JEE Main Part Test 2

Subject: Mathematics

1. Let P be any point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ such that the absolute difference of the distances of P from the two foci is 12. If the eccentricity of the hyperbola is 2, then the length of the latus rectum is

   A. $4\sqrt{3}$ unit

   B. 18 unit

   C. $2\sqrt{3}$ unit

   D. 36 unit

2. A rod of length l moves such that its ends A and B always lie on the lines $3x - y + 5 = 0$ and $y + 5 = 0$ respectively. The locus of the point P, which divides AB internally in the ratio $2:1$, is $l^2 = \frac{1}{k}(ax - by - 5)^2 + 9(y+5)^2$. Then

   A. $k = 4,\ a + b = 6$

   B. $k = 3,\ a + b = 5$

   C. $k = 4,\ a + b = 0$

   D. $k = 3,\ a + b = 4$

3. The number of non-negative integral values of b for which the origin and point $(1,1)$ lie on the same side of straight line $a^2x + aby + 1 = 0, \forall\, a \in \mathbb{R} - \{0\}$, is

   A. 1

   B. 3

   C. 2

   D. 5

4. Let from any point P on the line $y = x$, two tangents are drawn to the circle $(x-2)^2 + y^2 = 1$. Then the chord of contact of P with respect to given circle always passes through a fixed point, whose coordinates are given by

A. $\left(\frac{3}{2}, \frac{1}{4}\right)$

B. $\left(-\frac{3}{2}, \frac{1}{4}\right)$

C. $\left(-\frac{3}{2}, \frac{1}{2}\right)$

D. $\left(\frac{3}{2}, \frac{1}{2}\right)$

5. The line $4x + 3y - 4 = 0$ divides the circumference of the circle centred at $(5, 3)$, in the ratio $1 : 2$. Then the equation of the circle is

A. $x^2 + y^2 - 10x - 6y - 66 = 0$

B. $x^2 + y^2 - 10x - 6y + 100 = 0$

C. $x^2 + y^2 - 10x - 6y + 66 = 0$

D. $x^2 + y^2 - 10x - 6y - 100 = 0$

6. From the point $P(2, 1)$, a line of slope $m \in \mathbb{R}$ is drawn so as to cut the circle $x^2 + y^2 = 1$ in points A and B. If the slope m is varied, then the greatest possible value of $PA + PB$ is

A. $\frac{2}{\sqrt{5}}$

B. $\frac{10}{\sqrt{5}}$

C. $2\sqrt{5}$

D. $\frac{1}{\sqrt{5}}$

7. The locus of feet of perpendiculars drawn from the origin to the straight lines passing through $(2, 1)$ is

A. $x^2 + y^2 - 5y = 0$

B. $x^2 + y^2 - 2x - y = 0$

C. $2x + y - 5 = 0$

D. $x^2 + y^2 + 2x + y = 0$

8. $(2, 3)$ is a point on the side AB of $\triangle ABC$. The third vertex C moves such that the sides AC, BC are bisected by $x^2 - y^2 = 0$ at right angles. Then C lies on

A. $2x - 3y = 0$

B. $3x - 2y = 0$

C. $2x + 3y = 0$

D. $3x + 2y = 0$

9. Tangents are drawn to the ellipse $\frac{x^2}{36} + \frac{y^2}{9} = 1$ from any point on the parabola $y^2 = 4x$. The corresponding chord of contact will touch a parabola, whose equation is

A. $y^2 + 4x = 0$

B. $y^2 - 4x = 0$

C. $4y^2 + 9x = 0$

D. $y^2 + 9x = 0$

10. If z_1, z_2, z_3 are the solutions of $z^2 + \overline{z} = z$, then $z_1 + z_2 + z_3$ is equal to (z is a complex number on the Argand plane and $i = \sqrt{-1}$)

A. $2 + 2i$

B. $2 - 2i$

C. 0

D. 2

11. If the locus of the middle point of chords of an ellipse $\frac{x^2}{3} + \frac{y^2}{4} = 1$ passing through $(2, 0)$ is another ellipse A, then the length of latus rectum of the ellipse A is

A. $\frac{8}{3}$

B. $\sqrt{3}$

C. $\frac{1}{\sqrt{3}}$

D. $\frac{3}{8}$

12. If z is a complex number, not purely real such that imaginary part of $z - 1 + \frac{1}{z-1}$ is zero, then locus of z is

A. a straight line parallel to x-axis

B. a circle of radius 1 unit

C. a parabola with axis of symmetry parallel to x-axis

D. a hyperbola

13. Let z be an imaginary complex number satisfying $|z-1|=1$. If $\alpha=2z$, $\beta=2\alpha$ and $\gamma=2\beta$, then the value of $|z|^2+|\alpha|^2+|\beta|^2+|\gamma|^2+|z-2|^2+|\alpha-4|^2+|\beta-8|^2+|\gamma-16|^2$ is

A. 100

B. 320

C. 340

D. 400

14. The logical statement $[(p\wedge q)\to p]\to(q\wedge\sim q)$ is

A. a tautology

B. a contradiction

C. equivalent to $p\vee q$

D. neither a tautology nor a contradiction

15. An ellipse has eccentricity $\frac{1}{2}$ and one focus is at the point $P\left(\frac{1}{2},1\right)$. If the common tangent to the circle $x^2+y^2=1$ and hyperbola $x^2-y^2=1$ which is nearer to point P is directrix of the given ellipse, then the co-ordinates of centre of ellipse are

A. $\left(\frac{1}{3},\frac{1}{3}\right)$

B. $\left(\frac{2}{3},1\right)$

C. $\left(\frac{1}{3},1\right)$

D. $\left(1,\frac{1}{3}\right)$

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16. Complex numbers z_1, z_2, z_3 are the vertices A, B, C respectively, of an isosceles right-angled triangle with right angle at C. Then which of the following is true?

A. $(z_1 - z_2)^2 = (z_1 - z_3)(z_3 - z_2)$.

B. $(z_1 - z_2)^2 = 2(z_1 - z_3)(z_3 - z_2)$.

C. $(z_1 - z_2)^2 = 3(z_1 - z_3)(z_3 - z_2)$.

D. $(z_1 - z_2)^2 = 4(z_1 - z_3)(z_3 - z_2)$.

17. The statement $p \to (q \to p)$ is logically equivalent to

A. $p \to (p \to q)$

B. $p \to (q \vee p)$

C. $p \to (q \wedge p)$

D. $p \to (p \leftrightarrow q)$

18. Let PQ be a focal chord of parabola $y^2 = x$. If the coordinates of P is $(4, -2)$, then the slope of the tangent at Q is

A. 8

B. -4

C. $\frac{1}{8}$

D. 4

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19. For all real permissible values of m, if the straight line $y = mx + \sqrt{9m^2 - 4}$ is tangent to a hyperbola, then equation of the hyperbola can be

A. $9x^2 - 4y^2 = 64$

B. $4x^2 - 9y^2 = 64$

C. $9x^2 - 4y^2 = 36$

D. $4x^2 - 9y^2 = 36$

20. Let z be a complex number such that $|z - 2 + i| \leq 2$. If m and M denote the least and the greatest value of $|z|$ respectively, then the value of $m^2 + M^2$ is

A. 18

B. 9

C. $8\sqrt{5}$

D. $4\sqrt{5}$

21. If the coordinates of the foot of the perpendicular drawn from the point $(1, -2)$ on the line $y = 2x + 1$ is (α, β), then the value of $|\alpha + \beta|$ is

22. If $|z_1| = |z_2|$ and $\arg\left(\frac{z_1}{z_2}\right) = \pi$, then value of $z_1 + z_2$ is

23. If $y = x$ be the tangent to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ at ponit P such that the distance of P from origin is $4\sqrt{2}$, then the value of c is

24. The average marks of 10 students in a class was 60 with a standard deviation of 4, while the average marks of other ten students was 40 with a standard deviation of 6. If all the 20 students are taken together and σ is the combined standard deviation, then the value of $[\sigma]$ is
($[\cdot]$ represents the greatest integer function)

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25. If z is any complex number satisfying $|z-3-2i| \leq 2$, then the minimum value of $|2z-6+5i|$ is

26. If d_1 and d_2 are the longest and the shortest distances of the point $P(-7,2)$ from the circle $x^2+y^2-10x-14y-51=0$, then the value of $d_1^2+d_2^2$ is

27. The line $x+2y=36$ is normal to the parabola $x^2=12y$ at the point whose distance from the focus of the parabola is

28. Let P be a variable point on the ellipse $\frac{x^2}{100}+\frac{y^2}{64}=1$ with foci F_1 and F_2. If A is the area of triangle PF_1F_2, then the maximum possible value of A is

29. The minimum value of $f(x)=|x-6|+|x+3|+|x-8|+|x+4|+|x-3|,\ x \in \mathbb{R}$ is

30. If the line $y=mx+a$ meets the parabola $y^2=4ax$ at two points whose abscissa are x_1 and x_2, then the value of m for which $x_1+x_2=0$ is