

1. Let  $S_1$ ,  $S_2$  and  $S_3$  be three sets defined as

$$S_1 = \{z \in \mathbb{C} : |z - 1| \leq \sqrt{2}\}$$

$$S_2 = \{z \in \mathbb{C} : \operatorname{Re}((1 - i)z) \geq 1\}$$

$$S_3 = \{z \in \mathbb{C} : \operatorname{Im}(z) \leq 1\}$$

Then the set  $S_1 \cap S_2 \cap S_3$

- A. Has infinitely many elements
  - B. Has exactly two elements
  - C. Has exactly three elements
  - D. Is a singleton
2. Let a complex number be  $w = 1 - \sqrt{3}i$ . Let another complex number  $z$  be such that  $|zw| = 1$  and  $\arg(z) - \arg(w) = \frac{\pi}{2}$ . Then the area of the triangle with vertices origin,  $z$  and  $w$  is equal to:

A.  $\frac{1}{2}$

B. 4

C. 2

D.  $\frac{1}{4}$

3. The area of the triangle with vertices  $A(z)$ ,  $B(iz)$  and  $C(z + iz)$  is:

A.  $\frac{1}{2}|z + iz|^2$

B. 1

C.  $\frac{1}{2}$

D.  $\frac{1}{2}|z|^2$

4. If the equation  $a|z|^2 + \overline{\alpha z + \alpha \bar{z}} + d = 0$  represents a circle where  $a, d$  are real constants, then which of the following condition is correct?

- A.  $|\alpha|^2 - ad \neq 0$
- B.  $|\alpha|^2 - ad > 0$  and  $a \in \mathbb{R} - \{0\}$
- C.  $\alpha = 0, a, d \in \mathbb{R}^+$
- D.  $|\alpha|^2 - ad \geq 0$  and  $a \in \mathbb{R}$

5. If the equation,  $x^2 + bx + 45 = 0$  ( $b \in \mathbb{R}$ ) has conjugate complex roots and they satisfy  $|z + 1| = 2\sqrt{10}$ , then:

- A.  $b^2 + b = 12$
- B.  $b^2 - b = 42$
- C.  $b^2 - b = 30$
- D.  $b^2 + b = 72$

6. If  $z$  and  $\omega$  are two complex numbers such that  $|z\omega| = 1$  and  $\arg(z) - \arg(\omega) = \frac{3\pi}{2}$ , then  $\arg\left(\frac{1 - 2\bar{z}\omega}{1 + 3\bar{z}\omega}\right)$  is  
(Here  $\arg(z)$  denotes the principal argument of complex number  $z$ )

- A.  $\frac{3\pi}{4}$
- B.  $-\frac{3\pi}{4}$
- C.  $\frac{\pi}{4}$
- D.  $-\frac{\pi}{4}$

7. The region represented by  $z = x + iy \in \mathbb{C} : |z| - \operatorname{Re}(z) \leq 1$  is also given by the inequality:

**A.**  $y^2 \leq 2 \left( x + \frac{1}{2} \right)$

**B.**  $y^2 \leq x + \frac{1}{2}$

**C.**  $y^2 \leq 2(x + 1)$

**D.**  $y^2 \leq x + 1$

8. Which of the following boolean expression is a tautology?

**A.**  $(p \wedge q) \wedge (p \rightarrow q)$

**B.**  $(p \wedge q) \vee (p \vee q)$

**C.**  $(p \wedge q) \vee (p \rightarrow q)$

**D.**  $(p \wedge q) \rightarrow (p \rightarrow q)$

9. The statement among the following that is a tautology is:

**A.**  $A \wedge (A \vee B)$

**B.**  $B \rightarrow [A \wedge (A \rightarrow B)]$

**C.**  $A \vee (A \wedge B)$

**D.**  $[A \vee (A \rightarrow B)] \rightarrow B$

10. Let  $F_1(A, B, C) = (A \wedge \sim B) \vee [\sim C \wedge (A \vee B)] \vee \sim A$  and  $F_2(A, B) = (A \vee B) \vee (B \rightarrow \sim A)$  be two logical expressions. Then:

**A.**  $F_1$  is not a tautology but  $F_2$  is a tautology

**B.**  $F_1$  is a tautology but  $F_2$  is not a tautology

**C.**  $F_1$  and  $F_2$  both are tautologies

**D.** Both  $F_1$  and  $F_2$  are not tautologies

11. The statement  $A \rightarrow (B \rightarrow A)$  is equivalent to:

- A.**  $A \rightarrow (A \wedge B)$
- B.**  $A \rightarrow (A \vee B)$
- C.**  $A \rightarrow (A \rightarrow B)$
- D.**  $A \rightarrow (A \leftrightarrow B)$

12. The negation of the statement  $\sim p \wedge (p \vee q)$  is

- A.**  $\sim p \wedge q$
- B.**  $p \wedge \sim q$
- C.**  $\sim p \vee q$
- D.**  $p \vee \sim q$

13. The Boolean expression  $(p \wedge \sim q) \Rightarrow (q \vee \sim p)$  is equivalent to

- A.**  $q \Rightarrow p$
- B.**  $p \Rightarrow q$
- C.**  $p \Rightarrow \sim q$
- D.**  $\sim q \Rightarrow p$

14. If the truth value of the Boolean expression

$((p \vee q) \wedge (q \rightarrow r) \wedge (\sim r)) \rightarrow (p \wedge q)$  is false, then the truth values of the statements  $p, q, r$  respectively can be

- A.**  $F F T$
- B.**  $F T F$
- C.**  $T F F$
- D.**  $T F T$

15. Consider the statement "The match will be played only if the weather is good and ground is not wet". Select the correct negation from the following
- A.** The match will not be played or weather is good and ground is not wet
  - B.** The match will be played and weather is not good or ground is wet
  - C.** The match will not be played and weather is not good and ground is wet
  - D.** If the match will not be played, then either weather is not good or ground is wet
16. Consider the following three statements
- (A) If  $3 + 3 = 7$ , then  $4 + 3 = 8$ .
  - (B) If  $5 + 3 = 8$ , then earth is flat.
  - (C) If both (A) and (B) are true, then  $5 + 6 = 17$ .
- Then, which of the following statements is correct:
- A.** (A) and (B) are false while (C) is true
  - B.** (A) is false, but (B) and (C) are true
  - C.** (A) and (C) are true while (B) is false
  - D.** (A) is true while (B) and (C) are false
17. The mean and standard deviation of 20 observations were calculated as 10 and 2.5 respectively. It was found that by mistake one data value was taken as 25 instead of 35. If  $\alpha$  and  $\sqrt{\beta}$  are the mean and standard deviation respectively for correct data, then  $(\alpha, \beta)$  is
- A.** (11, 25)
  - B.** (11, 26)
  - C.** (10.5, 25)
  - D.** (10.5, 26)

18. Let the mean and variance of the frequency distribution

$$x : \quad x_1 = 2 \quad x_2 = 6 \quad x_3 = 8 \quad x_4 = 9$$

$$f : \quad 4 \quad 4 \quad \alpha \quad \beta$$

be 6 and 6.8 respectively. If  $x_3$  is changed from 8 to 7, then the mean for the new data will be

- A. 5
  - B. 4
  - C.  $\frac{17}{3}$
  - D.  $\frac{16}{3}$
19. The first of the two samples in a group has 100 items with mean 15 and standard deviation 3. If the whole group has 250 items with mean 15.6 and standard deviation  $\sqrt{13.44}$ , then the standard deviation of the second sample is
- A. 5
  - B. 6
  - C. 4
  - D. 8
20. If the mean and variance of the following data :

6, 10, 7, 13,  $a$ , 12,  $b$ , 12 are 9 and  $\frac{37}{4}$  respectively, then  $(a - b)^2$  is equal to

- A. 32
- B. 12
- C. 24
- D. 16

21. The mean and standard deviation (s.d.) of 10 observations are 20 and 2 respectively. Each of these 10 observations is multiplied by  $p$  and then reduced by  $q$ , where  $p \neq 0$  and  $q \neq 0$ . If the new mean and standard deviation become half of their original values, then  $q$  is equal to:
- A.  $-20$
  - B.  $-5$
  - C.  $10$
  - D.  $-10$
22. The mean and variance of 20 observations are found to be 10 and 4, respectively. On rechecking, it was found that an observation 9 was incorrect and the correct observation was 11. Then the correct variance is :
- A. 4.01
  - B. 3.99
  - C. 3.98
  - D. 4.02

1. If  $\left(\frac{1+i}{1-i}\right)^{m/2} = \left(\frac{1+i}{i-1}\right)^{n/3} = 1$ ,  $(m, n \in \mathbb{N})$ , then the greatest common divisor of the least values of  $m$  and  $n$  is
  
2. If the least and the largest real values of  $\alpha$ , for which the equation  $z + \alpha|z - 1| + 2i = 0$  ( $z \in \mathbb{C}$  and  $i = \sqrt{-1}$ ) has a solution, are  $p$  and  $q$  respectively; then  $4(p^2 + q^2)$  is equal to
  
3. Let  $z$  and  $w$  be two complex numbers such that  $w = z\bar{z} - 2z + 2$ ,  $\left|\frac{z+i}{z-3i}\right| = 1$  and  $\text{Re}(w)$  has minimum value. Then the minimum value of  $n \in \mathbb{N}$  for which  $w^n$  is real, is equal to
  
4. The sum of  $162^{\text{th}}$  power of the roots of the equation  $x^3 - 2x^2 + 2x - 1 = 0$  is
  
5. If the variance of 10 natural numbers  $1, 1, 1, \dots, 1, k$  is less than 10, then the maximum possible value of  $k$  is
  
6. Consider the statistics of two sets of observations as follows :
 

	Size	Mean	Variance
Observation I	10	2	2
Observation II	$n$	3	1

If the variance of the combined set of these two observations is  $\frac{17}{9}$ , then the value of  $n$  is equal to
  
7. Let the mean and variance of four numbers  $3, 7, x$  and  $y$  ( $x > y$ ) be 5 and 10 respectively. Then the mean of four numbers  $3 + 2x, 7 + 2y, x + y$  and  $x - y$  is
  
8. let  $n$  be an odd natural number such that the variance of  $1, 2, 3, 4, \dots, n$  is 14. Then  $n$  is equal to