\pi}{(2n+1)(2n+3)} + i \sin \frac{\pi}{(2n+1)(2n+3)}, n \in \mathbb{N}$, then the value of $\lim\limits_{k \to \infty} (z_1 \cdot z_2 \cdot z_3 \cdots z_k)$ is

A. 1

B. $\frac{\sqrt{3}}{2} + i\frac{1}{2}$

C. $\frac{1}{2} + i\frac{\sqrt{3}}{2}$

D. $\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}$

8. If the statement $(p \to q) \to (q \to r)$ is false, then truth values of statement p, q, r respectively, can be

A. $\mathrm{F \ T \ F}$

B. $\mathrm{T \ F \ T}$

C. $\mathrm{T \ F \ F}$

D. $\mathrm{F \ T \ T}$

9. The value of
$(\sqrt{3} + \tan 1^\circ)(\sqrt{3} + \tan 2^\circ)(\sqrt{3} + \tan 3^\circ) \cdots\cdots (\sqrt{3} + \tan 28^\circ)(\sqrt{3} + \tan 29^\circ)$
is

A. 2^{28}

B. $2^{15} + 2 - \sqrt{3}$

C. 2^{29}

D. $2^{14} + 2 - \sqrt{3}$

Full Syllabus Test 2

10. Let x_n, y_n, z_n, w_n denote n^{th} term of four different arithmetic progressions with positive terms. If $x_4 + y_4 + z_4 + w_4 = 8$ and $x_{10} + y_{10} + z_{10} + w_{10} = 20$, then maximum possible value of $x_{20} \cdot y_{20} \cdot z_{20} \cdot w_{20}$ is

A. 10^4

B. 10^8

C. 10^{10}

D. 10^{20}

11. The complete solution set of the inequality $\log_5(x^2 - 2) < \log_5\left(\frac{3}{2}|x| - 1\right)$ is

A. $\left(-\infty, -\frac{2}{3}\right) \cup \left(\frac{2}{3}, \infty\right)$

B. $\left(-2, -\frac{2}{3}\right) \cup \left(\frac{2}{3}, 2\right)$

C. $(-2, -\sqrt{2}) \cup (\sqrt{2}, 2)$

D. $(-2, -\sqrt{2}) \cup \left(\frac{2}{3}, 2\right)$

12. Let $y = f(x)$ be a real-valued differentiable function on the set of all real numbers $\mathbb{R}$ such that $f(1) = 1$. If $f(x)$ satisfies $xf'(x) = x^2 + f(x) - 2$, then

A. $f(x)$ is even function

B. $f(x)$ is odd function

C. minimum value of $f(x)$ is 0

D. $y = f(x)$ represent a parabola with focus $\left(1, \frac{5}{4}\right)$

Full Syllabus Test 2

13. A vector $(\vec{d})$ is equally inclined to three vectors $\vec{a} = \hat{i} - \hat{j} + \hat{k},\ \vec{b} = 2\hat{i} + \hat{j}$ and $\vec{c} = 3\hat{j} - 2\hat{k}$. Let $\vec{x}, \vec{y}, \vec{z}$ be three vectors in the plane of $\vec{a}, \vec{b}; \vec{b}, \vec{c}; \vec{c}, \vec{a}$, respectively. Then which of the following is INCORRECT?

A. $\vec{x} . \vec{d} = 0$

B. $\vec{y} . \vec{d} = 0$

C. $\vec{z} . \vec{d} = 0$

D. none of these

14. The eccentricity of the conic $4(2y - x - 3)^2 - 9(2x + y - 1)^2 = 80$ is

A. $\frac{2}{\sqrt{3}}$

B. $\frac{\sqrt{3}}{\sqrt{2}}$

C. $\sqrt{2}$

D. $\frac{\sqrt{13}}{3}$

15. The integral $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} e^x(\ln(\cos x))(\cos x \sin x - 1)\ \mathrm{cosec}^2 x\ dx$ is equal to

A. $\frac{e^{\pi/4}}{2}\left[2e^{\pi/4} + \ln 2 - 2\right]$

B. $\frac{e^{\pi/4}}{2}\left[e^{\pi/4} + \ln 2 - 2\right]$

C. $\frac{e^{\pi/4}}{2}\left[e^{\pi/4} - \ln 2 + 2\right]$

D. $\frac{e^{\pi/4}}{2}\left[2e^{\pi/4} - \ln 2 + 2\right]$

Full Syllabus Test 2

16. 10 is the mean of 25 observations. If $1, 3, 5, \ldots, 49$ are added to corresponding observations, then the new mean is

A. 32

B. 35

C. 39

D. 27

17. The value of $\lim\limits_{x \to \infty} x^2 \ln(x \cot^{-1} x)$ is

A. $-\frac{1}{3}$

B. $\frac{1}{3}$

C. $\frac{2}{3}$

D. $-\frac{2}{3}$

18. A random variable X has probability distribution

X	1	2	3	4	5	6	7	8
$P(X)$	0.13	0.22	0.12	0.21	0.13	0.08	0.06	0.05

If events are $E = \{x \text{ is an odd number}\}$,$F = \{x \text{ is divisible by } 3\}$ and $G = \{x \text{ is less than } 7\}$, then the value of $P(E \cup (F \cap G))$ is

A. 0.87

B. 0.77

C. 0.52

D. 0.82

Full Syllabus Test 2

19. If $f(x) = \sin^4 x + \cos^4 x - \frac{1}{2}\sin 2x$, then the range of $f(x)$ is

   A. $\left[0, \frac{3}{2}\right]$

   B. $\left[-\frac{1}{2}, \frac{7}{2}\right]$

   C. $\left[0, \frac{9}{8}\right]$

   D. $\left[\frac{3}{4}, \frac{7}{8}\right]$

20. If $x + y - 2 = 0,\ 2x - y + 1 = 0$ and $px + qy - r = 0$ are concurrent lines, then the slope of the member in the family of lines $2px + 3qy + 4r = 0$ which is farthest from the origin is

   A. $-\frac{1}{2}$

   B. -2

   C. $-\frac{2}{3}$

   D. $-\frac{3}{10}$

21. The number of solutions of the equation $\cos\theta + \cos 3\theta + \cos 5\theta + \cos 7\theta = 0$, where $\theta \in [0, 2\pi]$ is

22. If the equation of the plane containing the lines $x - y - z - 4 = 0,\ x + y + 2z - 4 = 0$ and parallel to the line of intersection of the planes $2x + 3y + z = 1$ and $x + 3y + 2z = 2$ is $x + Ay + Bz + C = 0$, then the value of $|A + B + C|$ is

23. If the term independent of x in $\left(1 + x + x^{-2} + x^{-3}\right)^{10}$ is n, then the last digit of $(n + 2)^3$ is

24. The number of integral values of a such that $x^2 + ax + a + 1 = 0$ has integral roots is

Full Syllabus Test 2

25. If $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then the sum of all the elements of matrix A is

26. Let $f(x) = \begin{cases} \dfrac{b^3+b-2b^2-2}{b^2+5b+6} - x^2\,; & 0 \le x < 1 \\ 3x-4\,; & 1 \le x \le 3 \end{cases}$
where $b \in \mathbb{R}$. If $f(x)$ has minimum value at $x=1$, then the least integral value of b is

27. Let $f(x) = [3 + 2\cos x], x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ where $[.]$ denotes the greatest integer function. The number of point(s) of discontinuity of $f(x)$ is

28. If $e^{xy^2} + y\cos x^2 = 5$, then absolute value of $y'(0) =$

29. If $\triangle = \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$ and $\triangle_1 = \begin{vmatrix} 1 & 1 & 1 \\ yz & zx & xy \\ x & y & z \end{vmatrix}$, then the value of $\triangle_1 + \triangle =$

30. If the area of the triangle ABC is Δ, such that $b^2 \sin 2C + c^2 \sin 2B = k\Delta$, then the value of k is