Subject: Mathematics

1. The solution of the differential equation
$x\frac{dy}{dx}+2y=x^2\ (x\neq 0)$ with $y(1)=1$ is

A. $y=\frac{3}{4}x^2+\frac{1}{4x^2}$

B. $y=\frac{4}{5}x^3+\frac{1}{5x^2}$

C. $y=\frac{x^2}{4}+\frac{3}{4x^2}$

D. $y=\frac{x^3}{5}+\frac{1}{5x^2}$

2. The value of the integral
$\int\frac{\sin\theta.\sin 2\theta(\sin^6\theta+\sin^4\theta+\sin^2\theta)\sqrt{2\sin^4\theta+3\sin^2\theta+6}}{1-\cos 2\theta}d\theta$ is
(where c is a constant of integration)

A. $\frac{1}{18}[9-2\sin^6\theta-3\sin^4\theta-6\sin^2\theta]^{\frac{3}{2}}+c$

B. $\frac{1}{18}[11-18\sin^2\theta+9\sin^4\theta-2\sin^6\theta]^{\frac{3}{2}}+c$

C. $\frac{1}{18}[11-18\cos^2\theta+9\cos^4\theta-2\cos^6\theta]^{\frac{3}{2}}+c$

D. $\frac{1}{18}[9-2\cos^6\theta-3\cos^4\theta-6\cos^2\theta]^{\frac{3}{2}}+c$

3. The general solution of the differential equation $(y^2-x^3)dx-xydy=0\ (x\neq 0)$ is : (where c is a constant of integration)

A. $y^2+2x^2+cx^3=0$

B. $y^2-2x^3+cx^2=0$

C. $y^2+2x^3+cx^2=0$

D. $y^2-2x^2+cx^3=0$

4.

Given $f(x) = \begin{cases} x, & 0 \le x < \dfrac{1}{2} \\ \dfrac{1}{2} & x = \frac{1}{2} \\ 1 - x & \dfrac{1}{2} < x < 1 \end{cases}$

and $g(x) = \left(x - \dfrac{1}{2}\right)^2, x \in R$. Then the area (in sq. units) of the region bounded by the curves $y = f(x)$ and $y = g(x)$ between the lines $2x = 1$ to $2x = \sqrt{3}$ is:

A. $\dfrac{\sqrt{3}}{4} - \dfrac{1}{3}$

B. $\dfrac{1}{3} + \dfrac{\sqrt{3}}{4}$

C. $\dfrac{1}{2} + \dfrac{\sqrt{3}}{4}$

D. $\dfrac{1}{2} - \dfrac{\sqrt{3}}{4}$

5.

The integral $\displaystyle\int_{\pi/6}^{\pi/4} \frac{dx}{\sin 2x(\tan^5 x + \cot^5 x)}$ equals :

A. $\dfrac{1}{10}\left[\dfrac{\pi}{4} - \tan^{-1}\left(\dfrac{1}{9\sqrt{3}}\right)\right]$

B. $\dfrac{1}{5}\left[\dfrac{\pi}{4} - \tan^{-1}\left(\dfrac{1}{3\sqrt{3}}\right)\right]$

C. $\dfrac{\pi}{40}$

D. $\dfrac{1}{20}\left[\tan^{-1}\left(\dfrac{1}{9\sqrt{3}}\right)\right]$

6. The value of $\int_{-\pi/2}^{\pi/2} \frac{dx}{[x]+[\sin x]+4}$, where $[t]$ denotes the greatest integer less than or equal to t, is :

A. $\frac{3}{20}(4\pi-3)$

B. $\frac{3}{10}(4\pi-3)$

C. $\frac{1}{12}(7\pi+5)$

D. $\frac{1}{12}(7\pi-5)$

7. The value of the integral $\int_{-2}^{2} \frac{\sin^2 x}{\left[\frac{x}{\pi}\right]+\frac{1}{2}} dx$

(where $[x]$ denotes the greatest integer less than or equal to x) is:

A. $4-\sin 4$

B. 0

C. 4

D. $\sin 4$

8. If $\int \frac{\cos x-\sin x}{\sqrt{8-\sin 2x}} dx = a \sin^{-1}\left(\frac{\sin x+\cos x}{b}\right)+c$, where c is a constant of integration, then the ordered pair (a, b) is equal to:

A. $(1,-3)$

B. $(1,3)$

C. $(-1,3)$

D. $(3,1)$

9. Let $f : R \to R$ be a continuously differentiable function such that $f(2) = 6$ and $f'(2) = \frac{1}{48}$.

If $\int_6^{f(x)} 4t^3 \, dt = (x-2)g(x)$, then $\lim_{x \to 2} g(x)$ is equal to

A. 12

B. 18

C. 24

D. 36

10. If $(2 + \sin x)\frac{dy}{dx} + (y+1)\cos x = 0$ and $y(0) = 1$, then $y\left(\frac{\pi}{2}\right)$ is equal to:

A. $\frac{1}{3}$

B. $-\frac{2}{3}$

C. $-\frac{1}{3}$

D. $\frac{4}{3}$

11. $\int f(x)dx = \psi(x)$, then $\int x^5 f(x^3)dx$ is equal to

A. $\frac{1}{3}\left[x^3\psi(x^3) - \int x^2\psi(x^3)dx\right] + c$

B. $\frac{1}{3}x^3\psi(x^3) - \int x^3\psi(x^3)dx + c$

C. $\frac{1}{3}x^3\psi(x^3) - \int x^2\psi(x^3)dx + c$

D. $\frac{1}{3}\left[x^3\psi(x^3) - \int x^3\psi(x^3)dx\right] + c$

12. If $\cos x \frac{dy}{dx} - y \sin x = 6x$, $\left(0 < x < \frac{\pi}{2}\right)$ and $y\left(\frac{\pi}{3}\right) = 0$, then $y\left(\frac{\pi}{6}\right)$ is equal to :

A. $-\frac{\pi^2}{2}$

B. $-\frac{\pi^2}{2\sqrt{3}}$

C. $-\frac{\pi^2}{4\sqrt{3}}$

D. $\frac{\pi^2}{2\sqrt{3}}$

13. If $\frac{dy}{dx} = \frac{xy}{x^2 + y^2}$; $y(1) = 1$; then a value of x satisfying $y(x) = e$ is:

A. $\sqrt{3}e$

B. $\frac{1}{2}\sqrt{3}e$

C. $\sqrt{2}e$

D. $\frac{e}{\sqrt{2}}$

14. If $y = y(x)$ is the solution of the differential equation $\frac{dy}{dx} = (\tan x - y)\sec^2 x$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, such that $y(0) = 0$, then $y\left(-\frac{\pi}{4}\right)$ is equal to :

A. $\frac{1}{2} - e$

B. $e - 2$

C. $2 + \frac{1}{e}$

D. $\frac{1}{e} - 2$

15. Let f and g be continuous function on $[0, a]$ such that $f(x) = f(a - x)$ and $g(x) + g(a - x) = 4$, then $\int_0^a f(x)g(x)dx$ is equal to :

A. $\int_0^a f(x)dx$

B. $2\int_0^a f(x)dx$

C. $4\int_0^a f(x)dx$

D. $-3\int_0^a f(x)dx$

16. The integral $\int \frac{e^{3\log_e 2x} + 5e^{2\log_e 2x}}{e^{4\log_e x} + 5e^{3\log_e x} - 7e^{2\log_e x}}dx,\ x > 0,$ is equal to :
(where c is a constant of integration)

A. $\log_e |x^2 + 5x - 7| + c$

B. $\frac{1}{4}\log_e |x^2 + 5x - 7| + c$

C. $4\log_e |x^2 + 5x - 7| + c$

D. $\log_e \sqrt{x^2 + 5x - 7} + c$

17. Area (in sq. units) of the region outside $\frac{|x|}{2} + \frac{|y|}{3} = 1$ and inside the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$ is:

A. $3(\pi - 2)$

B. $6(\pi - 2)$

C. $6(4 - \pi)$

D. $3(4 - \pi)$

18. Let $g(x) = \int_0^x f(t)\, dt$, where f is continuous function in $[0,3]$ such that $\frac{1}{3} \le f(t) \le 1$ for all $t \in [0,1]$ and $0 \le f(t) \le \frac{1}{2}$ for all $t \in (1,3]$. The largest possible interval in which $g(3)$ lies is :

A. $[1,3]$

B. $\left[1, -\frac{1}{2}\right]$

C. $\left[-\frac{3}{2}, -1\right]$

D. $\left[\frac{1}{3}, 2\right]$

19. Consider a region $R = \left\{(x,y) \in R^2 : x^2 \le y \le 2x\right\}$. If a line $y = \alpha$ divides the area of region R into two equal parts, then which of the following is true

A. $\alpha^3 - 6\alpha^2 + 16 = 0$

B. $3\alpha^2 - 8\alpha^{\frac{3}{2}} + 8 = 0$

C. $\alpha^3 - 6\alpha^{\frac{3}{2}} - 16$

D. $3\alpha^2 - 8\alpha + 8 = 0$

20. The value of $\lim\limits_{n\to\infty} \frac{1}{n} \sum_{r=0}^{2n-1} \frac{n^2}{n^2 + 4r^2}$ is

A. $\frac{1}{2}\tan^{-1}(2)$

B. $\frac{1}{2}\tan^{-1}(4)$

C. $\tan^{-1}(4)$

D. $\frac{1}{4}\tan^{-1}(4)$

21. The differential equation of the family of curves, $x^2 = 4b(y+b), b \in R$, is:

A. $xy'' = y'$

B. $x(y')^2 = x + 2yy'$

C. $x(y')^2 = x - 2yy'$

D. $x(y')^2 = 2yy' - x$

22. If
$$\int \frac{5\tan x}{\tan x - 2}dx = x + a\ln|\sin x - 2\cos x| + k$$
then a is equal to

A. -1

B. -2

C. 1

D. 2

23. Let $y = y(x)$ be the solution of the differential equation
$\sin x\frac{dy}{dx} + y\cos x = 4x,\ x \in (0,\pi)$. If $y\left(\frac{\pi}{2}\right) = 0$, then $y\left(\frac{\pi}{6}\right)$ is equal to

A. $-\frac{4}{9}\pi^2$

B. $\frac{4}{9\sqrt{3}}\pi^2$

C. $\frac{-8}{9\sqrt{3}}\pi^2$

D. $-\frac{8}{9}\pi^2$

24. The difference between degree and order of a differential equation that represents the family of curves given by $y^2 = a\left(x + \frac{\sqrt{a}}{2}\right), a > 0$ is

25. The integral $\int_0^2 ||x-1| - x|dx$ is equal to

26. Let $f(x)$ and $g(x)$ be two functions satisfying $f(x^2)+g(4-x)=4x^3$ and $g(4-x)+g(x)=0$, then the value of $\int\limits_{-4}^{4} f(x^2)\,dx$

27. Let $[T]$ denote the greatest integer less than or equal to T. Then the value of $\int_1^2 |2x-[3x]|\,dx$ is

28. The area of the region $S=\left\{(x,y):3x^2\le 4y\le 6x+24\right\}$ is

29. Let a and b respectively be the points of local maximum and local minimum of the function $f(x)=2x^3-3x^2-12x$.
If A is the total area of the region bounded by $y=f(x)$, the $x-$axis and the lines $x=a$ and $x=b$, then $4A$ is equal to .

30. Let $f:(0,2)\to R$ be defined as $f(x)=\log_2\left(1+\tan\left(\frac{\pi x}{4}\right)\right)$. Then,
$\lim\limits_{n\to\infty}\frac{2}{n}\left(f\left(\frac{1}{n}\right)+f\left(\frac{2}{n}\right)+\cdots+f(1)\right)$ is equal to