JEE Main Part Test 2

Subject: Mathematics

1. Let P be any point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ such that the absolute difference of the distances of P from the two foci is 12. If the eccentricity of the hyperbola is 2, then the length of the latus rectum is

A. $4\sqrt{3}$ unit

B. 18 unit

C. $2\sqrt{3}$ unit

D. 36 unit

Absolute difference of the distances of P from the two foci is equal to the length of transverse axis.
$\Rightarrow 2a = 12$
$\Rightarrow a = 6$

Given, $e = 2$
$\Rightarrow \sqrt{1 + \frac{b^2}{a^2}} = 2$
$\Rightarrow b^2 = 108$
Length of latus rectum $= \frac{2b^2}{a} = 36$

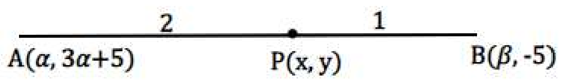
2. A rod of length l moves such that its ends A and B always lie on the lines $3x - y + 5 = 0$ and $y + 5 = 0$ respectively. The locus of the point P, which divides AB internally in the ratio $2 : 1$, is $l^2 = \frac{1}{k}(ax - by - 5)^2 + 9(y + 5)^2$. Then

A. $k = 4,\ a + b = 6$

B. $k = 3,\ a + b = 5$

C. $k = 4,\ a + b = 0$

D. $k = 3,\ a + b = 4$



By internal section formula,
$x = \frac{2\beta + \alpha}{3},\ y = \frac{-10 + 3\alpha + 5}{3}$
$\Rightarrow \alpha = \frac{3y + 5}{3}$ and $\beta = \frac{9x - 3y - 5}{6}$

Also, $l^2 = AB^2 = (\alpha - \beta)^2 + (3\alpha + 10)^2$
$\Rightarrow l^2 = \frac{1}{4}(3x - 3y - 5)^2 + 9(y + 5)^2$

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3. The number of non-negative integral values of b for which the origin and point $(1, 1)$ lie on the same side of straight line $a^2x + aby + 1 = 0, \forall\, a \in \mathbb{R} - \{0\}$, is

A. 1

B. 3

C. 2

D. 5

Putting $(0, 0)$ in $a^2x + aby + 1$, we get $1 > 0$
$(1, 1)$ and origin should lie on same side of $a^2x + aby + 1 = 0$
$\Rightarrow 1 \times a^2 + ab + 1 > 0$
$\Rightarrow a^2 + ab + 1 > 0$

For all values of a except 0, $a^2 + ab + 1 > 0$
$\Rightarrow D < 0$
$\Rightarrow b^2 - 4 < 0$
$\Rightarrow -2 < b < 2$
So, possible non-negative integral values of b are $0, 1$

4. Let from any point P on the line $y = x$, two tangents are drawn to the circle $(x-2)^2 + y^2 = 1$. Then the chord of contact of P with respect to given circle always passes through a fixed point, whose coordinates are given by

A. $\left(\frac{3}{2}, \frac{1}{4}\right)$

B. $\left(-\frac{3}{2}, \frac{1}{4}\right)$

C. $\left(-\frac{3}{2}, \frac{1}{2}\right)$

D. $\left(\frac{3}{2}, \frac{1}{2}\right)$

Given equation of the circle :
$(x-2)^2 + y^2 = 1$
$\Rightarrow x^2 + y^2 - 4x + 3 = 0$

Let (a, a) be any point on line $y = x$.
Chord of contact of this point w.r.t. circle is
$ax + ay - \frac{4}{2}(x + a) + 3 = 0$
$\Rightarrow a(x + y - 2) + (3 - 2x) = 0$
which always passes through the intersection of the lines $x + y - 2 = 0$ and $3 - 2x = 0$, which is given by $\left(\frac{3}{2}, \frac{1}{2}\right)$

5. The line $4x+3y-4=0$ divides the circumference of the circle centred at $(5,3)$, in the ratio $1:2$. Then the equation of the circle is

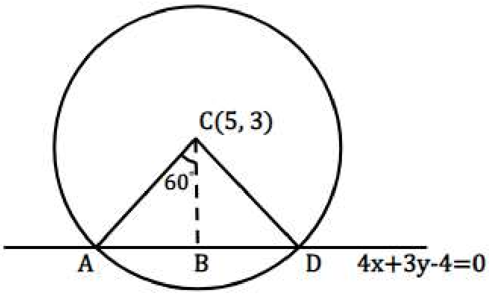
A. $x^2+y^2-10x-6y-66=0$

B. $x^2+y^2-10x-6y+100=0$

C. $x^2+y^2-10x-6y+66=0$

D. $x^2+y^2-10x-6y-100=0$

Let line $4x+3y-4=0$ make an angle α at the centre.
Then, $\alpha+2\alpha=360^\circ$
$\Rightarrow \alpha=120^\circ$
$\therefore \angle ACB=60^\circ$



Perpendicular distance of $(5,3)$ to line $4x+3y-4=0$ is
$BC=\left|\dfrac{4\times5+3\times3-4}{\sqrt{25}}\right|$
$\Rightarrow BC=5$
Now, $\cos 60^\circ=\dfrac{BC}{AC}$
$\therefore r=AC=10$
Hence, equation of the circle is
$(x-5)^2+(y-3)^2=10^2$
or, $x^2+y^2-10x-6y-66=0$

6. From the point $P(2,1)$, a line of slope $m\in\mathbb{R}$ is drawn so as to cut the circle $x^2+y^2=1$ in points A and B. If the slope m is varied, then the greatest possible value of $PA+PB$ is

A. $\dfrac{2}{\sqrt5}$

B. $\dfrac{10}{\sqrt5}$

C. $2\sqrt5$

D. $\dfrac{1}{\sqrt5}$

Using parametric form,
let $(2+r\cos\theta, 1+r\sin\theta)$ be any point on line through P having slope $m=\tan\theta$.
This point lies on the circle $x^2+y^2=1$
$\Rightarrow (2+r\cos\theta)^2+(1+r\sin\theta)^2=1$
We get quadratic equation $r^2+r(4\cos\theta+2\sin\theta)+4=0$
$\therefore PA+PB=|-4\cos\theta-2\sin\theta|\le 2\sqrt5$

7. The locus of feet of perpendiculars drawn from the origin to the straight lines passing through $(2, 1)$ is

A. $x^2 + y^2 - 5y = 0$

B. $x^2 + y^2 - 2x - y = 0$

C. $2x + y - 5 = 0$

D. $x^2 + y^2 + 2x + y = 0$

Let coordinates of the feet be (x, y).

Slope of the perpendiculars from the origin $= \dfrac{y-0}{x-0}$

Slope of the line passing through $(2, 1)$ and feet of the perpendiculars from the origin $= \dfrac{y-1}{x-2}$

Since the lines are perpendicular,

$\left(\dfrac{y-0}{x-0}\right)\left(\dfrac{y-1}{x-2}\right) = -1$

$\Rightarrow x^2 + y^2 - 2x - y = 0$

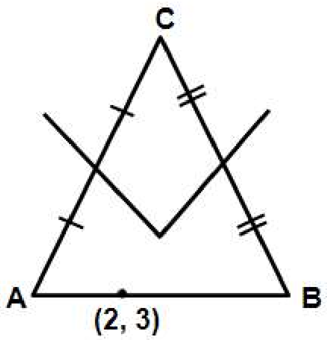
8. $(2, 3)$ is a point on the side AB of $\triangle ABC$. The third vertex C moves such that the sides AC, BC are bisected by $x^2 - y^2 = 0$ at right angles. Then C lies on

A. $2x - 3y = 0$

B. $3x - 2y = 0$

C. $2x + 3y = 0$

D. $3x + 2y = 0$



$x^2 - y^2 = 0$

$\Rightarrow (x-y)(x+y) = 0$

$\Rightarrow x - y = 0,\ x + y = 0$

Let $C \equiv (h, k)$

Then A, B are the images of C in $x - y = 0, x + y = 0$

$\therefore A \equiv (k, h), B \equiv (-k, -h)$

Slope of AB is $\dfrac{h}{k} = \dfrac{h-3}{k-2}$

$\Rightarrow 2h - 3k = 0$

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9. Tangents are drawn to the ellipse $\frac{x^2}{36}+\frac{y^2}{9}=1$ from any point on the parabola $y^2=4x$. The corresponding chord of contact will touch a parabola, whose equation is

A. $y^2+4x=0$

B. $y^2-4x=0$

C. $4y^2+9x=0$

D. $y^2+9x=0$

Let the tangent be drawn from $(t^2, 2t)$
We have, equation of ellipse as $\frac{x^2}{36}+\frac{y^2}{9}=1$
Chord of contact from an outside point to an ellipse is
$T=0$
$\Rightarrow \frac{xt^2}{36}+\frac{2yt}{9}=1$
$\Rightarrow xt^2+8yt-36=0$
Since the above equation touches a parabola,
$D=0$
$\Rightarrow 64y^2+4\times 36\times x=0$
$\therefore 4y^2+9x=0$

10. If z_1, z_2, z_3 are the solutions of $z^2+\bar{z}=z$, then $z_1+z_2+z_3$ is equal to
(z is a complex number on the Argand plane and $i=\sqrt{-1}$)

A. $2+2i$

B. $2-2i$

C. 0

D. 2

$z^2+\bar{z}=z$
Put $z=x+iy$
$(x+iy)^2+x-iy=x+iy$
$\Rightarrow x^2-y^2+i2xy+x-iy-x-iy=0$
$\Rightarrow x^2-y^2+i(2xy-2y)=0+0\cdot i$
Comparing real and imaginary parts, we get
$x^2-y^2=0$ and $2y(x-1)=0$
$\Rightarrow y=\pm x \quad \cdots(1)$
and $2y(x-1)=0$
If $y=0$, then $x=0 \quad$ [From (1)]
If $x-1=0$, then $y=\pm 1$
$\therefore z_1=0, z_2=1+i, z_3=1-i$
$z_1+z_2+z_3=0+1+i+1-i$
$\Rightarrow z_1+z_2+z_3=2$

11. If the locus of the middle point of chords of an ellipse $\frac{x^2}{3}+\frac{y^2}{4}=1$ passing through $(2,0)$ is another ellipse A, then the length of latus rectum of the ellipse A is

A. $\frac{8}{3}$

B. $\sqrt{3}$

C. $\frac{1}{\sqrt{3}}$

D. $\frac{3}{8}$

Given ellipse is $\frac{x^2}{3}+\frac{y^2}{4}=1$
Let the middle point of chord be (h,k)
Then equation of chord is $T=S_1$
$\Rightarrow \frac{xh}{3}+\frac{yk}{4}=\frac{h^2}{3}+\frac{k^2}{4}$
As chord passes through $(2,0)$,
$\frac{2h}{3}=\frac{h^2}{3}+\frac{k^2}{4}$
$\Rightarrow \frac{(h-1)^2}{1}+\frac{k^2}{4/3}=1$
Hence, equation of the ellipse A is
$\frac{(x-1)^2}{1}+\frac{y^2}{4/3}=1$
Here, $a=1, b=\frac{2}{\sqrt{3}}, b>a$
Length of latus rectum $=\frac{2a^2}{b}=\frac{2\times 1}{2/\sqrt{3}}=\sqrt{3}$

12. If z is a complex number, not purely real such that imaginary part of $z-1+\dfrac{1}{z-1}$ is zero, then locus of z is

A. a straight line parallel to x-axis

B. a circle of radius 1 unit

C. a parabola with axis of symmetry parallel to x-axis

D. a hyperbola

Imaginary part of $z-1+\dfrac{1}{z-1}$ is zero.

So,

$$z-1+\frac{1}{z-1}=\overline{z-1}+\frac{1}{\overline{z-1}}$$

$$\Rightarrow z-\bar{z}+\frac{1}{z-1}-\frac{1}{\bar{z}-1}=0$$

$$\Rightarrow z-\bar{z}+\frac{\bar{z}-1-z+1}{(z-1)(\bar{z}-1)}=0$$

$$\Rightarrow (z-\bar{z})\left(1-\frac{1}{(z-1)(\bar{z}-1)}\right)=0$$

$$\Rightarrow z=\bar{z} \text{ or } (z-1)(\bar{z}-1)-1=0$$

As z is not purely real, so

$(z-1)(\bar{z}-1)=1$

Assuming $z=x+iy$, then

$(x-1+iy)(x-1-iy)=1$

$\Rightarrow (x-1)^2+y^2=1$

The locus is circle whose centre is $(1,0)$ and radius is 1.

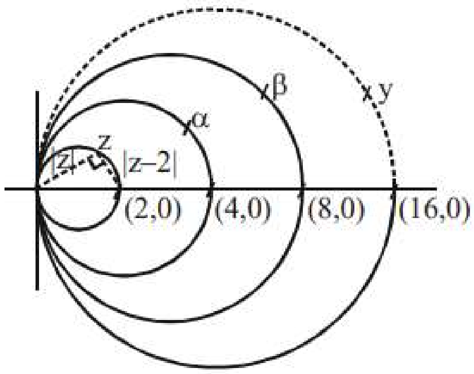
13. Let z be an imaginary complex number satisfying $|z-1|=1$. If $\alpha=2z$, $\beta=2\alpha$ and $\gamma=2\beta$, then the value of $|z|^2+|\alpha|^2+|\beta|^2+|\gamma|^2+|z-2|^2+|\alpha-4|^2+|\beta-8|^2+|\gamma-16|^2$ is

A. 100

B. 320

C. 340

D. 400



As we can see,
$|z|^2+|z-2|^2=4 \quad \cdots(1)$
Similarly,
$|\alpha|^2+|\alpha-4|^2=16 \quad \cdots(2)$
$|\beta|^2+|\beta-8|^2=64 \quad \cdots(3)$
$|\gamma|^2+|\gamma-16|^2=256 \quad \cdots(4)$

Adding all the above equations $(1),(2),(3)$ and (4),
$|z|^2+|\alpha|^2+|\beta|^2+|\gamma|^2+|z-2|^2+|\alpha-4|^2+|\beta-8|^2+|\gamma-16|^2$
$=340$

14. The logical statement $[(p\wedge q)\rightarrow p]\rightarrow(q\wedge\sim q)$ is

A. a tautology

B. a contradiction

C. equivalent to $p\vee q$

D. neither a tautology nor a contradiction

p	q	$p\vee q$	$p\wedge q$	$(p\wedge q)\rightarrow p$	$\sim q$	$q\wedge\sim q$	$[(p\wedge q)\rightarrow p]\rightarrow[q\wedge\sim q]$
T	T	T	T	T	F	F	F
T	F	T	F	T	T	F	F
F	T	T	F	T	F	F	F
F	F	F	F	T	T	F	F

The given compound statement is always false. So it is a contradiction.

15. An ellipse has eccentricity $\frac{1}{2}$ and one focus is at the point $P\left(\frac{1}{2}, 1\right)$. If the common tangent to the circle $x^2+y^2=1$ and hyperbola $x^2-y^2=1$ which is nearer to point P is directrix of the given ellipse, then the co-ordinates of centre of ellipse are

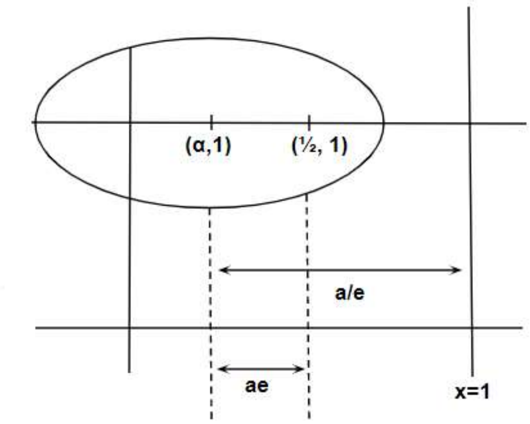
A. $\left(\frac{1}{3}, \frac{1}{3}\right)$

B. $\left(\frac{2}{3}, 1\right)$

C. $\left(\frac{1}{3}, 1\right)$

D. $\left(1, \frac{1}{3}\right)$

Given : Circle is $x^2+y^2=1$ and hyperbola is $x^2-y^2=1$
Therefore, the common tangents are
$x=\pm 1$



But $x=1$ is nearer to the point $P\left(\frac{1}{2}, 1\right)$.

$\therefore$ Directrix of the required ellipse is $x=1$

As one of the focus is $P\left(\frac{1}{2}, 1\right)$, so the centre is

$C=\left(\frac{1}{2}-ae, 1\right)=\left(\frac{1}{2}-\frac{a}{2}, 1\right)$

Distance from C to directrix $=\frac{a}{e}=2a$, so

$C=(1-2a, 1)$

Therefore,

$1-2a=\frac{1-a}{2}$

$\Rightarrow a=\frac{1}{3}$

$\therefore C=\left(\frac{1}{3}, 1\right)$

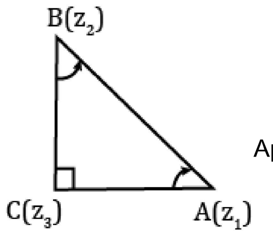
16. Complex numbers z_1, z_2, z_3 are the vertices A, B, C respectively, of an isosceles right-angled triangle with right angle at C. Then which of the following is true?

A. $(z_1 - z_2)^2 = (z_1 - z_3)(z_3 - z_2)$.

B. $(z_1 - z_2)^2 = 2(z_1 - z_3)(z_3 - z_2)$.

C. $(z_1 - z_2)^2 = 3(z_1 - z_3)(z_3 - z_2)$.

D. $(z_1 - z_2)^2 = 4(z_1 - z_3)(z_3 - z_2)$.



Applying rotation about point B,

$$\frac{z_1 - z_2}{z_3 - z_2} = \sqrt{2}e^{i\pi/4} \cdots (1)$$

Applying rotation about point A,

$$\frac{z_2 - z_1}{z_3 - z_1} = \sqrt{2}e^{-i\pi/4} \cdots (2)$$

Multiplying (1) and (2) , we get

$$\frac{(z_1 - z_2)(z_2 - z_1)}{(z_3 - z_2)(z_3 - z_1)} = 2$$

$$\Rightarrow (z_1 - z_2)^2 = -2(z_3 - z_2)(z_3 - z_1)$$

$$\therefore (z_1 - z_2)^2 = 2(z_1 - z_3)(z_3 - z_2)$$

17. The statement $p \to (q \to p)$ is logically equivalent to

A. $p \to (p \to q)$

B. $p \to (q \vee p)$

C. $p \to (q \wedge p)$

D. $p \to (p \leftrightarrow q)$

Truth table for given statement is as follows :

p	q	$q \to p$	$p \to (q \to p)$	$p \to q$	$p \to (p \to q)$	$q \vee p$	$p \wedge q$	$p \leftrightarrow q$	$p \to (q \vee p)$	$p \to q \wedge p$	$p \to (p \leftrightarrow q)$
T	T	T	T	T	T	T	T	T	T	T	T
T	F	T	T	F	F	T	F	F	T	F	F
F	T	F	T	T	T	T	F	F	T	T	T
F	F	T	T	T	T	F	F	T	T	T	T

$\therefore$ Using truth table, $p \to (q \to p)$ is logically equivalent to $p \to (q \vee p)$.

18. Let PQ be a focal chord of parabola $y^2 = x$. If the coordinates of P is $(4, -2)$, then the slope of the tangent at Q is

A. 8

B. -4

C. $\frac{1}{8}$

D. 4

Given parabola is $y^2 = x$

Any point on the parabola is $\left(\frac{t^2}{4}, \frac{t}{2}\right)$

Comparing with the coordinates of $P(t)$,

$\left(\frac{t^2}{4}, \frac{t}{2}\right) = (4, -2)$

$\Rightarrow t = -4$

As PQ is focal chord, so coordinates of $Q\left(-\frac{1}{t}\right)$ is $\left(\frac{1}{4t^2}, -\frac{1}{2t}\right) = \left(\frac{1}{64}, \frac{1}{8}\right)$

Now, equation of tangent at Q is $T = 0$

$\Rightarrow y \times \frac{1}{8} = \frac{1}{2}\left(x + \frac{1}{64}\right)$

$\Rightarrow y = 4x + \frac{1}{16}$

Hence, the slope of the tangent at Q is 4.

19. For all real permissible values of m, if the straight line $y = mx + \sqrt{9m^2 - 4}$ is tangent to a hyperbola, then equation of the hyperbola can be

A. $9x^2 - 4y^2 = 64$

B. $4x^2 - 9y^2 = 64$

C. $9x^2 - 4y^2 = 36$

D. $4x^2 - 9y^2 = 36$

Let the equation of tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is

$y = mx \pm \sqrt{a^2m^2 - b^2}$

Comparing it with $y = mx + \sqrt{9m^2 - 4}$, we have

$a^2 = 9$ and $b^2 = 4$

$\therefore$ Equation of the hyperbola is :

$\frac{x^2}{9} - \frac{y^2}{4} = 1$

$\Rightarrow 4x^2 - 9y^2 = 36$

20. Let z be a complex number such that $|z-2+i| \leq 2$. If m and M denote the least and the greatest value of $|z|$ respectively, then the value of m^2+M^2 is

A. 18

B. 9

C. $8\sqrt{5}$

D. $4\sqrt{5}$

Given, $|z-(2-i)| \leq 2$
$\Rightarrow z$ lies on or inside the circle having centre $C(2,-1)$ and radius $r=2$.
$|z|$ represents the distance of z from the origin.
Here, origin lies outside the circle.
$\therefore m = OC - r = \sqrt{5}-2$
and $M = OC + r = \sqrt{5}+2$
Therefore,
$m^2+M^2 = (\sqrt{5}-2)^2 + (\sqrt{5}+2)^2$
$\Rightarrow m^2+M^2 = 18$

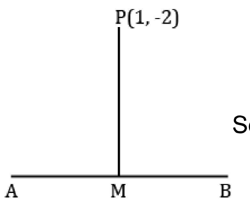
21. If the coordinates of the foot of the perpendicular drawn from the point $(1,-2)$ on the line $y=2x+1$ is (α, β), then the value of $|\alpha+\beta|$ is

Accepted Answers

2 2.0 2.00

Solution:

Let M be the foot of perpendicular drawn from the $P(1,-2)$ to $y=2x+1$
The equation of a line perpendicular to $y=2x+1$ is given by
$x+2y+\lambda=0 \quad \ldots(1)$
This passes through $P(1,-2)$.



So,

$1-4+\lambda=0 \Rightarrow \lambda=3$
On putting in equation (1), we get $x+2y+3=0$
Point M is the point of intersection of $2x-y+1=0$ and $x+2y+3=0$.
Solving these equations, we get
$x=-1,\ y=-1$
Therefore, the coordinates of the foot of perpendicular are $(-1,-1)$

Hence, the value of $|\alpha+\beta| = |-2| = 2$

22. If $|z_1| = |z_2|$ and $\arg\left(\frac{z_1}{z_2}\right) = \pi$, then value of $z_1 + z_2$ is

Accepted Answers

0 0.0 0.00 00

Solution:

We have,
$|z_1| = |z_2|$
$\Rightarrow \frac{|z_1|}{|z_2|} = 1$
$\Rightarrow \left|\frac{z_1}{z_2}\right| = 1$
And
$\arg\left(\frac{z_1}{z_2}\right) = \pi$
$\therefore \frac{z_1}{z_2} = \left|\frac{z_1}{z_2}\right| (\cos\pi + i\sin\pi)$
$\Rightarrow \frac{z_1}{z_2} = 1 \cdot (-1+0)$
$\Rightarrow z_1 = -z_2$
$\Rightarrow z_1 + z_2 = 0$

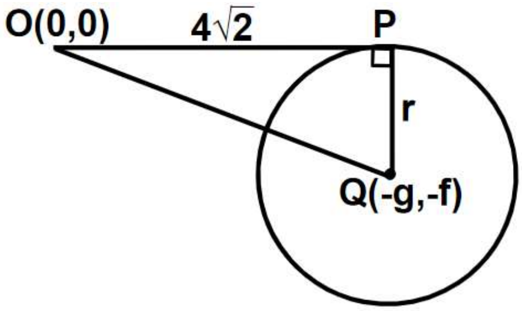
23. If $y = x$ be the tangent to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ at ponit P such that the distance of P from origin is $4\sqrt{2}$, then the value of c is

Accepted Answers

32 32.0 32.00

Solution:

$x^2 + y^2 + 2gx + 2fy + c = 0$
Let r be the radius and $Q(-g,-f)$ be the centre of the circle.
$r = \sqrt{g^2 + f^2 - c} = PQ$



From figure,
$OQ^2 = PQ^2 + PO^2$
$\Rightarrow (\sqrt{g^2+f^2})^2 = g^2 + f^2 - c + (4\sqrt{2})^2$
$\Rightarrow g^2 + f^2 = g^2 + f^2 - c + 32$
$\Rightarrow c = 32$

24. The average marks of 10 students in a class was 60 with a standard deviation of 4, while the average marks of other ten students was 40 with a standard deviation of 6. If all the 20 students are taken together and σ is the combined standard deviation, then the value of $[\sigma]$ is
([$\cdot$] represents the greatest integer function)

Accepted Answers

11 11.0 11.00

Solution:

$$\begin{aligned}
&\bar{x} = 60\\
&\Rightarrow \sum x_i = 600\\
&\sigma^2(x_i) = 4^2 = 16\\
&\Rightarrow 16 = \frac{\sum x_i^2}{10} - (60)^2\\
&\Rightarrow \sum x_i^2 = 36160
\end{aligned}$$

$$\begin{aligned}
&\bar{y} = 40\\
&\Rightarrow \sum y_i = 400\\
&\sigma^2(y_i) = 6^2 = 36\\
&\Rightarrow 36 = \frac{\sum y_i^2}{10} - (40)^2\\
&\Rightarrow \sum y_i^2 = 16360
\end{aligned}$$

$\therefore$ Combined variance,

$$\begin{aligned}
\sigma^2 &= \frac{1}{10+10}\left(\sum x_i^2 + \sum y_i^2\right) - \left(\frac{\sum x_i + \sum y_i}{10+10}\right)^2\\
&= \frac{36160 + 16360}{20} - \left(\frac{600+400}{20}\right)^2\\
&= 2626 - 2500\\
&= 126
\end{aligned}$$

$\therefore [\sigma] = 11$

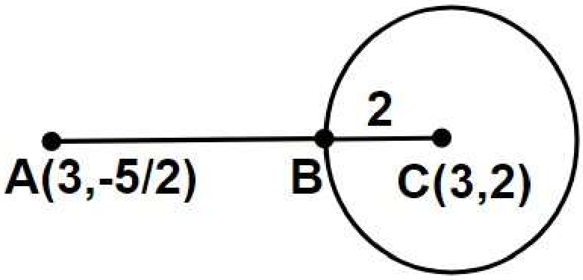
25. If z is any complex number satisfying $|z-3-2i| \leq 2$, then the minimum value of $|2z-6+5i|$ is

Accepted Answers

5 5.0 5.00

Solution:

$|z-3-2i| \leq 2$
It implies that z lies on or inside the circle of radius 2 and centre $(3,2)$.



$$|2z-6+5i|_{\min} = 2\left|z-3+\left(\frac{5}{2}\right)i\right|_{\min}$$
$$= 2(AB)$$
$$= 2(AC-BC)$$
$$= 2\left(\sqrt{(3-3)^2+\left(2+\frac{5}{2}\right)^2}-2\right)$$
$$= 2\left(2+\frac{5}{2}-2\right)$$
$$= 5$$

Alternate Solution :
Given $|z-3-2i| \leq 2 \quad \ldots(1)$
$|2z-6+5i| = |2(z-3-2i)+9i|$
$\geq ||2(z-3-2i)|-|9i|| \quad (\because |z_1+z_2| \geq ||z_1|-|z_2||)$
$\geq |2|z-3-2i|-|9i||$
$\Rightarrow |2z-6+5i| \geq |2|z-3-2i|-9|$
From equation (1),
$2|z-3-2i| \in [0,4]$
$\Rightarrow |2|z-3-2i|-9| \in [5,9]$
$\therefore |2z-6+5i| \geq 5$

26. If d_1 and d_2 are the longest and the shortest distances of the point $P(-7,2)$ from the circle $x^2+y^2-10x-14y-51=0$, then the value of $d_1^2+d_2^2$ is

Accepted Answers

588 588.0 588.00

Solution:

Given circle $S \equiv x^2+y^2-10x-14y-51=0$, P is $(-7,2)$
$S_1 = (-7)^2+2^2-10(-7)-14(2)-51 = 44 > 0$
$\Rightarrow P$ lies outside the circle.
Centre, $C = (-g,-f) = (5,7)$
Radius, $r = \sqrt{g^2+f^2-c} = \sqrt{25+49+51} = \sqrt{125}$
$CP = \sqrt{12^2+5^2} = 13$
$\therefore d_1 = CP + r = 13+\sqrt{125}$
$d_2 = CP - r = 13-\sqrt{125}$
$\therefore d_1^2+d_2^2 = (13+\sqrt{125})^2+(13-\sqrt{125})^2$
$= 2(169+125) = 588$

JEE Main Part Test 2

27. The line $x+2y=36$ is normal to the parabola $x^2=12y$ at the point whose distance from the focus of the parabola is

Accepted Answers

15 15.0 15.00

Solution:

Equation of normal to the parabola $x^2=4ay$ at $P(2at, at^2)$ in parametric form is $x+ty=2at+at^3$
For parabola $x^2=12y$, we get
$x+ty=6t+3t^3$
Given equation of normal is $x+2y=36$

Comparing both equations, we get
$t=2$
$\therefore$ Point of contact is $P(2\times3\times2, 3\times2^2)=P(12,12)$

Focus of the parabola is $F(0,3)$
$PF=\sqrt{12^2+9^2}=15$

Alternate :
$x^2=12y$
$\Rightarrow 2x=12\dfrac{dy}{dx}$
$\Rightarrow \dfrac{dy}{dx}=\dfrac{x}{6}$
$\therefore$ Slope of normal at $(x_1, y_1)$is $\dfrac{-6}{x_1}$

Now given equation is
$x+2y=36$
Comparing slope, we get
$-\dfrac{6}{x_1}=-\dfrac{1}{2}$
$\Rightarrow x_1=12$
$\Rightarrow y_1=\dfrac{x_1^2}{12}=12$

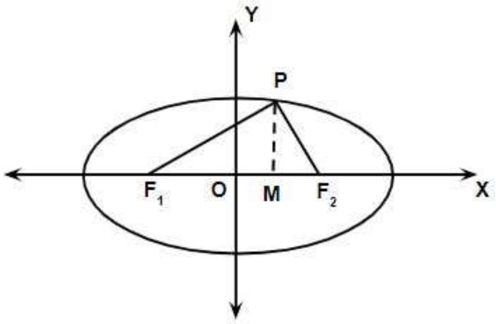
Therefore, the point P is $(12,12)$
Focus is $F(0,3)$
Required distance,
$PF=\sqrt{12^2+9^2}=15$

28. Let P be a variable point on the ellipse $\frac{x^2}{100}+\frac{y^2}{64}=1$ with foci F_1 and F_2. If A is the area of triangle PF_1F_2, then the maximum possible value of A is

Accepted Answers

48 48.0 48.00

Solution:



$a=10, b=8$
$\therefore c=\sqrt{10^2-8^2}=6$
Any point P on the ellipse is $P(10\cos\theta, 8\sin\theta)$.

$A=\frac{1}{2}(\text{Base}\times\text{height})$

$\Rightarrow A=\frac{1}{2}(F_1F_2\times PM)$

$\Rightarrow A=\frac{1}{2}(2\times 6\times 8\sin\theta)$

$\Rightarrow A=48\sin\theta$
For maximum value of $A,\ \theta=\frac{\pi}{2}$

and $A_{\max}=48$

29. The minimum value of $f(x)=|x-6|+|x+3|+|x-8|+|x+4|+|x-3|,\ x\in\mathbb{R}$ is

Accepted Answers

21 21.0 21.00

Solution:

$f(x)=|x-6|+|x+3|+|x-8|+|x+4|+|x-3|,\ x\in\mathbb{R}$
Critical points are $6,-3,8,-4,3$
Minimum value occurs at median of $-4,-3,3,6,8$
Median $=\left(\frac{5+1}{2}\right)^{th}$ term $=3$
$\therefore$ Minimum value $=|3-6|+|3+3|+|3-8|+|3+4|+|3-3|=21$

30. If the line $y = mx + a$ meets the parabola $y^2 = 4ax$ at two points whose abscissa are x_1 and x_2, then the value of m for which $x_1 + x_2 = 0$ is

Accepted Answers

2 2.00 2.0

Solution:

Given line is
$y = mx + a \quad \cdots(1)$
$y^2 = 4ax \quad \cdots(2)$

From equation (1) and equation (2), we get
$(mx + a)^2 = 4ax$
$\Rightarrow m^2x^2 + 2amx + a^2 = 4ax$
$\Rightarrow m^2x^2 + (2am - 4a)x + a^2 = 0$

Now for $x_1 + x_2 = 0$,
Sum of roots $= 0$
$\Rightarrow -\dfrac{2am - 4a}{m^2} = 0$
$\Rightarrow 4a - 2am = 0$
Since $a \neq 0$,
$\Rightarrow m = 2$