Topic : Electrostatics and capacitors

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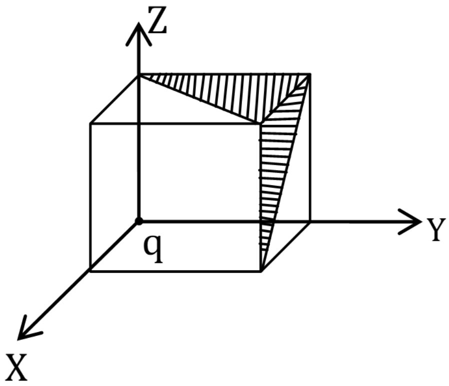
1. A charge $'q'$ is placed at one corner of a cube as shown in figure. The flux of electrostatic field $\vec{E}$ though the shaded area is:



A. $\frac{q}{48\epsilon_o}$

B. $\frac{q}{8\epsilon_o}$

C. $\frac{q}{24\epsilon_o}$

D. $\frac{q}{4\epsilon_o}$

Total flux through the cube $= \frac{q}{\epsilon_o} \times \frac{1}{8} = \frac{q}{8\epsilon_o}$

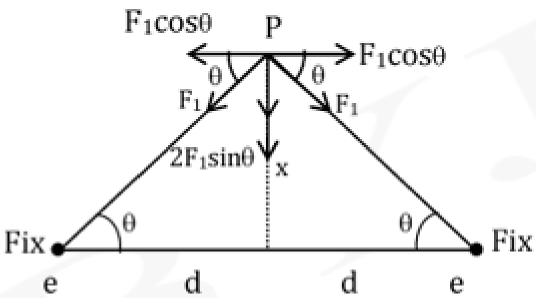
Total flux through one "outer" face of the cube $= \frac{q}{8\epsilon_o} \times \frac{1}{3} = \frac{q}{24\epsilon_o}$
[Because there is flux only through 3 faces]

Hence, total flux through shaded area,
$$\phi_T = \left(\frac{q}{24\epsilon_o} + \frac{q}{24\epsilon_o}\right) \times \frac{1}{2}$$
[half of each face is shaded]

$$\phi_T = \frac{q}{24\epsilon_o}$$

2. Two electrons having charge $(-e)$ each are fixed at a distance '2d'. A third charge proton placed at the midpoint is displaced slightly by a distance $x\ (x << d)$ perpendicular to the line joining the two fixed charges. Proton will execute simple harmonic motion having angular frequency:
$(m =$ mass of charged particle)

- ✓ A. $\left(\frac{e^2}{2\pi\varepsilon_o m d^3}\right)^{1/2}$
- ✗ B. $\left(\frac{\pi\varepsilon_o m d^3}{2e^2}\right)^{1/2}$
- ✗ C. $\left(\frac{2\pi\varepsilon_o m d^3}{e^2}\right)^{1/2}$
- ✗ D. $\left(\frac{2e^2}{\pi\varepsilon_o m d^3}\right)^{1/2}$



Restoring force on proton:-

$F_r = 2F_1 \sin\theta$, Where, $F_1 = \frac{-ke^2}{d^2+x^2}$

$$F_r = \frac{-2ke^2 x}{(d^2+x^2)^{3/2}}$$

$$\because x << d$$

$$F_r = \frac{-2ke^2 x}{d^3} = -Kx$$

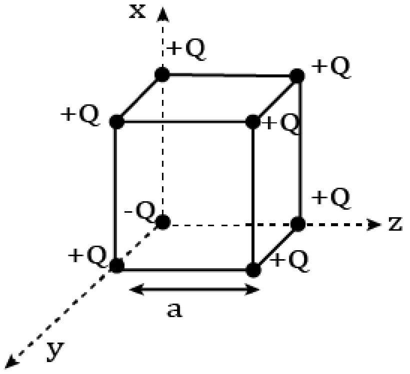
$$K = \frac{2ke^2}{d^3} = \frac{e^2}{2\pi\varepsilon_o d^3}$$

Angular Frequency :-

$$\omega = \sqrt{\frac{K}{m}}$$

$$\therefore \omega = \sqrt{\frac{e^2}{2\pi\varepsilon_o m d^3}}$$

3. A cube of side a has point charges, $+Q$ located at each of its vertices, except at the origin, where the charge is $-Q$. The electric field at the centre of the cube is :

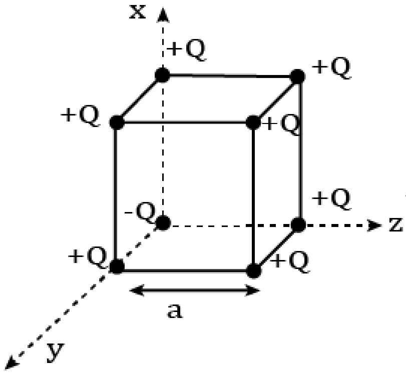


A. $\dfrac{2Q}{3\sqrt{3}\pi\varepsilon_0 a^2}(\hat{x}+\hat{y}+\hat{z})$

B. $\dfrac{Q}{3\sqrt{3}\pi\varepsilon_0 a^2}(\hat{x}+\hat{y}+\hat{z})$

C. $\dfrac{-2Q}{3\sqrt{3}\pi\varepsilon_0 a^2}(\hat{x}+\hat{y}+\hat{z})$

D. $\dfrac{-Q}{3\sqrt{3}\pi\varepsilon_0 a^2}(\hat{x}+\hat{y}+\hat{z})$



We can replace $-Q$ charge at origin by $+Q$ and $-2Q$.

Now, due to $+Q$ charge at every corner of the cube, the electric field at the centre of the cube, is zero.

So, net electric field at centre is only due to $-2Q$ charge at origin.

Position vector of centre of the cube,

$$\vec{r} = \frac{a}{2}(\hat{x} + \hat{y} + \hat{z})$$

Also, $r = \dfrac{\sqrt{3}a}{2}$

Further,

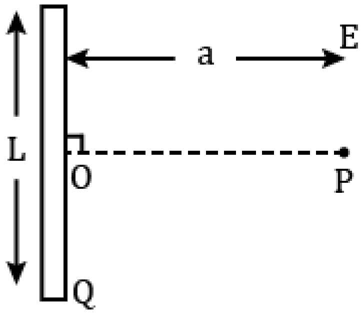
$$\vec{E} = \frac{q\vec{r}}{4\pi\varepsilon_0 r^3}$$

$$\vec{E} = \frac{-2Q \times \frac{a}{2}(\hat{x} + \hat{y} + \hat{z})}{4\pi\varepsilon_0\left(\frac{\sqrt{3}a}{2}\right)^3}$$

$$\vec{E} = \frac{-2Q}{3\sqrt{3}\pi\varepsilon_0 a^2}(\hat{x} + \hat{y} + \hat{z})$$

4. Find the electric field E at point P (as shown in figure) on the perpendicular bisector of a uniformly charged thin wire of length L carrying a charge Q.

The distance of the point P from the centre of the rod is $a = \frac{\sqrt{3}}{2}L$.

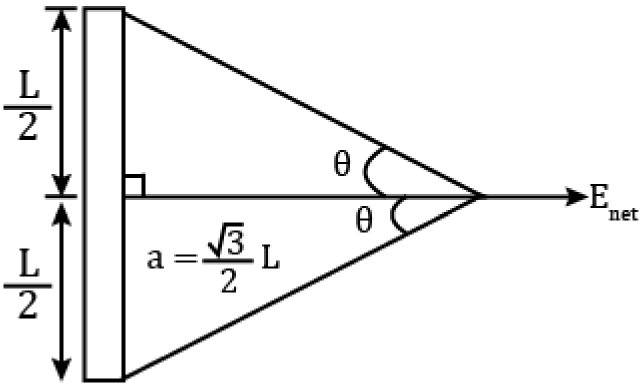


A. $\frac{Q}{2\sqrt{3}\pi\varepsilon_0 L^2}$

B. $\frac{\sqrt{3}Q}{4\pi\varepsilon_0 L^2}$

C. $\frac{Q}{3\pi\varepsilon_0 L^2}$

D. $\frac{Q}{4\pi\varepsilon_0 L^2}$



From the figure above,

$$\tan\ \theta = \frac{\frac{L}{2}}{\frac{\sqrt{3}}{2}L} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \theta = 30^{\circ}$$

$$E_{net} = \frac{k\lambda}{\frac{\sqrt{3}}{2}L}(\sin 30^{\circ} + \sin 30^{\circ}) = \frac{2KQ}{\sqrt{3}L^2}\left(\frac{1}{2}+\frac{1}{2}\right)$$

$$E_{net} = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{\sqrt{3}L^2}$$

$$E_{net} = \frac{Q}{2\sqrt{3}\pi\varepsilon_0 L^2}$$

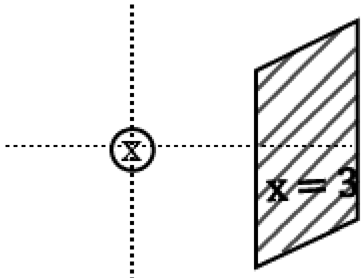
5. Find out the surface charge density at the intersection of point $x = 3\ \mathrm{m}$ plane and $x - \mathrm{axis}$ in the region of uniform line charge of $8\ \mathrm{nC/m}$ lying along the $z - \mathrm{axis}$ in free space.

A. $47.88\ \mathrm{C/m}$

B. $0.07\ \mathrm{nC\ m}^{-2}$

C. $0.424\ \mathrm{nC\ m}^{-2}$

D. $4.0\ \mathrm{nC\ m}^{-2}$



Electric field due to wire is given by,

$$E = \frac{2k\lambda}{r}$$

Electric field due to two dimensional lamina having surface charge density σ is given by,

$$E' = \frac{\sigma}{\varepsilon_o}$$

Equating both at the point of intersection, we get

$$\frac{2k\lambda}{r} = \frac{\sigma}{\varepsilon_0}$$

$$\Rightarrow\ 2 \times \frac{1}{4\pi\varepsilon_0} \times \frac{\lambda}{r} = \frac{\sigma}{\varepsilon_0}$$

$$\Rightarrow\ \frac{8 \times 10^{-9}}{2 \times 3.14 \times 3} = \sigma$$

$$\Rightarrow\ \sigma = 0.424\ \mathrm{nC\ m}^{-2}$$

6. Given below are two statements.

Statement I : An electric dipole is placed at the centre of a hollow sphere. The flux of electric field through the sphere is zero, but electric field is not zero anywhere in the sphere.

Statement II : If R is the radius of a solid metallic sphere and Q be the total charge on it. The electric field at a point on the spherical surface of radius $r\ (< R)$ is zero, but the electric flux passing through this closed spherical surface of radius r is not zero.

In the light of the above statements, choose the correct answer from the options given below.

- ✔ **A.** Statement I is true, but statement II is false.
- ✖ **B.** Statement I is false, but statement II is true.
- ✖ **C.** Both statement I and statement II are true.
- ✖ **D.** Both statement I and statement II are false.

We know that, electric flux,

$$\phi = \frac{q_{\text{enclosed}}}{\epsilon_o}$$

As $q_{\text{enclosed}} = q + (-q) = 0 \Rightarrow \phi = 0$

But, $E_{\text{inside}} \neq 0$ as there is net field due to both charges.

So, statement 1 is true.

Also, for a conducting sphere of radius R, all the charges reside on its surface.

Therefore, the electric flux passing through the closed spherical surface of radius $r(< R)$ is zero as $q_{\text{enclosed}} = 0$.

And, the electric field inside the conductor is zero.

So, statement 2 is false.

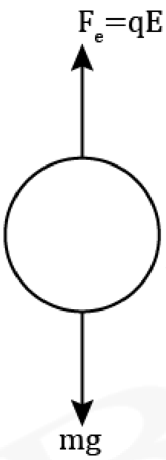
7. An oil drop of radius $2 \mathrm{~mm}$ with a density $3 \mathrm{~g} \mathrm{~cm}^{-3}$ is held stationary under a constant electric field $3.55 \times 10^{5} \mathrm{~V} \mathrm{~m}^{-1}$ in the Millikan's oil drop experiment. What is the number of excess electrons that the oil drop will possess? (Consider $g=9.81 \mathrm{~m} \mathrm{~s}^{-2}$)

A. 1.73×10^{10}

B. 48.8×10^{11}

C. 1.73×10^{12}

D. 17.3×10^{10}



Suppose the number of excess electrons is n.

Force due to electric field, $F_e=qE=(ne)E$
Weight of the oil drop, $W=mg$

Since drop is stationary, $F_e=W \Rightarrow neE=mg$

$$\Rightarrow n=\frac{mg}{eE}=\frac{\rho \times \frac{4}{3}\pi R^3 \times g}{eE}$$

$$\Rightarrow n=\frac{(3 \times 1000) \times \frac{4}{3} \times 3.14 \times (2 \times 10^{-3})^3 \times 9.8}{1.6 \times 10^{-19} \times 3.55 \times 10^5}$$

$$\Rightarrow n=\frac{984704 \times 10^5}{5.68}=1.73 \times 10^{10}$$

$$\Rightarrow n=1.73 \times 10^{10}$$

8. A solenoid of 1000 turns per metre has a core with relative permeability of 500. Insulated windings of the solenoid carry an electric current of $5\ \mathrm{A}$. The magnetic flux density produced by the solenoid is -

(Permeability of free space $= 4\pi \times 10^{-7}\ \mathrm{H/m}$)

A. $2\pi \times 10^{-3}\ \mathrm{T}$

B. $\frac{\pi}{5}\mathrm{T}$

C. $\pi \times 10^{-4}\ \mathrm{T}$

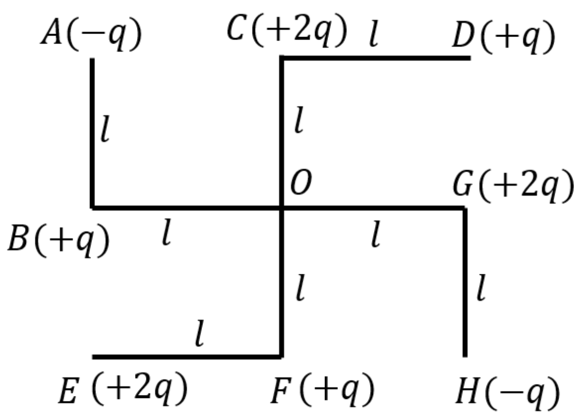
D. $\pi\ \mathrm{T}$

Given :
$I = 5\ \mathrm{A}$
$n = 1000\ \mathrm{m}^{-1}$
$\mu_r = 500$
$\mu_o = 4\pi \times 10^{-7}\ \mathrm{H/m}$

Magnetic flux density,
$B = \mu n I$
$\Rightarrow B = \mu_r \mu_0 n I$
$\Rightarrow B = 500 \times 4\pi \times 10^{-7} \times 10^3 \times 5$
$\Rightarrow B = \pi\ \mathrm{T}$

Hence, option (D) is the correct answer.

9. What will be the magnitude of electric field at point O as shown in figure? Each side of the figure is l and perpendicular to each other ?



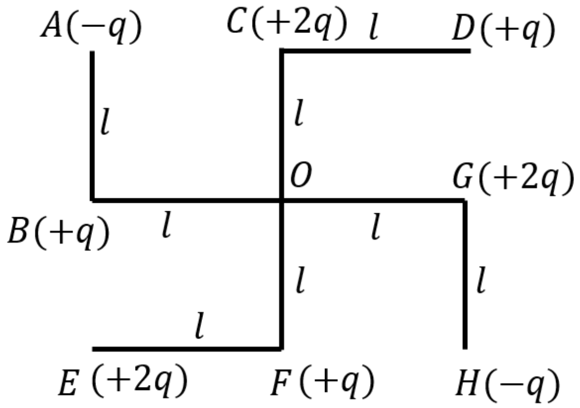
A. $\frac{1}{4\pi\epsilon_0}\frac{q}{l^2}$

B. $\frac{1}{4\pi\epsilon_0}\frac{q}{(2l^2)}\left(2\sqrt{2}-1\right)$

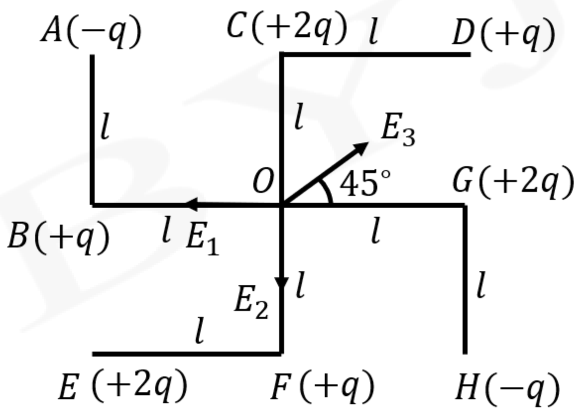
C. $\frac{q}{4\pi\epsilon_0(2l)^2}$

D. $\frac{1}{4\pi\epsilon_0}\frac{2q}{2l^2}\left(\sqrt{2}\right)$

Given,



If we talk about the charges at points B and G , the net electric field due to them, at point O will be towards B, as $2q > q$.
Similarly, if we talk about the charges at points C and F , the net electric field due to them, at point O will be towards F, as $2q > q$.
Again, if we talk about rest of the charges at different positions,
there will be no electric field at O in the alignment of AH as the magnitude of electric field due to charges at A and H are equal, so, they will cancle out each other.
There will be net electric field at point O along OD, as $2q > q$
Let us represent this situation pictorially,



Where, E_1, E_2 and E_3 are the net electric field at O as per the mentioned above conditions.
Thus,

$$E_1 = \frac{2kq}{l^2} - \frac{kq}{l^2} = \frac{kq}{l^2}$$

$$E_2 = \frac{2kq}{l^2} - \frac{kq}{l^2} = \frac{kq}{l^2}$$

$$E_3 = \frac{2kq}{\left(\sqrt{2}l\right)^2} - \frac{kq}{\left(\sqrt{2}l\right)^2} = \frac{kq}{2l^2}$$

After resolving these electric fields and solving it, we get the magnitude of resultant electric field at O as

$$E_R = \sqrt{2} \times \frac{kq}{l^2}\left(1 - \frac{1}{2\sqrt{2}}\right) = \frac{kq}{2l^2}\left(2\sqrt{2} - 1\right)$$

Or, $E_R = \frac{1}{4\pi\epsilon_0}\frac{q}{(2l^2)}\left(2\sqrt{2} - 1\right)$

Hence. option (b) is correct.

10. An electric field of $1000 \mathrm{~V/m}$, is applied to an electric dipole moment of $10^{-29}\ C \cdot m$. What is the potential energy of the electric dipole?

A. $-20 \times 10^{-18} J$

B. $-7 \times 10^{-27} J$

C. $-10 \times 10^{-29} J$

D. $-9 \times 10^{-20} J$

Potential energy of a dipole is given by,

$$\begin{aligned} U &= -\vec{P} \cdot \vec{E} \\ &= -PE \cos\ \theta \end{aligned}$$

Where, $\theta =$ angle between dipole moment vector and the field

$$U = -(10^{-29})(10^{3}) \cos 45^{\circ} \approx -7 \times 10^{-27} \mathrm{~J}$$

Hence, (B) is the correct answer.

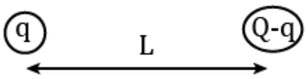
11. A certain charge Q is divided into two parts, q and $Q-q$. How should the charges be divided so that q and $Q-q$ placed at a certain distance apart, experience maximum electrostatic repulsion?

A. $Q=\frac{q}{2}$

B. $Q=2q$

C. $Q=4q$

D. $Q=3q$



$$F=\frac{kq(Q-q)}{L^2}=\frac{k}{L^2}(qQ-q^2)$$

$\frac{dF}{dq}=0$, when force is maximum.

$$\Rightarrow \frac{dF}{dq}=\frac{k}{L^2}[Q-2q]=0$$

$$\Rightarrow Q-2q=0 \Rightarrow Q=2q$$

Hence, option (B) is the correct answer.

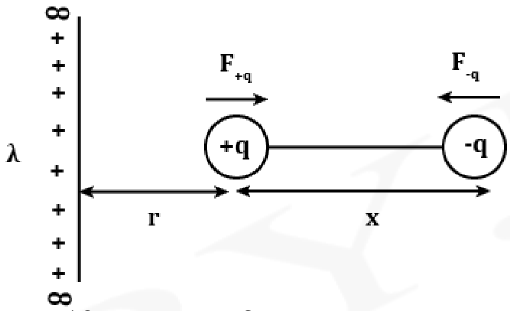
12. An electric dipole is placed on $x-$axis in proximity to a line charge of linear charge density $3.0 \times 10^{-6}\ \mathrm{C/m}$. Line charge is placed on $z-$axis and positive and negative charge of dipole is at a distance of $10\ \mathrm{mm}$ and $12\ \mathrm{mm}$ from the origin respectively. If total force of $4\ \mathrm{N}$ is exerted on the dipole, find out the amount of positive or negative charge of the dipole.

A. $815.1\ \mathrm{nC}$

B. $8.8\ \mu\mathrm{C}$

C. $0.485\ \mathrm{mC}$

D. $4.44\ \mu\mathrm{C}$



$r = 10\ \mathrm{mm},\ x = 2\ \mathrm{mm}$

Now forces on the charge,

$$|\overrightarrow{F_q}| = \frac{2k\lambda}{r}q$$

$$|\overrightarrow{F_{-q}}| = \frac{2k\lambda}{r+x}q$$

$$\Rightarrow |\overrightarrow{F_{net}}| = \frac{2k\lambda q}{r} - \frac{2k\lambda q}{r+x}$$

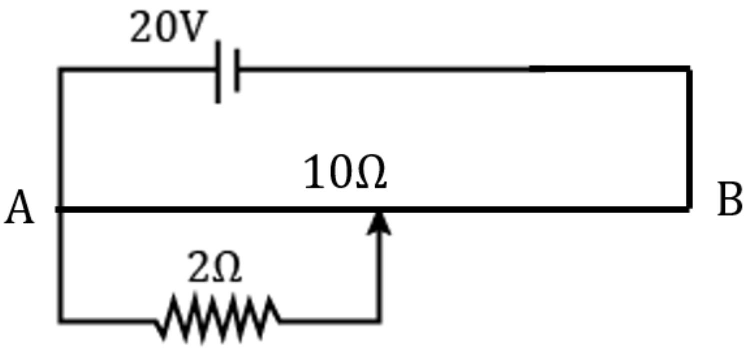
$$\Rightarrow |\overrightarrow{F_{net}}| = \frac{2k\lambda q.\,x}{r(r+x)}$$

$$\Rightarrow 4 = \frac{2 \times 9 \times 10^{9} \times 3 \times 10^{-6} \times q \times (2 \times 10^{-3})}{(10 \times 10^{-3}) \times (12 \times 10^{-3})}$$

$$\Rightarrow q = 4.44 \times 10^{-6}\ \mathrm{C} = 4.44\ \mu\mathrm{C}$$

Hence, (D) is the correct answer.

13. The given potentiometer has its wire of resistance $10\ \Omega$. When the sliding contact is in the middle of the potentiometer wire, the potential drop across $2\ \Omega$ resistor is :

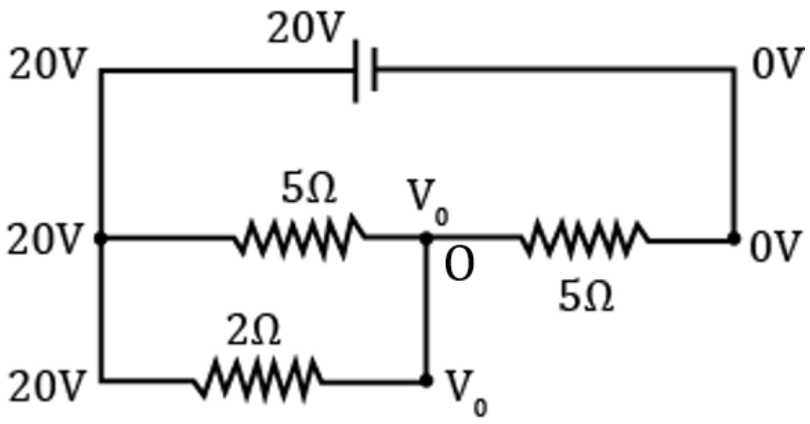


A. $10\ \text{V}$

B. $5\ \text{V}$

C. $\frac{40}{9}\text{V}$

D. $\frac{40}{11}\text{V}$



Applying Kirchhoff's junction law at O

$$\frac{20-V_0}{5}+\frac{20-V_0}{2}=\frac{V_0-0}{5}$$

$$\Rightarrow 4+10=\frac{2V_0}{5}+\frac{V_0}{2}$$

$$\Rightarrow 14=\frac{4V_0+5V_0}{10}$$

$$\Rightarrow V_0=\frac{140}{9}\mathrm{V}$$

Potential difference across 2Ω resistor is

$$V_{2\Omega}=20-V_0$$

$$\Rightarrow V_{2\Omega}=\left(20-\frac{140}{9}\right)\mathrm{V}$$

$$\Rightarrow V_{2\Omega}=\frac{40}{9}\mathrm{V}$$

Hence, option (C) is correct.

14. Two identical tennis balls each having mass $'m'$ and charge $'q'$ are suspended from a fixed point by threads of length $'l'$. What is the equilibrium separation when each thread makes a small angle θ with the vertical ?

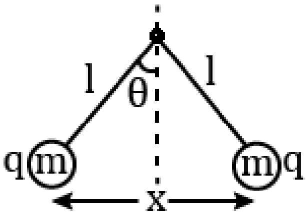
A. $x = \left(\frac{q^2 l}{2\pi\varepsilon_0 mg}\right)^{\frac{1}{2}}$

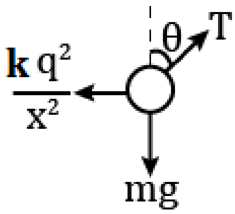
B. $x = \left(\frac{q^2 l}{2\pi\varepsilon_0 mg}\right)^{\frac{1}{3}}$

C. $x = \left(\frac{q^2 l^2}{2\pi\varepsilon_0 m^2 g}\right)^{\frac{1}{3}}$

D. $x = \left(\frac{q^2 l^2}{2\pi\varepsilon_0 m^2 g^2}\right)^{\frac{1}{3}}$

There are three forces, electostatic force (F_e), tension (T) and weight $(W = mg)$ acting on the charges.





The horizontal component of tension T gets balanced the electrostatic force while the vertical component by weight.

$$T \cos\theta = mg$$

$$T \sin\theta = \frac{kq^2}{x^2}$$

Taking the ratio of these two equations,

$$\tan\theta = \frac{kq^2}{x^2 mg}$$

Since, θ is small, so $\tan\theta \approx \sin\theta \approx \frac{x}{2l}$

$$\Rightarrow \frac{x}{2l} = \frac{kq^2}{x^2 mg}$$

Plugging the value of $k = \frac{1}{4\pi\varepsilon_0}$, we get,

$$x = \left(\frac{q^2 l}{2\pi\varepsilon_0 mg}\right)^{\frac{1}{3}}$$

Hence, option (B) is correct.

15. The two thin coaxial rings, each of radius a and having charges $+Q$ and $-Q$ respectively are separated by a distance of s. The potential difference between the centres of the two rings is :

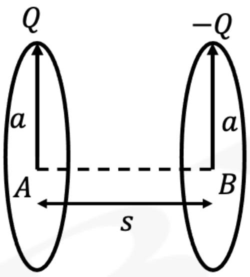
A. $\frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{a}-\frac{1}{\sqrt{s^2+a^2}}\right]$

B. $\frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{a}+\frac{1}{\sqrt{s^2+a^2}}\right]$

C. $\frac{Q}{2\pi\varepsilon_0}\left[\frac{1}{a}+\frac{1}{\sqrt{s^2+a^2}}\right]$

D. $\frac{Q}{2\pi\varepsilon_0}\left[\frac{1}{a}-\frac{1}{\sqrt{s^2+a^2}}\right]$

Clearly, the situation is as shown in the figure below:



$$V_A = \frac{KQ}{a} - \frac{KQ}{\sqrt{a^2+s^2}}$$

$$V_B = \frac{-KQ}{a} + \frac{KQ}{\sqrt{a^2+s^2}}$$

$$V_A - V_B = \frac{2KQ}{a} - \frac{2KQ}{\sqrt{a^2+s^2}}$$

$$V_A - V_B = \frac{Q}{2\pi\varepsilon_0}\left[\frac{1}{a}-\frac{1}{\sqrt{s^2+a^2}}\right]$$

Hence, option (D) is the correct answer.

16. Two equal capacitors are first connected in series and then in parallel. The ratio of the equivalent capacities in the two cases will be :

A. $2:1$

B. $1:4$

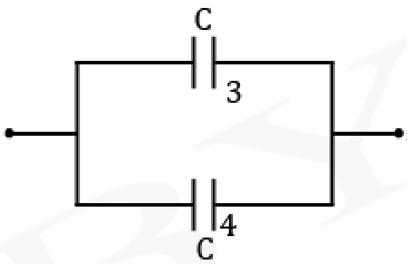
C. $3:1$

D. $3:2$

Given that the first connection is in series :

$$\Rightarrow \frac{1}{C_{12}} = \frac{1}{C} + \frac{1}{C} \Rightarrow C_{12} = \frac{C}{2}$$

The second connection is in parallel :



$$\Rightarrow C_{34} = C + C = 2C$$

Now,

$$\frac{C_{12}}{C_{34}} = \frac{\frac{C}{2}}{2C}$$

$$\Rightarrow \frac{C_{12}}{C_{34}} = \frac{1}{4} = 1:4$$

17. Consider the combination of 2 capacitors, C_1 and C_2, with $C_2 > C_1$. When connected in parallel, the equivalent capacitance is $\frac{15}{4}$ times the equivalent capacitance of the same capacitors connected in series. Calculate the ratio of capacitors, $\frac{C_2}{C_1}$.

A. $\frac{15}{11}$

B. $\frac{29}{15}$

C. $\frac{15}{4}$

D. Imaginary

Given:

$$C_{\text{eq}}(\text{parallel combination}) = \frac{15}{4} C_{\text{eq}}(\text{series combination})$$

$$\Rightarrow (C_1 + C_2) = \frac{15}{4}\left(\frac{C_1 C_2}{C_1 + C_2}\right)$$

$$\Rightarrow 15 C_1 C_2 = 4(C_1 + C_2)^2$$

$$\Rightarrow 15 C_1 C_2 = 4C_1^2 + 4C_2^2 + 8C_1 C_2$$

$$\Rightarrow 4C_1^2 + 4C_2^2 - 7C_1 C_2 = 0$$

$$\Rightarrow 4 + 4\left(\frac{C_2}{C_1}\right)^2 - 7\left(\frac{C_2}{C_1}\right) = 0$$

$$\Rightarrow 4 + 4x^2 - 7x = 0$$

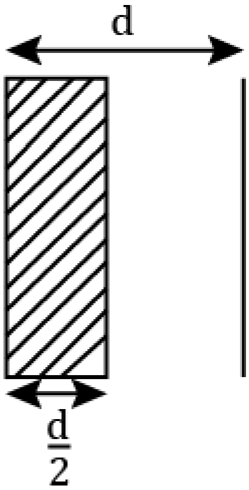
[Assuming, $x = \frac{C_2}{C_1}$]

$$\Rightarrow 4x^2 - 7x + 4 = 0$$

Discriminant,
$D = (-7)^2 - 4 \times 4 \times 4$
$\Rightarrow D = -15 =$ Negative

Hence,
$x = \frac{C_2}{C_1} =$ Imaginary

18. In a parallel plate capacitor set up, the plate area of capacitor is $2\ \mathrm{m}^2$ and the plates are separated by $1\ \mathrm{m}$. If the space between the plates are filled with a dielectric material of thickness $0.5\ \mathrm{m}$ and area $2\ \mathrm{m}^2$ (see figue) the capacitance of the set-up will be $n\epsilon_0$. The value of n is
(Dielectric constant of the material $= 3.2$) (Round off to the Nearest Integer)



Accepted Answers

3 3.0 3.00

Solution:

This capacitor can be thought as two capacitor connected in parallel. one with dilectric and another without dielectric.

$$C_1 = \frac{K\varepsilon_0 A}{d/2},\ C_2 = \frac{\varepsilon_0 A}{d/2}$$

Equivalent capacitance can be calculated as:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d}{2K\varepsilon_0 A} + \frac{d}{2\varepsilon_0 A}$$

$$\frac{1}{C_{eq}} = \frac{d}{2\varepsilon_0 A}\left(\frac{K+1}{K}\right)$$

$$C_{eq} = \frac{2\varepsilon_0 AK}{d(K+1)} = \frac{2 \times 2 \times 3.2}{1 \times 4.2}\varepsilon_0 = 3.04\varepsilon_0$$

19. A parallel plate capacitor whose capacitance C is $14\ \mathrm{pF}$ is charged by a battery to a potential difference $V = 12\ \mathrm{V}$ between its plates. The charging battery is now disconnected and a porcelain plate with $k = 7$ is inserted between the plates, then the porcelain plate would oscillate back and forth between the plates of capacitor, with a constant mechanical energy of _______ pJ.
(Assume no friction)

Accepted Answers

864 864.0 864.00

Solution:

Initial energy stored in capacitor is,

$$U_i = \frac{1}{2} C V^2$$

$$= \frac{1}{2} \times 14 \times (12)^2\ \mathrm{pJ}$$

$$= 1008\ \mathrm{pJ}$$

Final energy stored in capacitor is,

$$U_f = \frac{Q^2}{2kC}$$

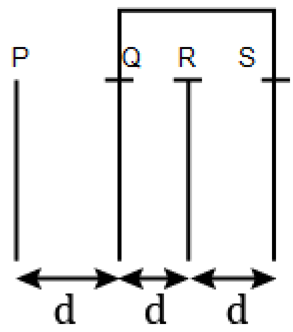
$$= \frac{(14 \times 12)^2}{2 \times 7 \times 14}$$

$$= 144\ \mathrm{pJ}$$

Oscillating energy $= U_i - U_f$
$= 1008 - 144$
$= 864\ \mathrm{pJ}$

20. Four identical rectangular plates with length, $l = 2\text{ cm}$ and breadth, $b = 3/2\text{ cm}$ are arranged as shown in the figure. The equivalent capacitance between P and R is $\dfrac{x \times 10^{-2}\epsilon_0}{d}$ where d is the distance between the plates in cm. The value of x is ______.

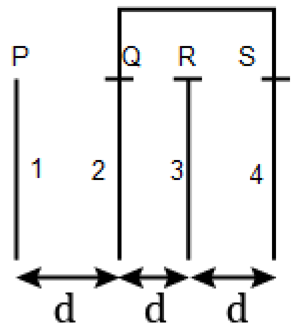
(Round off to the nearest integer)



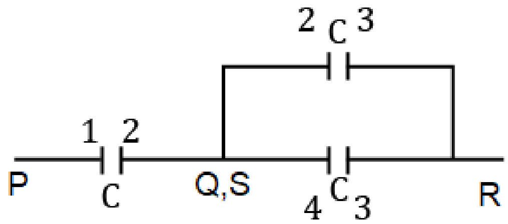
Accepted Answers

2 2.0 2.00

Solution:



The equivalent circuit can be drawn as,



Here,

$C = \frac{\epsilon_0 A}{d}$

Equivalent capacitance between P and R is given by,

$$C_{eq} = \frac{2C \times C}{2C + C} = \frac{2}{3}C = \frac{2}{3} \times \frac{\epsilon_0 A}{d}$$

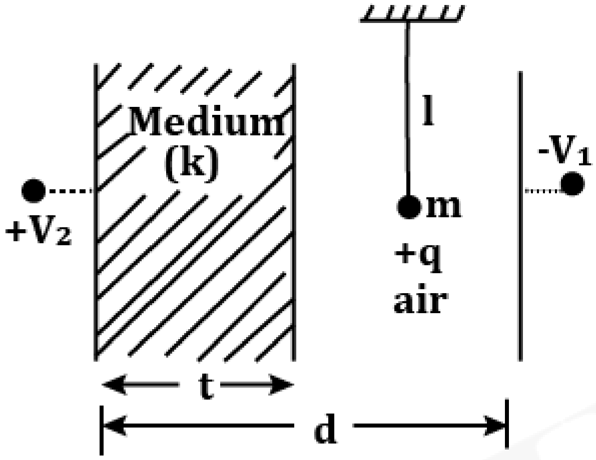
$$\Rightarrow C_{eq} = \frac{2}{3} \times \frac{\epsilon_0}{d \times 10^{-2}} \times \left(2 \times \frac{3}{2}\right) \times 10^{-4}$$

($\because$ Area of the plate = length $\times$ breadth)

$$\Rightarrow C_{eq} = 2 \times 10^{-2} \times \frac{\epsilon_0}{d} = \frac{x \times 10^{-2} \epsilon_0}{d}$$

$$\Rightarrow x = 2$$

21. A simple pendulum of mass $'m'$, length $'l'$ and charge $' + q'$ suspended in the electric field produced by two conducting parallel plates as shown in the figure. The value of deflection of pendulum in equilibrium position will be (C_1 and C_2 are the capacitance of capacitors formed by parallel plates, without medium in between and with medium in between, respectively.)



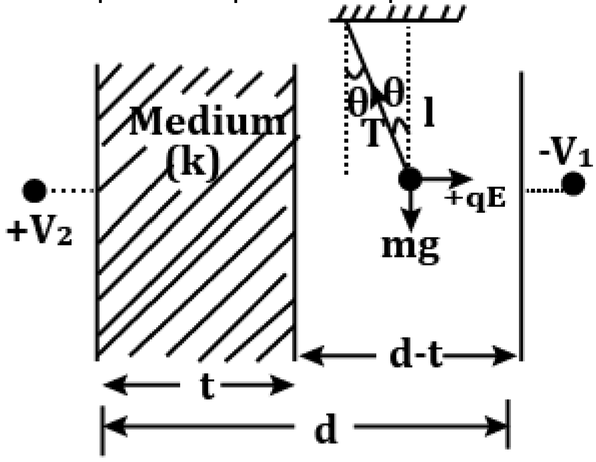
A. $\tan^{-1}\left[\frac{q}{mg} \times \frac{C_1(V_2 - V_1)}{(C_1 + C_2)(d - t)}\right]$

B. $\tan^{-1}\left[\frac{q}{mg} \times \frac{C_2(V_2 - V_1)}{(C_1 + C_2)(d - t)}\right]$

C. $\tan^{-1}\left[\frac{q}{mg} \times \frac{C_2(V_1 + V_2)}{(C_1 + C_2)(d - t)}\right]$

D. $\tan^{-1}\left[\frac{q}{mg} \times \frac{C_1(V_1 + V_2)}{(C_1 + C_2)(d - t)}\right]$

The equilibrium position of pendulum is shown in the figure.



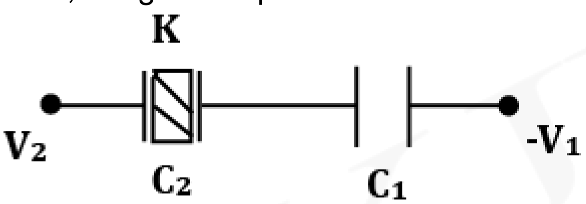
Let E be electric field in air

$T\sin\theta = qE$

$T\cos\theta = mg$

$\therefore \quad \tan\theta = \frac{qE}{mg} \,..(1)$

Now, the given capacitive circuit can be represented as shown below.



Both are in series, so equivalent capacitance can be written as

$$C_{eq} = \frac{C_1C_2}{C_1+C_2}$$

Net charge on equivalent cpacitor is given as

$$Q = C_{eq}\Delta V = \left[\frac{C_1C_2}{C_1+C_2}\right][V_2+V_1]$$

Also, we know that

$$E = \frac{Q}{A\epsilon_0} = \left[\frac{C_1C_2}{C_1+C_2}\right]\left[\frac{V_2+V_1}{A\epsilon_0}\right]$$

Also,

$$C_1 = \frac{\epsilon_0 A}{d-t} \Rightarrow E = \frac{C_2[V_2+V_1]}{(C_1+C_2)(d-t)}$$

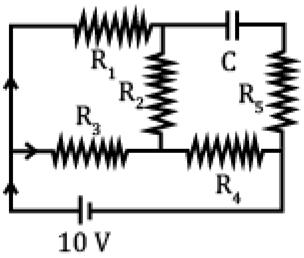
From eq (1), we have

$$\theta = \tan^{-1}\left[\frac{q.E}{mg}\right]$$

$$\Rightarrow \quad \theta = \tan^{-1}\left[\frac{q}{mg}\times\frac{C_2(V_1+V_2)}{(C_1+C_2)(d-t)}\right]$$

Hence, option (c) is correct.

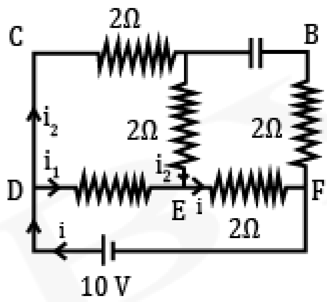
22. An ideal cell of emf $10\ \mathrm{V}$ is connected in circuit shown in figure. Each resistance is $2\ \Omega$. The potential difference (in V) across the capacitor when it is fully charged is



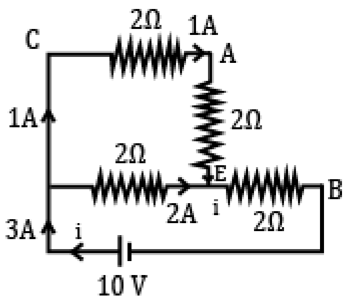
Accepted Answers

8 8.0 8.00

Solution:



As capacitor is fully charged no current will flow through it.



We have the current distribution as shown in the figure.

Equivalent resistance, $R_{eq} = \left(\frac{4 \times 2}{4+2}\right) + 2$

Net current, $i = \frac{10}{\frac{4}{3} + 2} = \frac{10 \times 3}{3} = 3\ \mathrm{A}$

Current division among resistors can be considered as
$i_1 = 2\ \mathrm{A}$ and $i_2 = 1\ \mathrm{A}$
Potential difference across capacitor is
$V_{AEB} = 1 \times 2 + 3 \times 2 = 8\ \mathrm{V}$.

23. AC voltage, $V(t) = 20\sin(\omega t)$, of frequency 50 Hz, is applied to a parallel plate capacitor. The separation between the plates is 2 mm and the area of the plates is $1\ \mathrm{m}^2$. The amplitude of the oscillating displacement current, for the applied AC voltage is -

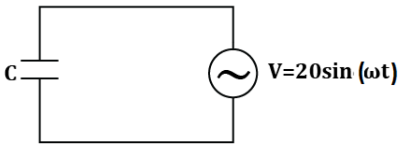
Take $\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{F/m}$

A. $21.14\ \mu\mathrm{A}$

B. $83.37\ \mu\mathrm{A}$

C. $27.79\ \mu\mathrm{A}$

D. $55.58\ \mu\mathrm{A}$



From the given information,

$$C = \frac{\epsilon_0 A}{d} = \frac{\epsilon_0}{2 \times 10^{-3}}\mathrm{F}$$

Further,

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi \times 50 \times \dfrac{\epsilon_0}{2 \times 10^{-3}}}$$

$$= \frac{2 \times 10^{-3}}{25 \times 4\pi\epsilon_0} = \frac{2 \times 10^{-3}}{25} \times 9 \times 10^9$$

$$\therefore X_C = 72 \times 10^4\ \mathrm{F}$$

Now,

$$i_0 = \frac{V_0}{X_C} = \frac{20}{72 \times 10^4} \approx 2.779 \times 10^{-5}\ \mathrm{A} = 27.79\ \mu\mathrm{A}$$

Hence, option (C) is the correct answer.

24. A current of $5 \mathrm{~A}$ is passing through a non-linear magnesium wire of cross-section $0.04 \mathrm{~m}^2$. At every point, the direction of current density is at an angle of 60°, with the unit vector of area of cross-section. The magnitude of electric field at every point of the conductor is :

Resistivity of magnesium is $44 \times 10^{-8} \Omega\text{-m}$.

A. $11 \times 10^{-2} \mathrm{~V/m}$

B. $11 \times 10^{-7} \mathrm{~V/m}$

C. $11 \times 10^{-5} \mathrm{~V/m}$ ✔

D. $11 \times 10^{-3} \mathrm{~V/m}$

We know that,

$$I = \vec{J} . \vec{A} = JA\cos\theta$$

$$\Rightarrow 5 = J \times 0.04 \times \cos 60^{\circ}$$

$$\Rightarrow J = 250 \mathrm{~A/m}^2$$

Now,

$$J = \sigma E$$

$$\Rightarrow E = \rho J$$

$$\Rightarrow E = 44 \times 10^{-8} \times 250 = 11 \times 10^{-5} \mathrm{~V/m}$$

Hence, option (C) is the correct answer.

25. A parallel plate capacitor, with plate area $'A'$ and distance of separation $'d'$, is filled with a dielectric. What is the capacity of the capacitor when permittivity of the dielectric varies as follows:

$\epsilon(x) = \epsilon_0 + kx$, for $\left(0 < x \leq \frac{d}{2}\right)$

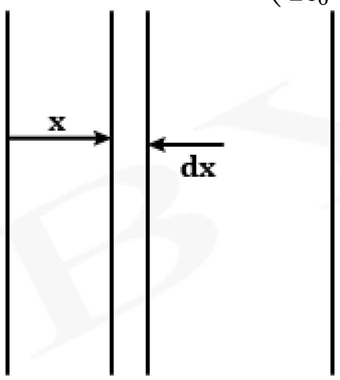
$\epsilon(x) = \epsilon_0 + k(d - x)$, for $\left(\frac{d}{2} \leq x \leq d\right)$

A. $\left(\epsilon_0 \frac{kd}{2}\right)^{\frac{2}{k\,a}}$

B. $\dfrac{kA}{2\,ln\left(\dfrac{2\epsilon_0 + kd}{2\epsilon_0}\right)}$

C. 0

D. $\dfrac{kA}{2} ln\left(\dfrac{2\epsilon_0}{2\epsilon_0 - kd}\right)$



The net capacity will be the effective capacity of series combination of two capacitors formed by the two halves of the dielectric.

i.e. $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$

Taking an element of width dx at a distance

x from left plate $\left(x < \frac{d}{2}\right)$

$$dC_1 = \frac{(\epsilon_0 + kx)A}{dx}$$

Capacitance of the first half of the capacitor is,

$$\frac{1}{C_1} = \int_0^{\frac{d}{2}} \frac{1}{dc} = \frac{1}{A}\int_0^{\frac{d}{2}} \frac{dx}{\epsilon_0 + kx}$$

$$\frac{1}{C_1} = \frac{1}{kA} ln\left(\frac{\epsilon_0 + \frac{kd}{2}}{\epsilon_0}\right)$$

Consider another element of width dx, at a distance

x from the center $\left(x > \frac{d}{2}\right)$

$$dC_2 = \frac{A(\epsilon_0 + k(d-x))}{dx}$$

Capacitance of the second half of the capacitor is,

$$\frac{1}{C_1} = \int_{\frac{d}{2}}^{d} \frac{1}{dC_2} = \frac{1}{A}\int_{\frac{d}{2}}^{d} \frac{dx}{\epsilon_0 + kd - kx}$$

$$\frac{1}{C_2} = \frac{1}{kA} ln\left(\frac{\epsilon_0 + \frac{kd}{2}}{\epsilon_0}\right)$$

As, $\frac{1}{C_1} = \frac{1}{C_2}$

$$C_{eq} = \frac{C_1}{2} = \frac{C_2}{2} = \frac{kA}{2\ ln\left(\frac{2\epsilon_0 + kd}{2\epsilon_0}\right)}$$

Hence, (B) is the correct answer.

26. If q_f is the free charge on the capacitor plates and q_b is the bound charge on the dielectric slab of dielectric constant K placed between the capacitor plates, then bound charge q_b can be expressed as

A. $q_b = q_f\left(1 - \frac{1}{\sqrt{K}}\right)$

B. $q_b = q_f\left(1 - \frac{1}{K}\right)$

C. $q_b = q_f\left(1 + \frac{1}{\sqrt{K}}\right)$

D. $q_b = q_f\left(1 + \frac{1}{K}\right)$

The electric field between the plates due to free charges only is given by,

$E_f = E_0$

When a dielectric is inserted in the capacitor, the bound charges decreases the Electric field,

$\therefore$ After introduction of dielectric, the net Electric field is given by,

$$E = \frac{E_0}{K}$$

$$\Rightarrow E = E_f - E_b$$

$$\Rightarrow E_b = E - E_f = E_0\left(1 - \frac{1}{K}\right)$$

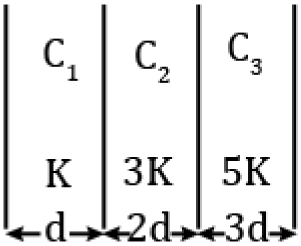
Electric field in terms of charge on the plates is given by,

$$E = \frac{q}{A\varepsilon_0}$$

$$\Rightarrow q_b = q_f\left(1 - \frac{1}{K}\right)$$

Hence, option (B) is correct.

27. In the reported figure, a capacitor is formed by placing a compound dielectric between the plates of parallel plate capacitor. The expression for the capacity of the said capacitor will be :
(Given area of plate $= A$)



A. $\frac{15}{34} \frac{K \varepsilon_0 A}{d}$

B. $\frac{15}{6} \frac{K \varepsilon_0 A}{d}$

C. $\frac{25}{6} \frac{K \varepsilon_0 A}{d}$

D. $\frac{9}{6} \frac{K \varepsilon_0 A}{d}$

Since the capacitors are connected in series, the effective capacitance is given by,

$$\frac{1}{C_{\text{eff}}}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}$$

Since, $C=\frac{K A \varepsilon_0}{d}$

$$\frac{1}{C_{\text{eff}}}=\frac{d}{K \varepsilon_0 A}+\frac{2d}{3K \varepsilon_0 A}+\frac{3d}{5K \varepsilon_0 A}$$

$$\therefore C_{\text{eff}}=\frac{15K \varepsilon_0 A}{34d}$$

Hence, option (A) is correct.

28. Two capacitors of capacities $2C$ and C are joined in parallel and charged up to potential V. The battery is removed and the capacitor of capacity C is filled completely with a medium of dielectric constant K. The potential difference across the capacitors will now be :

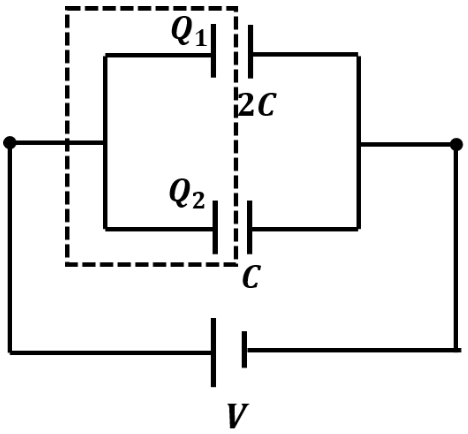
A. $\frac{V}{K+2}$

B. $\frac{V}{K}$

C. $\frac{3V}{K+2}$

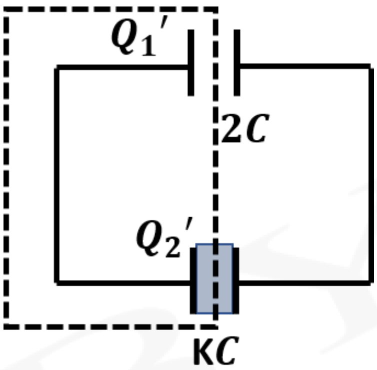
D. $\frac{3V}{K}$

Just after removing the battery, net charges on the positive plates of the capacitors is given by,



$Q_1 + Q_2 = (2C)V + CV = 3CV$

After insertion of dielectric material,



Let V_c be the common potential difference across both capacitors.

$Q_1' + Q_2' = (2C)V_c + (KC)V_c = CV_c(2+K)$

Applying conservation of charges,

$Q_1 + Q_2 = Q_1' + Q_2'$

$\Rightarrow 3CV = CV_c(2+K)$

$\therefore V_c = \dfrac{3V}{K+2}$

Hence, option (C) is correct.

29. The material filled between the plates of a parallel plate capacitor has resistivity $200\ \Omega\mathrm{m}$. The value of capacitance of the capacitor is $2\ pF$. If a potential difference of $40\ \mathrm{V}$ is applied across the plates of the capacitor, then the value of leakage current flowing out of the capacitor is :
[Given the value of relative permitivity of material $(k = 50)$]

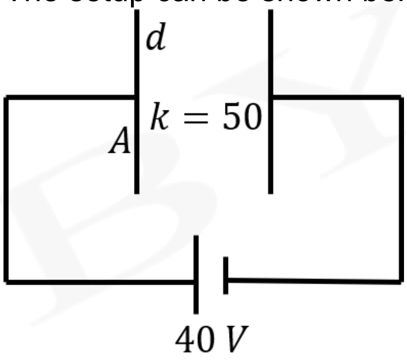
A. $9.0\ \mathrm{mA}$

B. $0.9\ \mathrm{mA}$

C. $0.9\ \mu\mathrm{A}$

D. $9.0\ \mu\mathrm{A}$

Given, resistivity of the filled material
$\rho = 200\ \Omega\mathrm{m}$
Capacitance of capacitor $C = 2\ pF = 2 \times 10^{-12}\ F$
$k = 50$
Potential difference applied across the capacitor, $V = 40\ \mathrm{V}$
The setup can be shown below as



Here, d is the length of the plate and A is the surface area of the plate.
So, the equivalent resistance is given by

$$R = \frac{\rho d}{A}$$

Also, we know that

$$C = \frac{k\epsilon_0 A}{d}$$

or, $\dfrac{d}{A} = \dfrac{k\epsilon_0}{C} = \dfrac{50 \times (8.85 \times 10^{-12})}{2 \times 10^{-12}}$

$$\Rightarrow \frac{d}{A} = 221.25\ \mathrm{m}^{-1}$$

So, we have

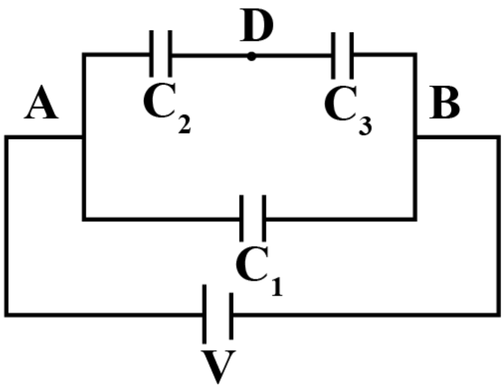
$$R = \frac{\rho d}{A} = 200 \times 221.25\ \Omega = 44250\ \Omega$$

Thus, the leakage current is given as

$$i_l = \frac{40}{R} = \frac{40}{44250} = 0.9\ \mathrm{mA}$$

Hence, option (b) is correct.

30. Three capacitors $C_1 = 2\mu\text{F},\ C_2 = 6\mu\text{F}$ and $C_3 = 12\mu\text{F}$ are connected as shown in figure. Find the ratio of the charges on capacitors $C_1,\ C_2$ and C_3 respectively



A. $3:4:4$

B. $2:3:3$

C. $2:1:1$

D. $1:2:2$

Given that,
$C_1 = 2\mu\text{F},$
$C_2 = 6\mu\text{F}$ and
$C_3 = 12\mu\text{F}$

As C_1 is connected to the battery, change on it is $q_1 = C_1V = 2V$

As C_2 & C_3 are in series, charge is equal on both the capacitors.
Effective capacitance for this series combination is $C_{eff} = \dfrac{C_2C_3}{C_2 + C_3} = 4\mu\text{F}$

Charge on C_2 & C_3 is $q_2 = q_3 = C_{eff}V = 4V$

Hence ratio of charges is $q_1 : q_2 : q_3 = 1:2:2$