COMPLEX NUMBER

1. Definition :

Complex numbers are defined as expressions of the form a + ib where a, b $\in$ R & i = $\sqrt{-1}$. It is denoted by z i.e.z = a + ib. 'a' is called real part of z (Re z) and 'b' is called imaginary part of z (Im z).

Every Complex Number Can Be Regarded As

Purely real	Purely imaginary	Imaginary
if $b = 0$	if $a = 0$	if $b \neq 0$

Note :

(i) The set R of real numbers is a proper subset of the complex numbers.
Hence the complex number system is
$N \subset W \subset I \subset Q \subset R \subset C.$

(ii) Zero is both purely real as well as purely imaginary but not imaginary.

(iii) i = $\sqrt{-1}$ is called the imaginary unit. Also $i^2 = -1$; $i^3 = -i$; $i^4 = 1$ etc.

(iv) $\sqrt{a}\sqrt{b} = \sqrt{ab}$, $a \geq 0$, $b \geq 0$ or either of them is non negative.

2. Conjugate complex:

If z = a + ib then its conjugate complex is obtained by changing the sign of its imaginary part & is denoted by $\bar{z}$. i.e. $\bar{z} = a - ib$.

Note that :

(i) $z + \bar{z} = 2\text{Re}(z)$ and If z is purely imaginary then $z + \bar{z} = 0$

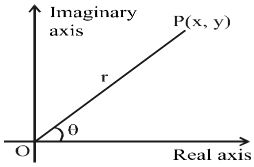
(ii) $z - \bar{z} = 2i\,\text{Im}(z)$ and If z is purely real then $z - \bar{z} = 0$

(iii) $z\bar{z} = a^2 + b^2$ which is real

3. Representation of complex number in various forms:

(a) Cartesian Form (Geometrical Representation) :

Every complex number z = x + iy can be represented by a point on the cartesian plane known as complex plane (Argand diagram) by the ordered pair (x,y).



Length OP is called **modulus** of the complex number denoted by $|z|$ & θ is called the **argument or amplitude.**

e.g. $|z| = \sqrt{x^2 + y^2}$ & $\theta = \tan^{-1}\dfrac{y}{x}$ (angle made by OP with positive x–axis)

Geometrically $|z|$ represents the distance of point P from origin. ($|z| \geq 0$)

(b) Trigonometric / Polar Representation :

$z = r(\cos\theta + i\sin\theta)$ where $|z| = r$; $\arg z = \theta$; $\bar{z} = r(\cos\theta - i\sin\theta)$

Note : $\cos\theta + i\sin\theta$ is also written as CiS θ.

Euler 's formula :

The formula $e^{ix} = \cos x + i\sin x$ is called Euler's formula.

Also $\cos x = \dfrac{e^{ix} + e^{-ix}}{2}$ & $\sin x = \dfrac{e^{ix} - e^{-ix}}{2i}$ are known as Eular's identities.

(c) Exponential Representation :

Let z be a complex number such that $|z| = r$ & $\arg z = \theta$, then $z = r\cdot e^{i\theta}$.

4. Important properties of conjugate:

(a) $\overline{(\bar{z})} = z$

(b) $\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$

(c) $\overline{z_1 - z_2} = \bar{z}_1 - \bar{z}_2$

(d) $\overline{z_1 . z_2} = \bar{z}_1 \cdot \bar{z}_2$

(e) $\overline{\left(\dfrac{z_1}{z_2}\right)} = \dfrac{\bar{z}_1}{\bar{z}_2}$; $z_2 \neq 0$

(f) If $f(\alpha + i\beta) = x + iy \Rightarrow f(\alpha - i\beta) = x - iy$

5. Imporant properties of modulus:

(a) $|z| \geq 0$

(b) $|z| \geq \text{Re}(z)$

(c) $|z| \geq \text{Im}(z)$

(d) $|z| = |\bar{z}| = |-z| = |-\bar{z}|$

(e) $z\bar{z} = |z|^2$

(f) $|z_1 z_2| = |z_1|.|z_2|$

(g) $\left|\dfrac{z_1}{z_2}\right| = \dfrac{|z_1|}{|z_2|}$, $z_2 \neq 0$

(h) $|z^n| = |z|^n$,

(i) $|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2 + 2\text{Re}(z_1\bar{z}_2)$ or $|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2 + 2|z_1|\,|z_2|\cos(\theta_1 - \theta_2)$

(j) $|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2\left[|z_1|^2 + |z_2|^2\right]$

(k) $\left|\,|z_1| - |z_2|\,\right| \le |z_1 + z_2| \le |z_1| + |z_2|$ **[Triangle Inequality]**
(l) $\left|\,|z_1| - |z_2|\,\right| \le |z_1 - z_2| \le |z_1| + |z_2|$ **[Triangle Inequality]**

(m) If $\left|z + \frac{1}{z}\right| = a\ (a > 0)$, then max $|z| = \frac{a + \sqrt{a^2 + 4}}{2}$ & min $|z| = \frac{1}{2}\left(\sqrt{a^2 + 4} - a\right)$

6. Important properties of amplitude:

(a) amp $(z_1.z_2)$ = amp z_1 + amp z_2 + $2k\pi$; $k \in I$

(b) amp $\left(\frac{z_1}{z_2}\right)$ = amp z_1 − amp z_2 + $2k\pi$; $k \in I$

(c) amp (z^n) = n amp (z) + $2k\pi$, where proper value of k must be chosen so that RHS lies in $(-\pi, \pi]$.

(d) $\ln(z) = \ln(re^{i\theta}) = \ln r + i\theta = \ln|z| + i$ amp (z)

7. Demoiver's theorem :

If n is an ainteger then the value of $(\cos\theta + i\sin\theta)^n$ is $\cos n\theta + i\sin n\theta$ and if n is a ratyional number then $(\cos\theta + i\sin n\theta)$ is one of it's value.

Note : Continued product of roots of a complex quantity should be determined using theory of equations.

8. Cube roots of unity

(a) The cube roots of unity are 1, $\omega = \frac{-1 + i\sqrt{3}}{2} = e^{i2\pi/3}$ & $\omega^2 = \frac{-1 - i\sqrt{3}}{2} = e^{i4\pi/3}$

(b) $1 + \omega^1 + \omega^2 = 0$, $\omega^3 = 1$, in genral $1 + \omega^r + \omega^{2r} = \begin{cases} 0 & r \text{ is not integral multiple of } 3 \\ 3 & r \text{ is multiple of } 3 \end{cases}$

(c) $a^2 + b^2 + c^2 - ab - bc - ca = (a + b\omega + c\omega^2)(a + b\omega^2 + c\omega)$

$a^3 + b^3 = (a + b)(a\omega + b\omega^2)(a\omega^2 + b\omega)$

$a^3 - b^3 = (a - b)(a - \omega b)(a - \omega^2 b)$

$x^2 + x + 1 = (x - \omega)(x - \omega^2)$

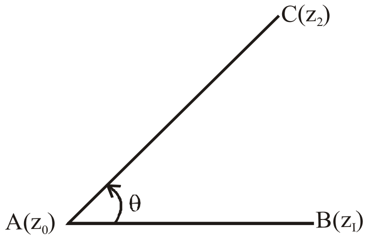
9. Square root of complex number

$\sqrt{a + ib} = \pm\left\{\frac{\sqrt{|z| + a}}{2} + i\frac{\sqrt{|z| - a}}{2}\right\}$ for $b > 0$ & $\pm\left\{\frac{\sqrt{|z| + a}}{2} - i\frac{\sqrt{|z| - a}}{2}\right\}$ for $b < 0$ where

$|z| = \sqrt{a^2 + b^2}$

10. Rotation :

$$\frac{z_2 - z_0}{|z_2 - z_0|} = \frac{z_1 - z_0}{|z_1 - z_0|}\, e^{i\theta}.$$



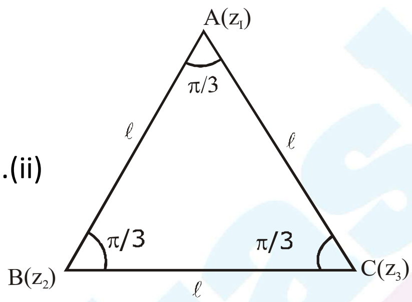
Take θ in anticlockwise direction

11. Results related to triangle :

(a) Equilateral triangle :

$$\frac{z_1 - z_2}{l} = \frac{z_3 - z_2}{l}\, e^{i\pi/3} \quad ....(i)$$

Also $\frac{z_2 - z_3}{l} = \frac{z_1 - z_3}{l}\, e^{i\pi/3}$ ....(ii)

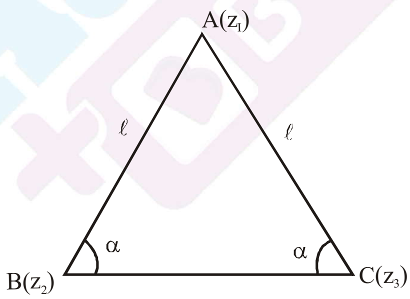


from (i) & (ii)

$z_1^2 + z_2^2 + z_3^2 = z_1z_2 + z_2z_3 + z_3z_1$. or $\frac{1}{z_1 - z_2} + \frac{1}{z_2 - z_3} + \frac{1}{z_3 - z_1} = 0$

(b) Isosceles triangle :

$4\cos^2\alpha\,(z_1 - z_2)(z_3 - z_1) = (z_3 - z_2)^2$.



(c) Area of Δ ABC = $\frac{1}{4}\begin{vmatrix} z_1 & \bar{z}_1 & 1 \\ z_2 & \bar{z}_2 & 1 \\ z_3 & \bar{z}_3 & 1 \end{vmatrix}$

12. Equation of line through points z_1 & z_2:

$$\begin{vmatrix} z & \bar{z} & 1 \\ z_1 & \bar{z}_1 & 1 \\ z_2 & \bar{z}_2 & 1 \end{vmatrix} = 0$$

$\Rightarrow z(\bar{z}_1 - \bar{z}_2) + \bar{z}\ (z_2 - z_1) + z_1\bar{z}_2 - \bar{z}_1 z_2 = 0$

$\Rightarrow z(\bar{z}_1 - \bar{z}_2)\, i + \bar{z}\ (z_2 - z_1)\, i + (z_1\bar{z}_2 - \bar{z}_1\ z_2)\, i = 0$

Let $(z_2 - z_1)i = a$, then equation of line is $\bar{a}\,z + a\,\bar{z} + b = 0$ where $a \in C$ & $b \in R$.

Note :

(i) Complex slope of line $\bar{a}\,z + a\,\bar{z} + b = 0$ is $= -\dfrac{a}{\bar{a}}$

(ii) Two lines with slope μ_1 & μ_2 are parallel or perpendicular if $\mu_1 = \mu_2$ or $\mu_1 + \mu_2 = 0$ respectively.

(iii) Lengh of perpendicular from point $A(\alpha)$ to line $\bar{a}\,z + a\,\bar{z} + b = 0$ is $\dfrac{|\bar{a}\alpha + a\bar{\alpha} + b|}{2|a|}$

13. Equation of circle:

(a) Circle whose centre is z_0 & radii $= r$ $|z - z_0| = r$

(b) General equation of circle

$z\bar{z} + a\bar{z} + \bar{a}z + b = 0$ (b is real)

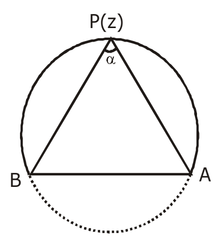
centre '$-a$'; & radii $= \sqrt{|a|^2 - b}$

(c) Diameter form $(z - z_1)(\bar{z} - \bar{z}_2) + (z - z_2)(\bar{z} - \bar{z}_1) = 0$

or $\arg\left(\dfrac{z - z_1}{z - z_2}\right) = \pm\dfrac{\pi}{2}$ (z_1 & z_2 are ends of diameter)

(d) Equation $\left|\dfrac{z - z_1}{z - z_2}\right| = k$ represent a circle if $k \neq 1$ and a straight line if $k = 1$.

(e) Equation $|z - z_1|^2 + |z - z_2|^2 = k$ represent circle if $k \geq \dfrac{1}{2}\ |z_1 - z_2|^2$.



(f) $\arg\left(\dfrac{z - z_1}{z - z_2}\right) = \alpha;\ 0 < \alpha < \pi,\ \alpha \neq \dfrac{\pi}{2}$ represent a segment of cirlce passing through $A(z_1)$ & $B(z_2)$

14. Standard loci:

(a) $|z-z_1| + |z-z_2| = 2k$ (a constant represent)

(i) if $2k > |z_1-z_2| \Rightarrow$ An ellipse

(ii) if $2k = |z_1-z_2| \Rightarrow$ An line segment

(iii) if $2k < |z_1-z_2| \Rightarrow$ No solution

(b) Equation $||z-z_1| - |z-z_2|| = 2k$ (a constant represent)

(i) if $2k < |z_1-z_2| \Rightarrow$ A hyperbola

(ii) if $2k = |z_1-z_2| \Rightarrow$ A line ray

(iii) $2k > |z_1-z_2| \Rightarrow$ No solution

MATHEMATICAL REASONING

1. Statement :

A sentence which is either true or false but cannot be both is called a statement. A sentence which is an exclamatory or a wish or an imperative or an interrogative can not be a statement.
If a statement is true then its truth value is T and if it is false then its truth value is F

2. Simple statement :

Any statement whose truth value does not depend on other statement is called simple statement

3. Compound statement :

A statement which is a combination of two or more simple statements are called compound statement
Here the simple statements which form a compound statement are known as its sub statements

4. Logical Connectives :

The words or phrases which combine simple statements to form a compound statement are called logical connectives.

S.N.	Connectives	symbol	use	operation
1.	and	$\wedge$	$p \wedge q$	conjunction
2.	or	$\vee$	$p \vee q$	disjunction
3.	not	$\sim$ or '	$\sim p$ or p'	negation
4.	If .... then	$\Rightarrow$ or $\rightarrow$	$p \Rightarrow q$ or $p \rightarrow q$	Implication or conditional
5.	If and only if (iff)	$\Leftrightarrow$ or $\leftrightarrow$	$p \Leftrightarrow q$ or $p \leftrightarrow q$	Equivalence or Bi-conditional

5. Truth Table :

Conjunction

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Negation

p	$(\sim p)$
T	F
F	T

Conditional

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Biconditional

p	q	$p \to q$	$q \to p$	$(p \to q) \wedge (q \to p)$ or $p \leftrightarrow q$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Note : If the compound statement contain n sub statements then its truth table will contain 2^n rows.

6. Logical Equivalence :

Two compound statements $S_1(p, q, r...)$ and $S_2(p, q, r ....)$ are said to be logically equivalent or simply equivalent if they have same truth values for all logically possibilities

Two statements S_1 and S_2 are equivalent if they have identical truth table i.e. the entries in the last column of their truth table are same. If statements S_1 and S_2 are equivalent then we write $S_1 \equiv S_2$

i.e. $\boxed{p \to q \equiv \sim p \vee q}$

7. Tautology and contradiction :

(i) Tautology : A statement is said to be a tautology if it is true for all logical possibilities. i.e. its truth value always T. it is denoted by t.

(ii) Contradiction : A statement is a contradiction if it is false for all logical possibilities. i.e. its truth value always F. It is denoted by C.

Note : The negation of a tautology is a contradiction and negation of a contradiction is a tautology

8. Duality:

Two compound statements S_1 and S_2 are said to duals of each other if one can be obtained from the other by replacing $\wedge$ by $\vee$ and $\vee$ by $\wedge$ if a compound statement contains the special variable t (tautology) and c (contradiction) then obtain its dual we replaced t by c and c by t in addition to replacing $\wedge$ by $\vee$ and $\vee$ by $\wedge$.

Note :

(i) the connectives $\wedge$ by $\vee$ are also called dual of each other.

(ii) If S^* (p,q) is the dual of the compound statement S(p,q) then

(a) $S^*(\sim p, \sim q) \equiv \sim S(p,q)$ (b) $\sim S^*(p,q) \equiv S(\sim p, \sim q)$

9. Converse, inverse and contrapositive of the conditional statement (p→q):

(i) Converse : The converse of the conditional statement $p \to q$ is defined as $q \to p$

(ii) Inverse : The inverse of the conditional statement $p \to q$ is defined as $\sim p \to \sim q$

(iii) Contrapositive : The contrapositive of conditional statement $p \to q$ is defiened as $\sim q \to \sim p$

10. Negation of compound statments:

if p and q are two statements then

(i) Negation of conjunction : $\sim (p \wedge q) \equiv \sim p \vee \sim q$

(ii) Negation of disjunction : $\sim (p \vee q) \equiv \sim p \wedge \sim q$

(iii) Negation of conditional : $\sim (p \to q) \equiv p \wedge \sim q$

(iv) Negation of biconditional : $\sim (p \leftrightarrow q) \equiv (p \wedge \sim q) \vee (\sim p \wedge q)$

we know that $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

$$\therefore \sim (p \leftrightarrow q) \equiv \sim [(p \to q) \wedge (q \to p)]$$

$$\equiv \sim (p \to q) \vee \sim (q \to p)$$

$$\equiv (p \wedge \sim q) \vee (q \wedge \sim p)$$

Note : The above result also can be proved by preparing truth table for

$\sim (p \leftrightarrow q)$ and $(p \wedge \sim q) \vee (q \wedge \sim p)$

11. Algebra of statements :

If p, q, r are any three statements then

(i) Idempotent Laws :

(a) $p \wedge p \equiv p$ (b) $p \vee p \equiv p$

(ii) Commutative laws :

(a) $p \wedge q \equiv q \wedge p$ (b) $p \vee q \equiv q \vee p$

(iii) Associative laws :

(a) $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

(b) $(p \vee q) \vee r \equiv p \vee (q \vee r)$

(iv) Distributive laws : (a) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

(b) $p \wedge (q \wedge r) \equiv (p \wedge q) \wedge (p \wedge r)$

(c) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

(d) $p \vee (q \vee r) \equiv (p \vee q) \vee (p \vee r)$

(v) De Morgan Laws : (a) $\sim (p \wedge q) \equiv \sim p \vee \sim q$

(b) $\sim (p \vee q) \equiv \sim p \wedge \sim q$

(vi) Involution laws (or Double negation laws) : $\sim (\sim p) \equiv p$

(vii) Identity Laws : If p is a statement and t and c are tautology and contradiction respectively then

(a) $p \wedge t \equiv p$ (b) $p \vee t \equiv t$ (c) $p \wedge c \equiv c$ (d) $p \vee c \equiv p$

(viii) Complement Laws :

(a) $p \wedge (\sim p) \equiv c$ (b) $p \vee (\sim p) \equiv t$ (c) $(\sim t) \equiv c$ (d) $(\sim c) \equiv t$

(ix) Contrapositive laws : $p \rightarrow q \equiv \sim q \rightarrow \sim p$

12. Quantified statements and quantifiers:

The words or phrases "All", "Some", "None". There exists a" are examples of quantifiers.

A statement containing one or more of these words (or phrases) is a quantified statement.

Note : Phrases "There exists a" and" Atleast one" and the word "some" have the same meaning.

Negation of Quantified Statements :

(1) 'None' is the negation of 'at least one or 'some' or 'few'

Similarly negation of 'some' is 'none'

(2) The equation of "some A are B" or "There exist A which is B" is "No A are (is) B" or "There does not exist any A which is B"

(3) Negation of "All A are B" is " Some A are not B".

STATISTICS

MEASURES OF CENTRAL TENDENCY

An average value or a central value of a distribution is the value of variable which is representative of the entire distribution, this representative value are called the measures of central tendency.

Generally the following five measures of central tendency.

(a) Mathematical average

(i) Arithmetic mean (ii) Geometric mean (iii) Harmonic mean

(b) Positional average

(i) Median (ii) Mode

1. Arithmetic mean :

(i) For ungrouped dist. : If $x_1, x_2, \ldots\ldots x_n$ are n values of variate x_i then their A.M. $\bar{x}$ is defined as

$$\bar{x} = \frac{x_1 + x_2 + \ldots.. + x_n}{n} = \frac{\sum_{i=1}^{n} x_i}{n}$$

$\Rightarrow \Sigma x_i = n\bar{x}$

(ii) For ungrouped and grouped freq. dist. : If $x_1, x_2, \ldots. x_n$ are values of variate with corresponding frequencies $f_1, f_2, \ldots f_n$ then their A.M. is given by

$$\bar{x} = \frac{f_1x_1 + f_2x_2 + \ldots. + f_nx_n}{f_1 + f_2 + \ldots. + f_n} = \frac{\sum_{i=1}^{n} f_ix_i}{N}, \quad \text{where } N = \sum_{i=1}^{n} f_i$$

(iii) By short method :

Let $d_i = x_i - a$

$\therefore \quad \bar{x} = a + \frac{\Sigma f_id_i}{N}$, where a is assumed mean

(iv) By step deviation method : Sometime during the application of short method of finding the A.M. If each deviation d_i are divisible by a common number h(let)

Let $u_i = \frac{d_i}{h} = \frac{x_i - a}{h}$

$\therefore \quad \bar{x} = a + \left(\frac{\Sigma f_iu_i}{N}\right)h$

(v) Weighted mean : If $w_1, w_2, \ldots\ldots w_n$ are the weights assigned to the values $x_1, x_2, \ldots\ldots x_n$ respectively then their weighted mean is defined as

$$\text{Weighted mean} = \frac{w_1x_1 + w_2x_2 + \ldots.. + w_nx_n}{w_1 + \ldots.. + w_n} = \frac{\sum_{i=1}^{n} w_ix_i}{\sum_{i=1}^{n} w_i}$$

(vi) Combined mean : If $\bar{x}_1$ and $\bar{x}_2$ be the means of two groups having n_1 and n_2 terms respectively then the mean (combined mean) of their composite group is given by

$$\text{combined mean} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$$

If there are more than two groups then, combined mean = $\frac{n_1\bar{x}_1 + n_1\bar{x}_2 + n_3\bar{x}_3 + \ldots.}{n_1 + n_2 + n_3 + \ldots.}$

(vii) Properties of Arithmetic mean :

- Sum of deviations of variate from their A.M. is always zero i.e. $\Sigma(x_i - \bar{x}) = 0$, $\Sigma f_i(x_i - \bar{x}) = 0$
- Sum of square of deviations of variate from their A.M. is minimum i.e. $\Sigma(x_i - \bar{x})^2$ is minimum
- If $\bar{x}$ is the mean of variate x_i then A.M. of $(x_i + \lambda) = \bar{x} + \lambda$
  A.M. of $(\lambda x_i) = \lambda\bar{x}$
  A.M. of $(ax_i + b) = a\bar{x} + b$

(where λ, a, b are constant)

- A.M. is independent of change of assumed mean i.e. it is not affected by any change in assumed mean.

2. Median :

The median of a series is the value of middle term of the series when the values are written in ascending order. Therefore median, divided an arranged series into two equal parts.

Formulae of median :

(i) For ungrouped distribution : Let n be the number of variate in a series then

$$\text{Median} = \begin{cases} \left(\frac{n+1}{2}\right)^{th} \text{term}, \text{ (when n is odd)} \\ \text{Mean of} \left(\frac{n}{2}\right)^{th} \text{and} \left(\frac{n}{2}+1\right)^{th} \text{terms}, \text{ (when n is even)} \end{cases}$$

(ii) For ungrouped freq. dist. : First we prepare the cumulative frequency (c.f.) column and Find value of N then

$$\text{Median} = \begin{cases} \left(\frac{N+1}{2}\right)^{th} \text{term, (when N is odd)} \\ \text{Mean of } \left(\frac{N}{2}\right)^{th} \text{ and } \left(\frac{N}{2}+1\right)^{th} \text{ terms, (when N is even)} \end{cases}$$

(iii) For grouped freq. dist : Prepare c.f. column and find value of $\frac{N}{2}$ then find the class which contain value of c.f. is equal or just greater to N/2, this is median class

$$\therefore \text{ Median} = \ell + \frac{\left(\frac{N}{2} - F\right)}{f} \times h$$

where
ℓ — lower limit of median class
f — freq. of median class
F — c.f. of the class preceeding median class
h — Class interval of median class

3. Mode :

In a frequency distribution the mode is the value of that variate which have the maximum frequency

Method for determining mode :

(i) For ungrouped dist. : The value of that variate which is repeated maximum number of times

(ii) For ungrouped freq. dist. : The value of that variate which have maximum frequency.

(iii) For grouped freq. dist. : First we find the class which have maximum frequency, this is model calss

$$\therefore \text{ Mode} = \ell + \frac{f_0 - f_1}{2f_0 - f_1 - f_2} \times h$$

where
ℓ — lower limit of model class
f_0 — freq. of the model class
f_1 — freq. of the class preceeding model class
f_2 — freq. of the class succeeding model class
h — class interval of model class

4. Relation between Mean, Median and Mode :

In a moderately asymmetric distribution following relation between mean, median and mode of a distribution. It is known as empirical formula.

Mode = 3 Median – 2 Mean

Note (i) Median always lies between mean and mode

(ii) For a symmetric distribution the mean, median and mode coincide.

5. Measures of dispersion :

The dispersion of a statistical distribution is the measure of deviation of its values about their average (central) value.

It gives an idea of scatteredness of different values from the average value.

Generally the following measures of dispersion are commonly used.

(i) Range (ii) Mean deviation

(iii) Variance and standard deviation

(i) Range : The difference between the greatest and least values of variate of a distribution, is called the range of that distribution.

If the distribution is grouped distribution, then its range is the difference between upper limit of the maximum class and lower limit of the minimum class.

Also, coefficient of range = $\dfrac{\text{difference of extreme values}}{\text{sum of extreme values}}$

(ii) Mean deviation (M.D.) : The mean deviation of a distribution is, the mean of absolute value of deviations of variate from their statistical average (Mean, Median, Mode).

If A is any statistical average of a distribution then mean deviation about A is defined as

Mean deviation = $\dfrac{\sum_{i=1}^{n} |x_i - A|}{n}$ (for ungrouped dist.)

Mean deviation = $\dfrac{\sum_{i=1}^{n} f_i |x_i - A|}{N}$ (for freq. dist.)

Note :- Mean deviation is minimum when it taken about the median

Coefficient of Mean devation = $\dfrac{\text{Mean deviation}}{A}$

(where A is the central tendency about which Mean deviation is taken)

(iii) Variance and standard deviation : The variance of a distribution is, the mean of squares of deviation of variate from their mean. It is denoted by σ^2 or var(x).

The positive square root of the variance are called the standard deviation. It is denoted by σ or S.D.

Hence standard deviation = $+\sqrt{\text{variance}}$

Formulae for variance :

(i) for ungrouped dist. :

$$\sigma_x^2 = \frac{\Sigma(x_i - \bar{x})^2}{n}$$

$$\sigma_x^2 = \frac{\Sigma x_i^2}{n} - \bar{x}^2 = \frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2$$

$$\sigma_d^2 = \frac{\Sigma d_i^2}{n} - \left(\frac{\Sigma d_i}{n}\right)^2, \quad \text{where } d_i = x_i - a$$

(ii) For freq. dist. :

$$\sigma_x^2 = \frac{\Sigma f_i(x_i - \bar{x})^2}{N}$$

$$\sigma_x^2 = \frac{\Sigma f_i x_i^2}{N} - (\bar{x})^2 = \frac{\Sigma f_i x_i^2}{N} - \left(\frac{\Sigma f_i x_i}{N}\right)^2$$

$$\sigma_d^2 = \frac{\Sigma f_i d_i^2}{N} - \left(\frac{\Sigma f_i d_i}{N}\right)^2$$

$$\sigma_u^2 = h^2\left[\frac{\Sigma f_i u_i^2}{N} - \left(\frac{\Sigma f_i u_i}{N}\right)^2\right] \quad \text{where } u_i = \frac{d_i}{h}$$

(iii) Coefficient of S.D. = $\frac{\sigma}{\bar{x}}$

Coefficient of variation = $\frac{\sigma}{\bar{x}} \times 100$ (in percentage)

Note :- $\sigma^2 = \sigma_x^2 = \sigma_d^2 = h^2 \sigma_u^2$

6. Mean square deviation :

The mean square deviation of a distrubution is the mean of the square of deviations of variate from assumed mean. It is denoted by S^2

Hence $S^2 = \frac{\Sigma(x_i - a)^2}{n} = \frac{\Sigma d_i^2}{n}$ (for ungrouped dist.)

$S^2 = \frac{\Sigma f_i(x_i - a)^2}{N} = \frac{\Sigma f_i d_i^2}{N}$ (for freq. dist.), where $d_i = (x_i - a)$

7. Relation between variance and mean square deviation :

$\because \quad \sigma^2 = \frac{\Sigma f_i d_i^2}{N} - \left(\frac{\Sigma f_i d_i}{N}\right)^2$

$\Rightarrow \quad \sigma^2 = s^2 - d^2, \qquad$ where $d = \bar{x} - a = \frac{\Sigma f_i d_i}{N}$

$\Rightarrow \quad s^2 = \sigma^2 + d^2 \Rightarrow s^2 \geq \sigma^2$

Hence the variance is the minimum value of mean square deviation of a distribution

8. Mathematical properties of variance:

Var.$(x_1+\lambda)$ = Var.(x_1)
Var.(λx_1) = λ^2Var.(x_1)
Var $(ax_1 + b) = a^2$. Var (x_1)
where λ, a,b, are constant

If means of two series containing n_1, n_2 terms are $\bar{X}_1, \bar{X}_2$ and their variance's are σ_1^2, σ_2^2 respectively and their combined mean is $\bar{X}$ then the variance σ^2 of their combined series is given by following formula

$$\sigma^2 = \frac{n_1(\sigma_1^2 + d_1^2) + n_2(\sigma_2^2 + d_2^2)}{(n_1 + n_2)} \quad \text{where } d_1 = \bar{X}_1 - \bar{X},\ d_2 = \bar{X}_2 - \bar{X}$$

i.e. $\sigma^2 = \frac{n_1\sigma_1^2 + n_2\sigma_2^2}{n_1 + n_2} + \frac{n_1 n_2}{(n_1 + n_2)^2}\left(\bar{X}_1 - \bar{X}_2\right)^2$