Subject: Mathematics

1. All possible values of $\theta \in [0, 2\pi]$ for which $\sin 2\theta + \tan 2\theta > 0$ lie in :

A. $\left(0, \frac{\pi}{2}\right) \cup \left(\pi, \frac{3\pi}{2}\right)$

B. $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{5\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{7\pi}{4}\right)$

C. $\left(0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$

D. $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{11\pi}{6}\right)$

2. If $0 < x, y < \pi$ and $\cos x + \cos y - \cos(x+y) = \frac{3}{2}$, then $\sin x + \cos y$ is equal to :

A. $\frac{1+\sqrt{3}}{2}$

B. $\frac{1-\sqrt{3}}{2}$

C. $\frac{\sqrt{3}}{2}$

D. $\frac{1}{2}$

3. The number of roots of the equation, $(81)^{\sin^2 x} + (81)^{\cos^2 x} = 30$ in the interval $[0, \pi]$ is equal to:

A. 3

B. 2

C. 4

D. 8

4. The number of solutions of the equation $x+2\tan x=\frac{\pi}{2}$ in the interval $[0,\ 2\pi]$ is :

A. 5

B. 2

C. 4

D. 3

5. The angle of elevation of a jet plane from a point A on the ground is 60°. After a flight of 20 seconds at the speed of $432\ \mathrm{km/hour}$, the angle of elevation changes to 30°. If the jet plane is flying at a constant height, then its height is

A. $1200\sqrt{3}$ m

B. $1800\sqrt{3}$ m

C. $3600\sqrt{3}$ m

D. $2400\sqrt{3}$ m

6. Two vertical poles are $150m$ apart and the height of one is three times that of the other. If from the middle point of the line joining their feet, an observer finds the angles of elevation of their tops to be complementary, then the height of the shorter pole (in meters) is:

A. 25

B. $20\sqrt{3}$

C. 30

D. $25\sqrt{3}$

7. The value of $\cot \frac{\pi}{24}$ is

A. $3\sqrt{2}-\sqrt{3}-\sqrt{6}$

B. $\sqrt{2}-\sqrt{3}-2+\sqrt{6}$

C. $\sqrt{2}+\sqrt{3}+2-\sqrt{6}$

D. $\sqrt{2}+\sqrt{3}+2+\sqrt{6}$

8. Let $f_k(x)=\frac{1}{k}\left(\sin^k x+\cos^k x\right)$ for $k=1,2,3,\ldots$. Then for all $x \in \mathbb{R}$, the value of $f_4(x)-f_6(x)$ is equal to:

A. $\frac{1}{12}$

B. $\frac{-1}{12}$

C. $\frac{1}{4}$

D. $\frac{5}{12}$

9. Consider a triangular plot ABC with sides $AB=7m, BC=5m$ and $CA=6m$. A vertical lamp-post at the mid point D of AC subtends an angle 30° at B. The height (in m) of the lamp-post is :

A. $\frac{3}{2}\sqrt{21}$

B. $7\sqrt{3}$

C. $2\sqrt{21}$

D. $\frac{2}{3}\sqrt{21}$

10. In a ΔPQR, if $3\sin P + 4\cos Q = 6$ and $4\sin Q + 3\cos P = 1$, then the angle R is equal to

A. $\dfrac{5\pi}{6}$

B. $\dfrac{\pi}{6}$

C. $\dfrac{\pi}{4}$

D. $\dfrac{3\pi}{4}$

11. A spherical gas balloon of radius 16 meter subtends an angle 60° at the eye of the observer A while the angle of elevation of its center from the eye of A is 75°. Then the height (in meter) of the top most point of the balloon from the level of the observer's eye is

A. $8(2 + 2\sqrt{3} + \sqrt{2})$

B. $8(\sqrt{6} - \sqrt{2} + 2)$

C. $8(\sqrt{2} + 2 + \sqrt{3})$

D. $8(\sqrt{6} + \sqrt{2} + 2)$

12. Two vertical poles $AB = 15$m and $CD = 10$m are standing apart on a horizontal ground with points A and C on the ground. If P is the point of intersection of BC and AD, then the height of P(in m) above the line AC is

A. 5

B. $\dfrac{20}{3}$

C. $\dfrac{10}{3}$

D. 6

13. If $\cos(\alpha+\beta)=\frac{3}{5}, \sin(\alpha-\beta)=\frac{5}{13}$ and $0<\alpha,\beta<\frac{\pi}{4},$ then $\tan(2\alpha)$ is equal to:

A. $\frac{21}{16}$

B. $\frac{63}{16}$

C. $\frac{63}{52}$

D. $\frac{33}{52}$

14. The value of $\cos\frac{\pi}{2^2}\cdot\cos\frac{\pi}{2^3}\cdot\ \ldots\ \cdot\cos\frac{\pi}{2^{10}}\cdot\sin\frac{\pi}{2^{10}}$ is :

A. $\frac{1}{1024}$

B. $\frac{1}{2}$

C. $\frac{1}{512}$

D. $\frac{1}{256}$

15. If the equation $\cos^4\theta+\sin^4\theta+\lambda=0$ has real solutions for θ, then λ lies in the interval:

A. $\left(-\frac{1}{2},-\frac{1}{4}\right]$

B. $\left[-1,-\frac{1}{2}\right]$

C. $\left[-\frac{3}{2},-\frac{5}{4}\right]$

D. $\left(-\frac{5}{4},-1\right)$

16. The maximum value of

$$3\cos\theta + 5\sin\left(\theta - \frac{\pi}{6}\right)$$

for any real value of θ is:

A. $\frac{\sqrt{79}}{2}$

B. $\sqrt{19}$

C. $\sqrt{31}$

D. $\sqrt{34}$

17. The expression $\frac{\tan A}{1-\cot A} + \frac{\cot A}{1-\tan A}$ can be written as :

A. $\sin A \cdot \cos A + 1$

B. $\sec A \cdot \operatorname{cosec} A + 1$

C. $\tan A + \cot A$

D. $\sec A + \operatorname{cosec} A$

18. If $L = \sin^2\left(\frac{\pi}{16}\right) - \sin^2\left(\frac{\pi}{8}\right)$ and $M = \cos^2\left(\frac{\pi}{16}\right) - \sin^2\left(\frac{\pi}{8}\right)$, then:

A. $M = \frac{1}{2\sqrt{2}} + \frac{1}{2}\cos\frac{\pi}{8}$

B. $M = \frac{1}{4\sqrt{2}} + \frac{1}{4}\cos\frac{\pi}{8}$

C. $L = -\frac{1}{2\sqrt{2}} + \frac{1}{2}\cos\frac{\pi}{8}$

D. $L = \frac{1}{4\sqrt{2}} - \frac{1}{4}\cos\frac{\pi}{8}$

19. If $\cos(\alpha+\beta)=\frac{3}{5}, \sin(\alpha-\beta)=\frac{5}{13}$ and $0<\alpha,\beta<\frac{\pi}{4}$, then $\tan(2\alpha)$ is equal to:

A. $\frac{21}{16}$

B. $\frac{63}{16}$

C. $\frac{63}{52}$

D. $\frac{33}{52}$

20. The value of $\cos\frac{\pi}{2^2}\cdot\cos\frac{\pi}{2^3}\cdot\ \ldots\ \cdot\cos\frac{\pi}{2^{10}}\cdot\sin\frac{\pi}{2^{10}}$ is :

A. $\frac{1}{1024}$

B. $\frac{1}{2}$

C. $\frac{1}{512}$

D. $\frac{1}{256}$

21. A bird is sitting on the top of a vertical pole $20\ m$ high and its elevation from a point O on the ground is 45°. It flies off horizontally straight away from the point O. After one second, the elevation of the bird from O is reduced to 30°. Then the speed (in m/s) of the bird is

A. $40(\sqrt{2}-1)$

B. $40(\sqrt{3}-\sqrt{2})$

C. $20\sqrt{2}$

D. $20(\sqrt{3}-1)$

22. If $5(\tan^2 x - \cos^2 x) = 2\cos 2x + 9$, then the value of $\cos 4x$ is:

A. $\frac{-3}{5}$

B. $\frac{1}{3}$

C. $\frac{2}{9}$

D. $\frac{-7}{9}$

23. Let a vertical tower AB have its end A on the level ground. Let C be the mid-point of AB and P be a point on the ground such that $AP = 2AB$. If $\angle BPC = \beta$, then $\tan\beta$ is equal to:

A. $\frac{6}{7}$

B. $\frac{1}{4}$

C. $\frac{2}{9}$

D. $\frac{4}{9}$

24. PQR is a triangular park with $PQ = PR = 200$ m.
A T.V. tower stands at the mid-point of QR. If the angles of elevation of the top of the tower at P, Q and R are respectively $45^\circ, 30^\circ$ and 30°, then the height of the tower (in m) is:

A. $50\sqrt{2}$

B. 100

C. 50

D. $100\sqrt{3}$

Subject: Mathematics

1. The number of integral values of k for which the equation $3\sin x + 4\cos x = k + 1$ has a solution, $k \in \mathbb{R}$ is

2. The number of distinct solutions of the equation, $\log_{\frac{1}{2}}|\sin x| = 2 - \log_{\frac{1}{2}}|\cos x|$ in the interval $[0, 2\pi]$, is

3. The numbers of solutions of the equation $|\cot x| = \cot x + \frac{1}{\sin x}$ in the interval $[0, 2\pi]$ is

4. Let S be the sum of all solutions (in radians) of the equation $\sin^4\theta + \cos^4\theta - \sin\theta\cos\theta = 0$ in $[0, 4\pi]$. Then $\frac{8S}{\pi}$ is equal to

5. Let AD and BC be two vertical poles at A and B respectively on a horizontal ground. If $AD = 8m, BC = 11m$ and $AB = 10m$; then the distance (in meters) of a point M on AB from the point A such that $MD^2 + MC^2$ is minimum is

6. The angle of elevation of the top of a hill from a point on the horizontal plane passing through the foot of the hill is found to be 45°. After walking a distance of 80 meters towards the top, up a slope inclined at an angle of 30° to the horizontal plane, the angle of elevation of the top of the hill becomes 75°. Then the height of the hill (in meters) is