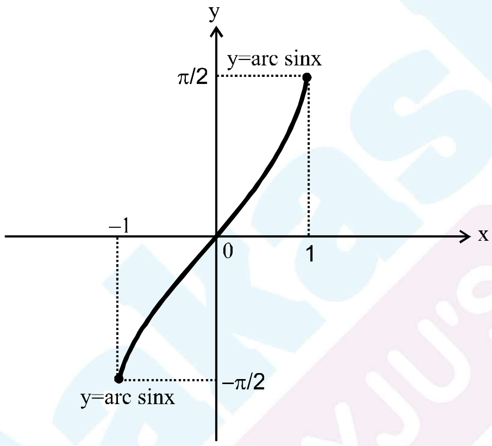
INVERSE TRIGONOMETRIC FUNCTIONS

1. Domain range & Graph of Inverse trigonometric functions :

(a) $f^{-1} : [-1, 1] \rightarrow [-\pi/2, \pi/2]$

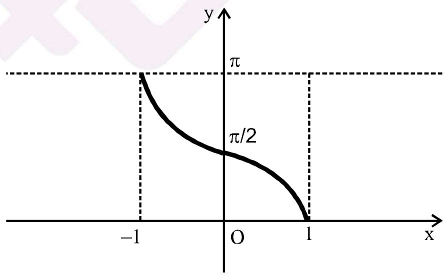
$f^{-1}(x) = \sin^{-1}(x)$



(y= $\sin^{-1}$x)

(b) $f^{-1} : [-1, 1] \rightarrow [0, \pi]$

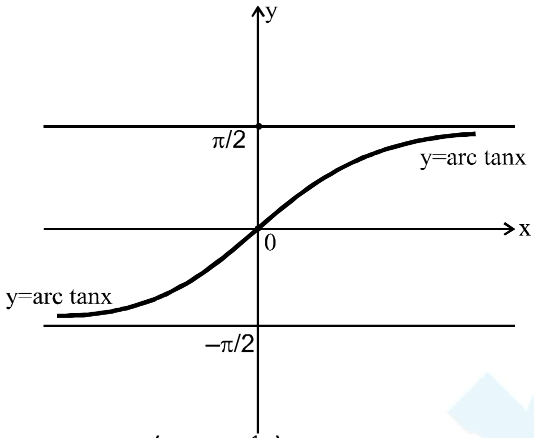
$f^{-1}(x) = \cos^{-1} x$



(y= $\cos^{-1}$x)

(c) $f^{-1} : R \to (-\pi/2, \pi/2)$

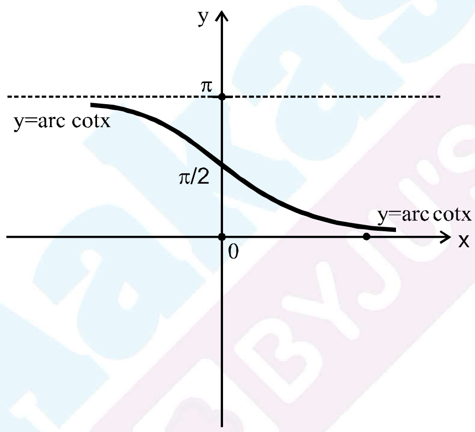
$f^{-1}(x) = \tan^{-1} x$



(y= $\tan^{-1}$x)

(d) $f^{-1} : R \to (0, \pi)$

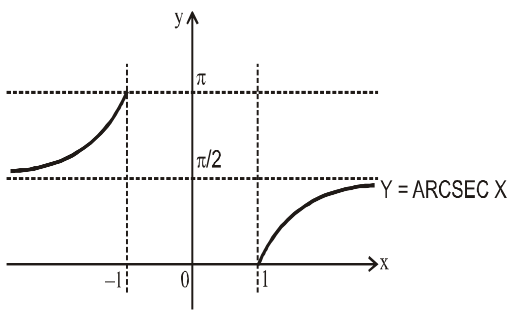
$f^{-1}(x) = \cot^{-1} x$



(y= $\cot^{-1}$x)

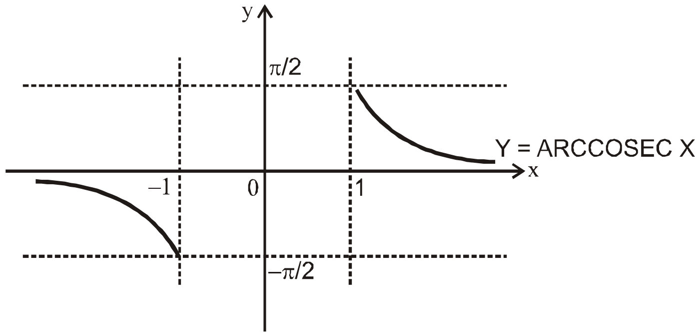
(e) $f^{-1} : (-\infty, -1] \cup [1, \infty) \to [0, \pi/2) \cup (\pi/2, \pi]$

$f^{-1}(x) = \sec^{-1} x$



(f) $f^{-1} : (-\infty, -1] \cup [1, \infty) \to [-\pi/2, 0) \cup (0, \pi/2]$

$f^{-1}(x) = \text{cosec}^{-1} x$



2. Properties of inverse circular functions:

P-1

(i) $y = \sin(\sin^{-1}x) = x$, $x \in [-1,1]$, $y \in [-1,1]$, y is periodic

(ii) $y = \cos(\cos^{-1} x) = x$, $x \in [-1,1]$, $y \in [-1,1]$, y is periodic

(iii) $y = \tan(\tan^{-1} x) = x$, $x \in R$, $y \in R$, y is periodic

(iv) $y = \cot(\cot^{-1} x) = x$, $x \in R$; $y \in R$, y is periodic

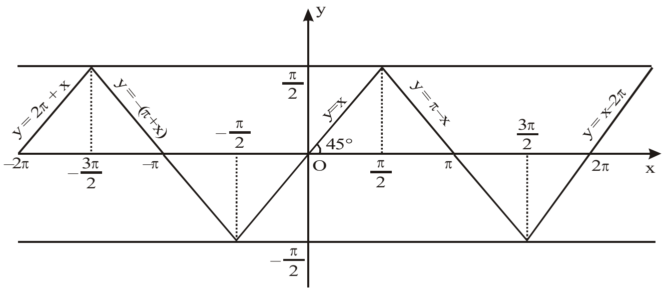
(v) $y = \text{cosec}(\text{cosec}^{-1} x) = x$, $|x| \geq 1$, $|y| \geq 1$, y is periodic

(vi) $y = \sec(\sec^{-1} x) = x$, $|x| \geq 1$; $|y| \geq 1$, y is periodic

P-2

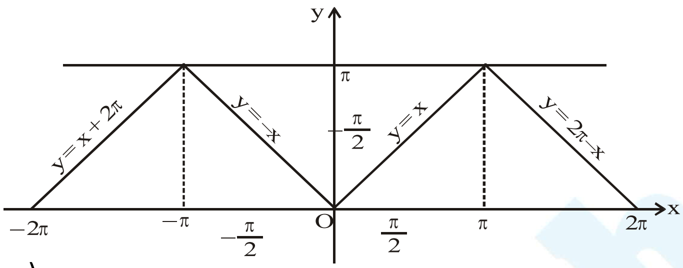
(i) $y = \sin^{-1}(\sin x)$, $x \in R$, $y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ periodic with period 2π.

$$\sin^{-1}(\sin x) = \begin{cases} -\pi - x, & -\frac{3\pi}{2} \leq x \leq -\frac{\pi}{2} \\ x, & -\frac{\pi}{2} < x < \frac{\pi}{2} \\ \pi - x, & \frac{\pi}{2} \leq x \leq \frac{3\pi}{2} \\ x - 2\pi, & \frac{3\pi}{2} \leq x \leq \frac{5\pi}{2} \\ 3\pi - x, & \frac{5\pi}{2} \leq x \leq \frac{7\pi}{2} \\ x - 4\pi, & \frac{7\pi}{2} \leq x \leq \frac{9\pi}{2} \end{cases}$$



(ii) $y = \cos^{-1}(\cos x)$, $x \in R$, $y \in [0,\pi]$, periodic with period 2π and it is an even function.

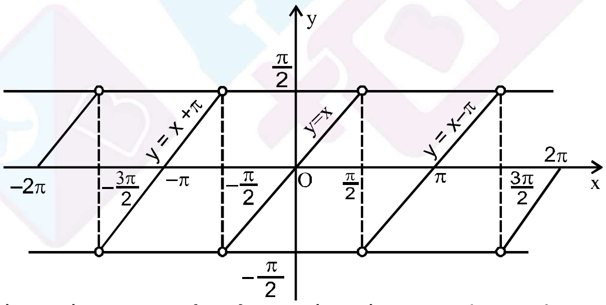
$$\cos^{-1}(\cos x) = \begin{cases} -x & , -\pi \le x \le 0 \\ x & , 0 \le x \le \pi \\ 2\pi - x & , \pi \le x \le 2\pi \\ x - 2\pi & , 2\pi \le x \le 3\pi \\ 4\pi - x & , 3\pi \le x \le 4\pi \end{cases}$$



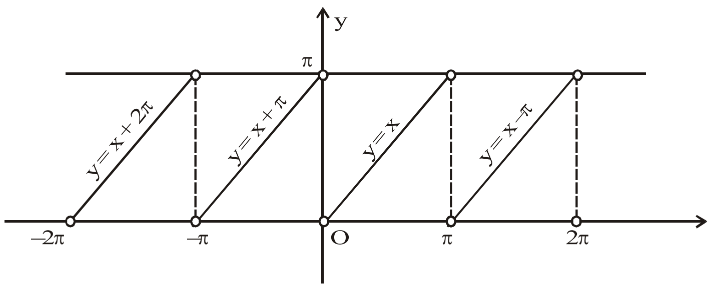
(iii) $y = \tan^{-1}(\tan x)$

$x \in R - \left\{(2n-1)\frac{\pi}{2}, n \in I\right\}$; $y \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, periodic with period π and it is an odd function.

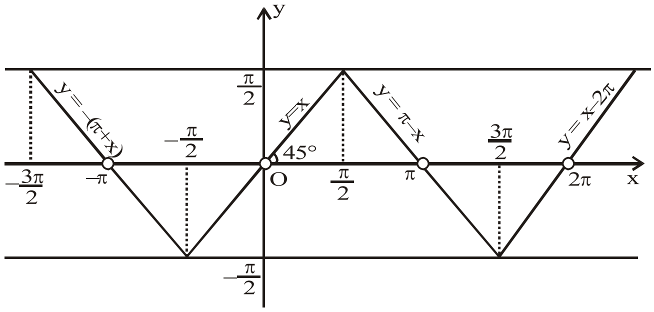
$$\tan^{-1}(\tan x) = \begin{cases} x + \pi & , -\frac{3\pi}{2} < x < -\frac{\pi}{2} \\ x & , -\frac{\pi}{2} < x < \frac{\pi}{2} \\ x - \pi & , \frac{\pi}{2} < x < \frac{3\pi}{2} \\ x - 2\pi & , \frac{3\pi}{2} < x < \frac{5\pi}{2} \\ x - 3\pi & , \frac{5\pi}{2} < x < \frac{7\pi}{2} \end{cases}$$



(iv) $y = \cot^{-1}(\cot x)$, $x \in R - \{n\pi\}$, $y \in (0, \pi)$, periodic with period π



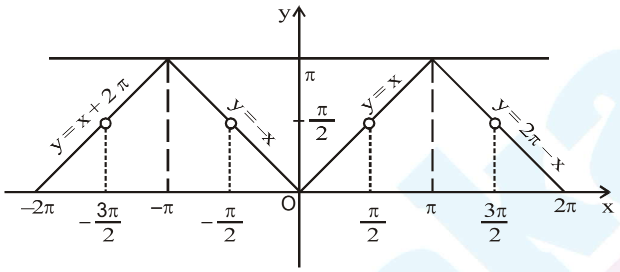
(v) y = cosec^{-1} (cosec x), $x \in R - \{n\pi, n \in I\}$ $y \in \left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$, is periodic with period 2π



(vi) y = sec^{-1} (sec x), y is periodic with period 2π

and it is an even function.

$x \in R - \left\{(2n-1)\frac{\pi}{2} n \in I\right\}$, $y \in \left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$



P-3

(i) cosec^{-1} x = sin^{-1} $\frac{1}{x}$: $x \le -1, x \ge 1$

(ii) sec^{-1} x = cos^{-1} $\frac{1}{x}$: $x \le -1, x \ge 1$

(iii) cot^{-1}x = tan^{-1} $\frac{1}{x}$: $x > 0$

= π + tan^{-1} $\frac{1}{x}$; $x < 0$

P-4

(i) sin^{-1} (–x) = – sin^{-1} x , $-1 \le x \le 1$
(ii) tan^{-1} (–x) = – tan^{-1} x , $x \in R$
(iii) cos^{-1} (–x) = π – cos^{-1} x , $-1 \le x \le 1$
(iv) cot^{-1} (–x) = π – cot^{-1} x , $x \in R$
(v) sec^{-1} (–x) = π – sec^{-1} x , $x \le -1$ or $x \ge 1$
(iv) cosec^{-1}(–x) = – cosec^{-1} x, $x \le -1$ or $x \ge 1$

P-5:

(i) $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$; $-1 \leq x \leq 1$

(ii) $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$; $x \in R$

(iii) $\operatorname{cosec}^{-1} x + \sec^{-1} x = \frac{\pi}{2}$; $|x| \geq 1$

P-6:

(i) $\tan^{-1} x + \tan^{-1} y = \tan^{-1} \frac{x+y}{1-xy}$ where $x > 0, y > 0$ & $xy < 1$

$= \pi + \tan^{-1}\frac{x+y}{1-xy}$, where $x > 0, y > 0$ & $xy > 1$

$= \frac{\pi}{2}$, where $x > 0, y > 0$ & $xy = 1$

$= -\pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right), x < 0, y < 0, xy > 1$

(ii) $\tan^{-1} x - \tan^{-1} y = \tan^{-1} \frac{x-y}{1+xy}$ where $x > 0, y > 0$

(iii) $\sin^{-1} x + \sin^{-1} y = \sin^{-1} [x\sqrt{1-y^2} + y\sqrt{1-x^2}\,]$ where $-1 \leq x, y \leq 1$, $(x^2 + y^2) \leq 1$

Note that : $x^2 + y^2 < 1 \Rightarrow 0 < \sin^{-1} x + \sin^{-1} y < \frac{\pi}{2}$

(iv) $\sin^{-1} x + \sin^{-1} y = \pi - \sin^{-1} [x\sqrt{1-y^2} + y\sqrt{1-x^2}\,]$ where $x, y > 0, x^2 + y^2 > 1$

Note that : $x^2 + y^2 > 1 \Rightarrow \frac{\pi}{2} < \sin^{-1} x + \sin^{-1} y < \pi$.

(v) $\sin^{-1} x - \sin^{-1} y = \sin^{-1} [x\sqrt{1-y^2} - y\sqrt{1-x^2}\,]$ where $-1 \leq x, y \leq 1$ and $x^2 + y^2 \leq 1$

(vi) $\cos^{-1} x + \cos^{-1} y = \cos^{-1} [xy - \sqrt{1-y^2}\,\sqrt{1-x^2}\,]$ where $-1 <= x, y < 1, x + y >= 0$

$2\pi - \cos^{-1}\left(xy - \sqrt{1-x^2}\,\sqrt{1-y^2}\right)$ if $-1 \leq x, y \leq 1$ and $x + y \leq 0$

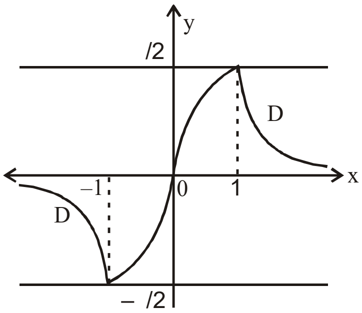
(b) $\cos^{-1} x - \cos^{-1} y = \begin{cases} \cos^{-1}\left(xy + \sqrt{1-x^2}\,\sqrt{1-y^2}\right) ; & x \leq y, -1 \leq x,\ y \leq 1,\ x + y \geq 0 \\ -\cos^{-1}\left(xy + \sqrt{1-x^2}\,\sqrt{1-y^2}\right); & x >= y,\ -1 \leq y \leq 0,\ 0 < x \leq 1 \end{cases}$

(viii) $\tan^{-1}x + \tan^{-1}y + \tan^{-1} z = \tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-yz-zx}\right]$ if $x, y, z > 0$ & $xy + yz + zx < 1$

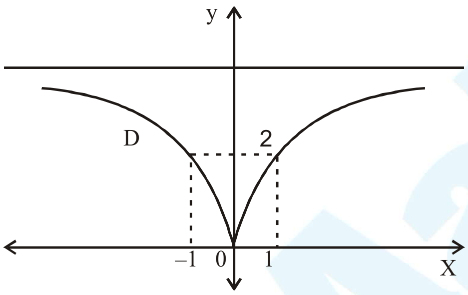
Note : In the above results x & y are taken positive. In case if these are given as negative, we first apply P-4 and then use above results.

3. Simplified inverse trigonometric functions :

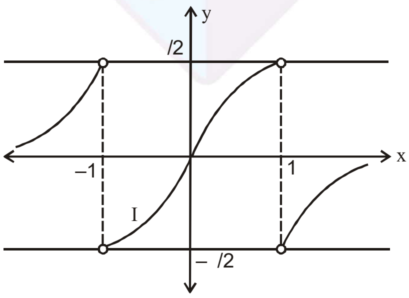
(a) $y = f(x) = \sin^{-1}\left(\dfrac{2x}{1+x^2}\right) = \begin{cases} 2\tan^{-1}x & \text{if } |x| \le 1 \\ \pi - 2\tan^{-1}x & \text{if } x > 1 \\ -(\pi + 2\tan^{-1}x) & \text{if } x < -1 \end{cases}$



(b) $y = f(x) = \cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right) = \begin{cases} 2\tan^{-1}x & \text{if } x \ge 0 \\ -2\tan^{-1}x & \text{if } x < 0 \end{cases}$

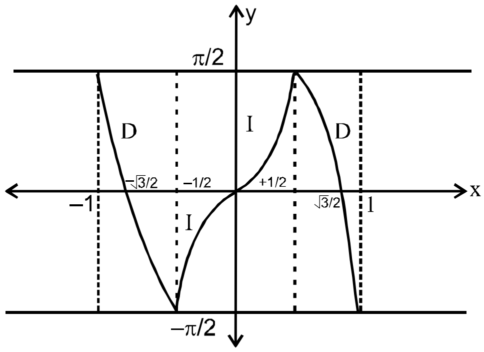


(c) $y = f(x) = \tan^{-1}\dfrac{2x}{1-x^2} = \begin{cases} 2\tan^{-1}x & \text{if } |x| < 1 \\ \pi + 2\tan^{-1}x & \text{if } x < -1 \\ -(\pi - 2\tan^{-1}x) & \text{if } x > 1 \end{cases}$



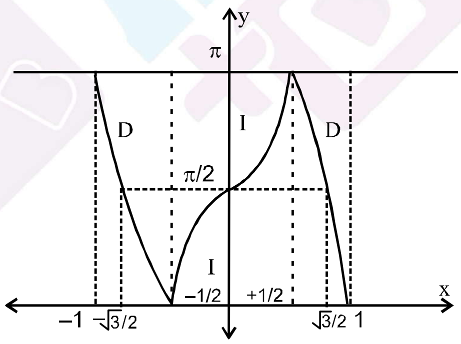
(d) $y = f(x) = \sin^{-1}(3x - 4x^3)$

$$= \begin{cases} -(\pi + 3\sin^{-1}x) & \text{if} \quad -1 \le x \le -\dfrac{1}{2} \\ 3\sin^{-1}x & \text{if} \quad -\dfrac{1}{2} \le x \le \dfrac{1}{2} \\ \pi - 3\sin^{-1}x & \text{if} \quad \dfrac{1}{2} \le x \le 1 \end{cases}$$

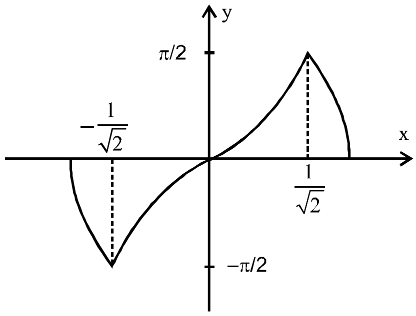


(e) $y = f(x) = \cos^{-1}(4x^3 - 3x)$

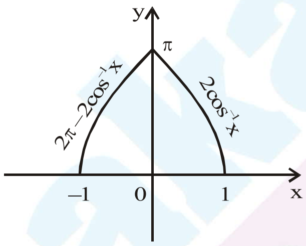
$$= \begin{cases} 3\cos^{-1}x - 2\pi & \text{if} \quad -1 \le x \le -\dfrac{1}{2} \\ 2\pi - 3\cos^{-1}x & \text{if} \quad -\dfrac{1}{2} \le x \le \dfrac{1}{2} \\ 3\cos^{-1}x & \text{if} \quad \dfrac{1}{2} \le x \le 1 \end{cases}$$



(f) $\sin^{-1}\left(2x\sqrt{1-x^2}\right)=\begin{cases}-\left(\pi+2\sin^{-1}x\right) & -1\le x\le -\dfrac{1}{\sqrt{2}}\\ 2\sin^{-1}x & -\dfrac{1}{\sqrt{2}}\le x\le \dfrac{1}{\sqrt{2}}\\ \pi-2\sin^{-1}x & \dfrac{1}{\sqrt{2}}\le x\le 1\end{cases}$



(g) $\cos^{-1}\left(2x^2-1\right)=\begin{cases}2\cos^{-1}x & 0\le x\le 1\\ 2\pi-2\cos^{-1}x & -1\le x\le 0\end{cases}$



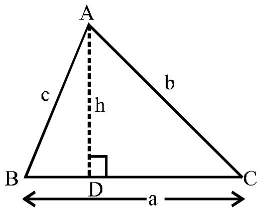
SOLUTIONS OF TRIANGLE

1. Sine formulae:

In any triangle ABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = \lambda = \frac{abc}{2\Delta} = 2R$$

where R is circumaradius and Δ is area of triangle



2. Cosine Formulae:

(a) $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ or $a^2 = b^2 + c^2 - 2bc \cos A$

(b) $\cos B = \frac{c^2 + a^2 - b^2}{2ac}$

(c) $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

3. Projection formulae:

(a) b cos C + c cos B = a
(b) c cos A + a cos C = b
(c) a cos B + b cos A = c

4. Napier's analogy (Tangent Rule):

(a) $\tan\left(\frac{B-C}{2}\right) = \frac{b-c}{b+c}\cot\frac{A}{2}$

(b) $\tan\left(\frac{C-A}{2}\right) = \frac{c-a}{c+a}\cot\frac{B}{2}$

(c) $\tan\left(\frac{A-B}{2}\right) = \frac{a-b}{a+b}\cot\frac{C}{2}$

5. Half Angle formulae:

$s = \frac{a+b+c}{2}$ = semi-permeter of triangle

(a) (i) $\sin\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$ (ii) $\sin\frac{B}{2} = \sqrt{\frac{(s-c)(s-a)}{ca}}$

(iii) $\sin\frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{ab}}$

(b) (i) $\cos\frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}$ (ii) $\cos\frac{B}{2} = \sqrt{\frac{s(s-b)}{ca}}$

(iii) $\cos\frac{C}{2} = \sqrt{\frac{s(s-c)}{ab}}$

(c) (i) $\tan\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}} = \frac{\Delta}{s(s-a)}$

(ii) $\tan\frac{B}{2} = \sqrt{\frac{(s-c)(s-a)}{s(s-b)}} = \frac{\Delta}{s(s-b)}$

(iii) $\tan\frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}} = \frac{\Delta}{s(s-c)}$

(d) Area of Triangle

$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

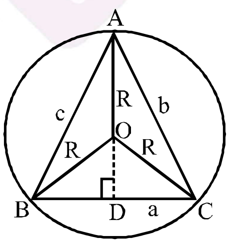
$= \frac{1}{2}$ bc sin A $= \frac{1}{2}$ ca sin B $= \frac{1}{2}$ ab sin C

$= \frac{1}{4}\sqrt{2(a^2b^2+b^2c^2+c^2a^2)-a^4-b^4-c^4}$

6. Radius of the circumcircle 'R':

Circumcentre is the point of concurrence of perpendicular bisectors of the sides and distance between circumcentre & vertex of triangle is called circumradius 'R'.

$$R = \frac{a}{2\sin A} = \frac{b}{2\sin B} = \frac{c}{2\sin C} = \frac{abc}{4\Delta}.$$



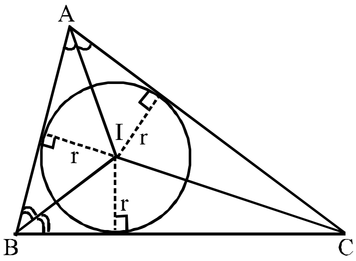
7. Radius of the incircle 'r':

Point of intersection of internal angle bisectors is incentre and perpen dicular distance of incentre from any side is called inradius 'r'

$$r = \frac{\Delta}{s} = (s-a)\tan\frac{A}{2} = (s-b)\tan\frac{B}{2}$$

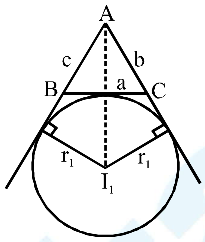
$= (s-c)\tan\frac{C}{2} = 4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

$= a\,\frac{\sin\frac{B}{2}\sin\frac{C}{2}}{\cos\frac{A}{2}} = b\,\frac{\sin\frac{A}{2}\sin\frac{C}{2}}{\cos\frac{B}{2}} = c\,\frac{\sin\frac{B}{2}\sin\frac{A}{2}}{\cos\frac{C}{2}}$



8. Radii of the Ex-Circles:

Point of intersection of two external angle and one internal angle bisectors is excentre and perpendicular distance of excentre from any side is called exradius.If r_1 is the radius of in-scribed circle opposite to angle A of ΔABC and so on then :



(a) $r_1 = \frac{\Delta}{s-a} = s\tan\frac{A}{2} = 4R\sin\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2} = \frac{a\cos\frac{B}{2}\cos\frac{C}{2}}{\cos\frac{A}{2}}$

(b) $r_2 = \frac{\Delta}{s-b} = s\tan\frac{B}{2} = 4R\cos\frac{A}{2}\sin\frac{B}{2}\cos\frac{C}{2} = \frac{b\cos\frac{A}{2}\cos\frac{C}{2}}{\cos\frac{B}{2}}$

(c) $r_3 = \frac{\Delta}{s-c} = s\tan\frac{C}{2} = 4R\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2} = \frac{c\cos\frac{A}{2}\cos\frac{B}{2}}{\cos\frac{C}{2}}$

9. Length of angle bisector, Medians & Altitude:

If m_a, β_a & h_a are the lengths of a median, an angle bisector & altiude from the angle A then,

$\frac{1}{2}\sqrt{b^2+c^2+2bc\cos A} = m_a = \frac{1}{2}\sqrt{2b^2+2c^2-a^2}$ and $\beta_a = \frac{2bc\cos\frac{A}{2}}{b+c}$, $h_a = \frac{a}{\cot B+\cot C}$

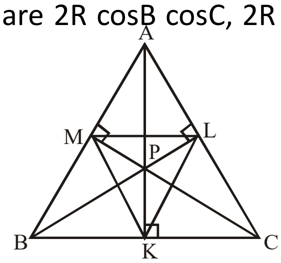
Note that $m_a^2 + m_b^2 + m_c^2 = \frac{3}{4}(a^2+b^2+c^2)$

10. Orthocentre and Pedal triangle:

(a) The point of intersection of altitudes is orthocentre & the triangle KLM which is formed by joining the feet of the altitudes is called the edal triangle.

(b) The distances of the orthocentre from the angular points of the ΔABC are 2R cosA, 2R cosB & 2R cosC.

(c) The distance of orthocentre from sides are 2R cosB cosC, 2R cosC cosA and 2R cosA cosB.



(d) The sides of the pedal triangle are a cos A (=R sin 2A) bcosB = (R sin2B) and c cos C (=R sin 2C) and its angles are $\pi - 2A$, $\pi - 2B$ and $\pi - 2C$

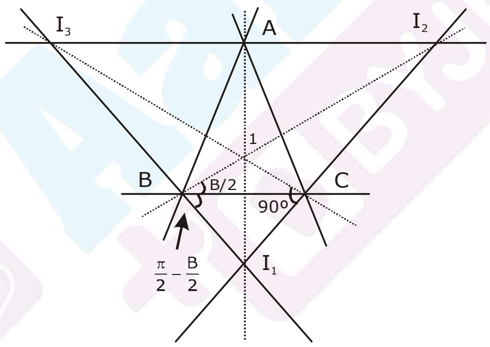
(e) Circumradii of the triangles PBC, PCA, PAB and ABC are equal

(f) Area of pedal triangle = $2\Delta \cos A \cos B \cos C$

$= \frac{1}{2} R^2 \sin 2A \sin 2B \sin 2C$

(g) Circumradii of pedal triangle = R/2

11. Ex-Central Tariangle:



(a) The trianlge formed by joining the three excentres I_1,I_2 and I_3 of ΔABC is called the excentral or excentric triangle.

(b) Incentre I of Δ ABC is the orthocentre of the excentral $\Delta\, I_1I_2I_3$.

(c) Δ ABC is the pedal triangle of the $\Delta I_1I_2I_3$.

(d) The sides of the excentral triangle are

$4R\cos\frac{A}{2}, 4R\cos\frac{B}{2}$ and $4R\cos\frac{C}{2}$ and its angles are $\frac{\pi}{2}-\frac{A}{2}, \frac{\pi}{2}-\frac{B}{2}$ and $\frac{\pi}{2}-\frac{C}{2}$

(e) $II_1 = 4R\sin\frac{A}{2}; II_2 = 4R\sin\frac{B}{2}; II_3 = 4R\sin\frac{C}{2}$

12. The distance between the special points:

(a) The distance between circumcentre and orthocentre is

$$= R\sqrt{1-8\cos A\cos B\cos C}$$

(b) The distance between circumcentre and incentre is

$$=\sqrt{R^2-2Rr}$$

(c) The distance between incentre and orthocentre is

$$=\sqrt{2r^2-4R^2\cos A\cos B\cos C}$$

(d) The distances between circumcentre & excentres are

$$OI_1=R\sqrt{1+8\sin\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}}=\sqrt{R^2+2Rr_1}$$ & so on.

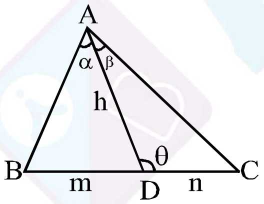
(e) Distance between circumcentre and centroid

$$OG = \sqrt{R^2-\frac{1}{9}(a^2+b^2+c^2)}$$

13. m-n Theorem:

$(m + n) \cot \theta = m \cot \alpha - n \cot \beta$

$(m + n) \cot \theta = n \cot \beta - m \cot \alpha.$



14. Important Points:

(a) (i) If a cos B = b cos A, then the triangle is isosceles.

(ii) If a cos A = b cos B, then the triangle is isosceles or right angled.

(b) In right angled triangle

(i) $a^2 + b^2 + c^2 = 8R^2$

(ii) $\cos^2 A + \cos^2 B + \cos^2 C = 1$

(c) In equilateral triangle

(i) $R = 2r$ (ii) $r_1 = r_2 = r_3 = \frac{3R}{2}$ (iii) $r : R : r_1 = 1 : 2 : 3$

(iv) area $= \dfrac{\sqrt{3}a^2}{4}$ (v) $R = \dfrac{a}{\sqrt{3}}$

(d) (i) The circumcentre lies

(1) inside an acute angled triangle

(2) outside an obtuse angled triangle &

(3) mid point of the hypotenuse of right angled triangle.

(ii) The orthocentre of right angled triangle is the vertex at the right angle.

(iii) The orthocentre, centroid & circumcentre are collinear & centroid divides the line segment joining orthocentre & circumcentre internally in the ratio 2 : 1 except in case of equilateral triangle all these centres coincide

15. Regular Polygon:

Consider a 'n' sided regular polygon of side length 'a'

(a) Radius of incircle of this polygon of side length $r = \dfrac{a}{2}\cot\dfrac{\pi}{n}$

(b) Radius of circumcircle of this polygon $R = \dfrac{a}{2}\operatorname{cosec}\dfrac{\pi}{n}$

(c) Perimeter & area of regular polygon perimeter $= na = 2nr\tan\dfrac{\pi}{n} = 2nR\sin\dfrac{\pi}{n}$

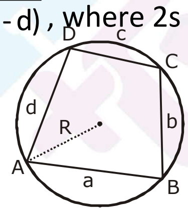
Area $= \dfrac{1}{2}nR^2\sin\dfrac{2\pi}{n} = nr^2\tan\dfrac{\pi}{n} = \dfrac{1}{4}na^2\cot\dfrac{\pi}{n}$

16. Cyclic Quadrilateral:

(a) Quadriateral ABCD is cyclic if $\angle A + \angle C = \pi = \angle B + \angle D$

(opposite angle are supplementary angles)

(b) Area $= \sqrt{(s-a)(s-b)(s-c)(s-d)}$, where $2s = a + b + c + d$,



(c) $\cos B = \dfrac{a^2+b^2-c^2-d^2}{2(ab+cd)}$ & similarly other angles

(d) Ptolemy's theorem : If ABCD is cyclic quadrilateral, then $AC \cdot BD = AB \cdot CD + BC \cdot AD$

17. Solution of Triangles:

Case-I : If three sides are given then to find out three angles use

$\cos A = \dfrac{b^2+c^2-a^2}{2bc}$, $\cos B = \dfrac{c^2+a^2-b^2}{2ac}$ & $\cos C = \dfrac{a^2+b^2-c^2}{2ab}$

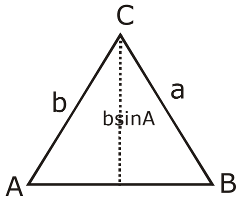
Case-II : Two sides & included angle are given :

Let sides a, b & angle C are given then use $\tan\frac{A-B}{2} = \frac{a-b}{a+b}\cot\frac{C}{2}$ and find value of A – B .... (i)

& $\frac{A+B}{2} = 90º - \frac{C}{2}$ .... (ii) $c = \frac{a\sin C}{\sin A}$ .... (iii)

Case - III:

Two sides a, b & angle A opposite to one of then is given



(a) If a < b sin A No triangle exist

(b) If a = bsin A & A is acute, then one triangle exist which is right angle.

(c) a > b sin A, a < b & A is acute, then two triangle exist

(d) a > b sinA, a > b & A is acute,then one triangle exist

(e) a > b sin A & A is obtuse, then there is one triangle if a > b & no triangle if a < b.

HEIGHTS AND DISTANCES

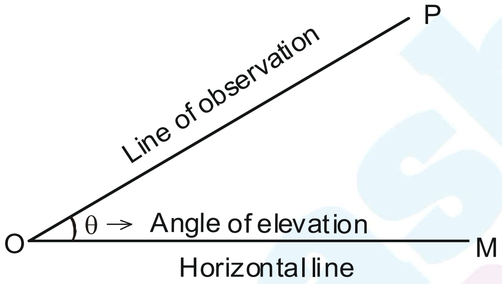
Introduction

One of the important applications of trigonometry is to find the height and distance of the point which are not directly measurable. This is done with the help of trigonometric ratios.

DEFINITIONS

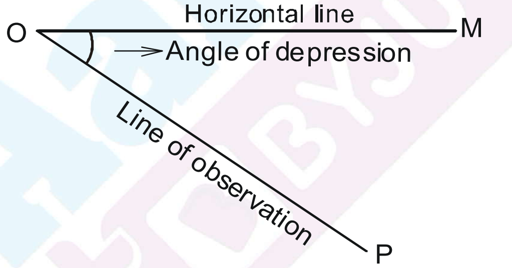
Angle of elevation :

Let O and P be two points where P is at a higher level than O. Let O be the position of the observer and P be the position of the object. Draw a horizontal line OM through the point O. OP is called the line of observation or line of sight. Then $\angle POM = \theta$ is called the angle of elevation of P as observed from O.



Angle of depression :

In the figure, if P be at a lower level than O, then $\angle MOP = \theta$ is called the angle of depression.



Note :

(1) The angle of elevation or depression is the angle between the line of observation and the horizontal line according as the object is at a higher or lower level than the observer.

(2) The angle of elevation or depression is always measured from horizontal line through the point of observation.

Some useful Results

In a triangle ABC,

$\sin\theta = \frac{p}{h}$, $\cos\theta = \frac{b}{h}$

$\tan\theta = p/b$