GRAVITATION

Newton's law of Gravitation

$$m_1 \xleftarrow{\qquad r \qquad} m_2$$
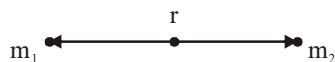
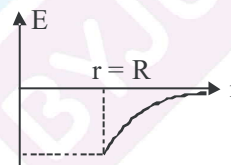Force of attraction between two point masses $F = \dfrac{Gm_1m_2}{r^2}$

Directed along the line joining the point masses.

- It is a conservative force field. Mechanical energy is conserved.
- It is a central force field. Angular momentum is conserved.

Gravitational field due to spherical shell

(i) Outside the shell $E_g = -\dfrac{GM}{r^2}$, where r > R

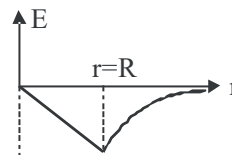(ii) On the surface $E_g = -\dfrac{GM}{R^2}$, where r = R

(iii) Inside the shell $E_g = 0$ where r < R

[Note : Direction always towards the centre of the sphere]

Gravitational field due to solid sphere

(i) Outside the sphere $E_g = -\dfrac{GM}{r^2}$, where r > R

(ii) On the surface $E_g = -\dfrac{GM}{R^2}$, where r = R

(iii) Inside the sphere $E_g = -\dfrac{GMr}{R^3}$, where r < R

- **Acceleration due to gravity** $g = \dfrac{GM}{R^2}$

- **At height h** $g_h = \dfrac{GM}{(R+h)^2}$ If $h \ll R$; $g_h \approx g_s\left(1 - \dfrac{2h}{R}\right)$

- **At depth d** $g_d = \dfrac{GM(R-d)}{R^3} = g_s\left(1 - \dfrac{d}{R}\right)$

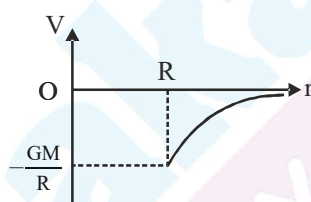- **Effect of rotation on g,** $g' = g - \omega^2 R\cos^2\lambda$, where λ is angle of latitude

Gravitational potential

Due to a point mass at a distance $\quad V = -\dfrac{GM}{r}$

Gravitational potential due to spherical shell

(i) Outside the shell $\quad V = -\dfrac{GM}{r}, r > R$

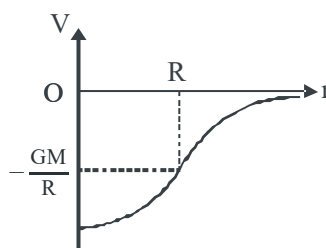(ii) Inside / on the surface the shell $\quad V = -\dfrac{GM}{R}, r \leq R$

Potential due to solid sphere

(i) Outside the sphere $\qquad V = -\dfrac{GM}{r}, r > R$

(ii) On the surface $\qquad V = -\dfrac{GM}{R}, r = R$

(iii) Inside the sphere $\qquad V = -\dfrac{GM(3R^2 - r^2)}{2R^3}, r < R$

Potential on the axis of a thin ring at a distance x

$V = -\dfrac{GM}{\sqrt{R^2 + x^2}}$

GRAVITATION

Escape velocity from a planet of mass M and radius R

$$v_e = \sqrt{\frac{2GM}{R}}$$

Orbital velocity of satellite

$$v_0 = \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM}{(R+h)}}$$

For nearby satellite

$$v_0 = \sqrt{\frac{GM}{R}} = \frac{v_e}{\sqrt{2}}$$

Here V_e = escape velocity on earth surface.

Time period of satellite

$$T = \frac{2\pi r}{v} = \frac{2\pi r^{3/2}}{\sqrt{GM}}$$

Energies of a satellite

(i) Potential energy $\quad U = -\frac{GMm}{r}$

(ii) Kinetic energy $\quad K = \frac{1}{2}mv^2 = \frac{GMm}{2r}$

(iii) Mechanical energy $\quad E = U + K = -\frac{GMm}{2r}$

(iv) Binding energy $\quad BE = -E = \frac{GMm}{2r}$

Kepler's laws

(i) law of orbitals.

Path of a planet is elliptical with the sun at a focus.

(ii) Law of areas

Areal velocity $\frac{dA}{dt}$ = constant $= \frac{L}{2m}$

(iii) Law of periods

$T^2 \propto a^3$ or $T^2 \propto \left(\frac{r_{max} + r_{min}}{2}\right)^3 \propto$ (mean radius)^3. For circular orbits $\quad T^2 \propto R^3$

KEY POINTS

- At the centre of earth, a body has centre of mass, but no centre of gravity.

- The centre of mass and centre of gravity of a body coincide if gravitation field is uniform.

- You will not experience gravitational force in daily life due to objects of same size as value of G is very small.

- Moon travellers tie heavy weight at their back before landing on Moon due to smaller value of g at Moon.

- Space rockets are usually launched in equatorial line from West to East because g is minimum at equator and earth rotates from West to East about its axis.

- Angular momentum in gravitational field is conserved because gravitational force is a central force.

- Kepler's second law or constancy of areal velocity is a consequence of conservation of angular momentum.

CURRENT ELECTRICITY AND HEATING EFFECT OF CURRENT

ELECTRIC CURRENT :

(i) Electric charges in motion constitute an electric current. Any medium having practically free electric charges (i.e. free to migrate) is a conductor of electricity.

(ii) The electric charge flows from higher potential energy state to lower potential energy state.

(iii) Positive charge flows from higher to lower potential and negative charge flows from lower to higher. Metals such as gold, silver, copper, aluminium etc. are good conductors.

ELECTRIC CURRENT IN A CONDUCTOR

(i) In absence of potential difference across a conductor, no net current flows through a cross section. When a potential difference is applied across a conductor the charge carriers (electrons in case of metallic conductors) flow in a definite direction which constitutes a net current in it.

(ii) These electrons are not accelerated by electric field in the conductor produced by potential difference across the conductor. They move with a constant drift velocity.

(iii) The direction of current is along the flow of positive charge (or opposite to flow of negative charge). $i = n v_d e A$, where V_d = drift velocity

ELECTRIC CURRENT AND CURRENT DENSITY

The strength of the current i is the rate at which the electric charges are flowing. If a charge Q coulomb passes through a given cross section of the conductor in t second the current I through the conductor is given by

$$I = \frac{Q \ (coulomb)}{t \ (second)} (ampere)$$

Ampere is the unit of current. If i is not constant then $i = \dfrac{dq}{dt}$, where dq is net charge transported at a section in time dt. In a current carrying conductor we can define a vector which gives the direction as current per unit normal, cross sectional area & is known as current density.

Thus, $\vec{J} = \dfrac{I}{S}\hat{n}$ or $I = \vec{J}.\vec{S}$

Where $\hat{n}$ is the unit vector in the direction of the flow of current.

For random J or S, we use I = $\int \vec{j}.\vec{ds}$

RELEATION IN J, E AND V_D:

In conductors drift velocity of electrons is proportional to the electric field inside the conductor as ; $V_d = \mu E$

where μ is the mobility of electrons

current density is given as $J = \dfrac{I}{A} = ne\, V_d = ne\,(\mu E) = \sigma E$

where $\sigma = ne\mu$ is called conductivity of material and we can also write

$\rho = \dfrac{I}{\sigma} \rightarrow$ resistivity of material.

Thus **E = ρ J.** It is called as differential form of Ohm's Law.

SOURCES OF POTENTIAL DIFFERENCE & ELECTROMOTIVE FORCE

Dry cells, secondary cells, generator and thermo couple are the devices used for producing potential difference in an electric circuit. The potential difference between the two terminals of a source when no energy is drawn from it, is called the" Electromotive force" or " EMF " of the source. The unit of potential difference is volt.

1 volt = 1 Ampere × 1 Ohm.

ELECTRICAL RESISTANCE

The property of a substance which opposes the flow of electric current through it, is termed as electrical resistance. Electrical resistance depends on the size, geometery, temperature and internal structure of the conductor.

LAW OF RESISTANCE

The resistance R offered by a conductor depends on the following factors:

$R \propto I$ (length of the conductor) ; $R \propto \dfrac{1}{A}$ (cross section area of the conductor)

(At a given temperature $R = \rho \dfrac{\ell}{A}$. Where ρ is the resistivity of the material). It is also known as specific resistance of the material & it depends upon nature of conductor.

DEPENDENCE OF RESISTANCE ON TEMPERATURE

(i) The resistance of most conductors and all pure metals increases with temperature, but there are a few in which resistance decreases with temperature.

(ii) If R_o & R be the resistance of a conductor at 0°C and θ°C, then it is found that R= R_o(1 + α θ).

Here we assume that the dimensions of resistance do not change with temperature if expansion coefficient of material is considerable. Then instead of resistance we use same property for resistivity as $\rho = \rho_o (1 + \alpha\theta)$.

(iii) The materials for which resistance decreases with temperature, the temperature coefficient of resistance is negative.

Where α is called the temperature co-efficient of resistance .

(iv) The unit of α is K^{-1} or $°C^{-1}$.

(v) Reciprocal of resistivity is called conductivity and reciprocal of resistance is called conductance (G) . S.I. unit of G is mho.

OHM'S LAW

Ohm's law is the most fundamental law of all the laws in electricity.

It says that the current through the cross section of the conductor is proportional to the applied potential difference under the given physical condition. V = R I . Ohm's law is applicable to only metalic conductors.

KIRCHHOFF'S LAW'S

(i) Junction law or Nodal Analysis : This law is based on law of conservation of charge. It states that" The algebric sum of the currents meeting at a point is zero" or total currents entering a junction is equals to total current leaving the junction.
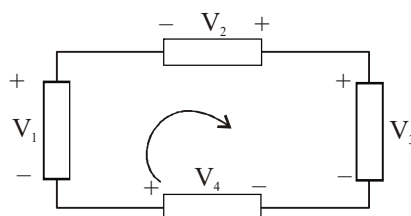
$$\sum I_{in} = \sum I_{out}$$

It is also known as KCL (Kirchhoffs current law) .

(ii) Loop analysis : The algebric sum of all the voltages in closed circuit is zero.

$$\sum IR + \sum EMF = 0$$ in a closed loop.

The closed loop can be traversed in any direction. While traversing a loop if higher potential point is entered, put a + ve sign in expression or if lower potential point is entered put a negative sign.
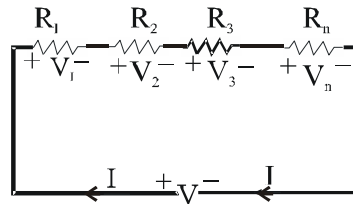

$-V_1 - V_2 + V_3 - V_4 = 0$. Boxes may contain resistor or battery or any other element (linear or non-linear).

It is also known as **KVL (Kirchhoff's voltage law) .**

COMBINATION OF RESISTANCES

A number of resistance can be connected and all the complited combinations can be reduced to two different types, namely series and parallel.

(i) RESISTANCE IN SERIES

When the resistances are connected end to end then they are said to be in series. The current through each resistor is same. The effective resistance appearing across the battery;
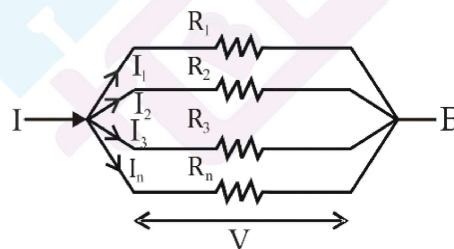
$R = R_1 + R_2 + R_3 + ... + R_n$ and

$V = V_1 + V_2 + V_3 + ... + V_n$

The voltage across a resistor is proportional to the resistance

$$V_1 = \frac{R_1}{R_1 + R_2 + ... + R_n} V : V_2 = \frac{R_2}{R_1 + R_2 + ... + R_n} V \ etc$$

(ii) RESISTANCE IN PARALLEL

A parallel circuit of resistors is one, in which the same voltage is applied across all the components in a parallel grouping of resistors $R_1, R_2, R_3 .... R_n$.

Conclusion:

(a) Potnetial difference across each resistor is same

(b) $I = I_1 + I_2 + I_3 + ... I_n$.

(c) Effective resistance (R) then $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3} + ... \dfrac{1}{R_n}$

(d) Current in different resistors is inversally proportional to the resistance.

$$I_1 : I_2 : .... I_n = \frac{1}{R_1} : \frac{1}{R_2} : \frac{1}{R_3} : .... \frac{1}{R_n}$$

where G = $\dfrac{1}{R}$ = conductance of a resistor

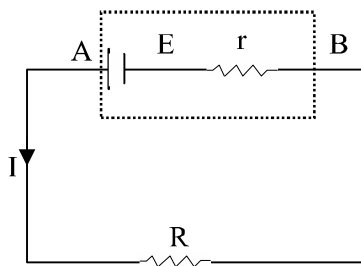$I_1 = \dfrac{G_1}{G_1 + G_2 + ...G_n}I, I_2 = \dfrac{G_2}{G_1 + G_2 + ...G_n}I$ etc.

Emf Of A cell & Its internal resistance

If a cell emf E and internal resistance r be connected with a resistance R the total resistance of the circuit is (R+ r)

$I = \dfrac{E}{R+r}$; $V_{AB} = E - I\left(R+r\right)$

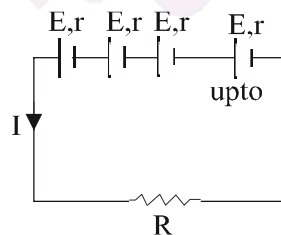where V_{AB} = Terminal voltage of the battery.

If $r \rightarrow 0$, cell is Ideal & $V \rightarrow E$ & $r = R\left(\dfrac{E}{v} - 1\right)$

Grouping of cells

(i) Cells In Series : Let there be n cells each of emf E, arranged in series. Let r be the internal resistance of each cell. The total emf = n E. Current in the circuit I = $\dfrac{nE}{R+nr}$
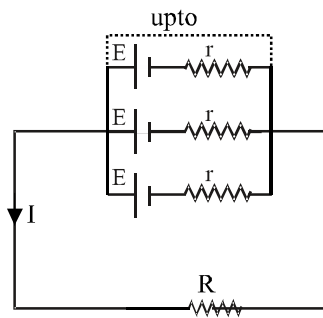
If nr << R then I = $\dfrac{nE}{R} \rightarrow$ Series combination should be used

If nr >> R then I = $\dfrac{E}{r} \rightarrow$ Series combination should not be used.

(ii) Cells In parallel : If m cells each of emf E & internal resitance r be connected in parallel and if this comination be connected to an external resistance then

the emf of the circuit = E Internal resistance of the circuit = $\dfrac{r}{m}$.

$$I = \frac{E}{R + \dfrac{r}{m}} = \frac{mE}{mR + r}$$

If mR << r then $I = \dfrac{mE}{r} \rightarrow$ Parallel combination should be used.

If mR >> r then $I = \dfrac{E}{R} \rightarrow$ Parallel combination will be used.

(iii) Cells In Multiple ARC :
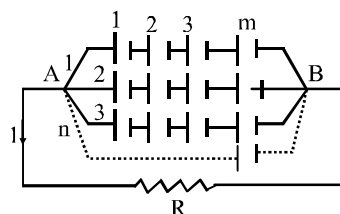
mn = number identical cells.

n = number of rows

m = number of cells in each rows:

The combination of cells is equivalent to single cell of :

(a) emf = mE & (b) internal resistance = $\dfrac{mr}{n}$

Current $I = \dfrac{mE}{R + \dfrac{mr}{n}}$

For maximum current

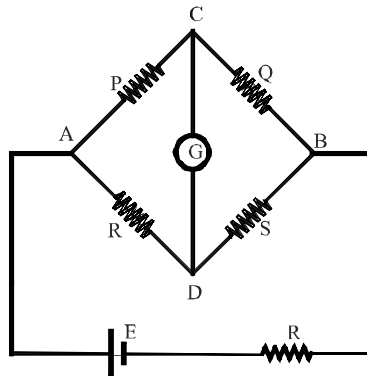nR = mr or R = $\dfrac{mr}{n}$ so $I_{max} = \dfrac{nE}{2r} = \dfrac{mE}{2R}$

For a cell to deliver maximum power across the load internal resistance = load resistance

WHEAT STONE NETWORK

When current through the galvanometer is zero

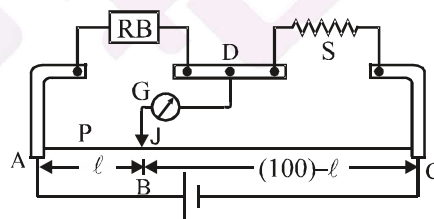(null point or balance point) $\dfrac{P}{Q} = \dfrac{R}{S}$

When

PS > QR, $V_C < V_D$ & PS < QR, $V_C > V_D$

or PS = QR $\Rightarrow$ products of opposite arms are equal. Potential difference between C & D at null point is zero . The null point is not affected by resistance of G & E. It is not affected even if the positions of G & E are interchanged.

$I_{CD} \propto (QR - PS)$

Metre Bridge

At balance condition ; $\dfrac{P}{Q} = \dfrac{R}{S} \Rightarrow \dfrac{r\ell}{r(100-\ell)} = \dfrac{R}{S} \Rightarrow S = \dfrac{(100-\ell)}{\ell}R$
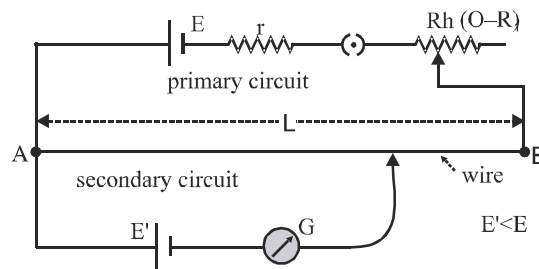

POTENTIOMETER

A potentiometer is a linear conductor of unifonn cross-section with a steady current set up in it. This maintains a uniform potential gradient along the length of the wire . Any potential difference which is less than the potential difference maintained across the potentiometer wire can be measured using this . The potentiometer equation is $\dfrac{E_1}{E_2} = \dfrac{\ell_1}{\ell_2}$
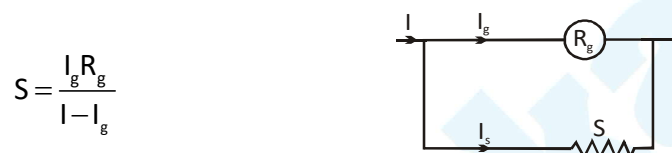
Circuits of potentiometer:

$$X = \frac{V}{L} = \frac{current \times resistance\,of\,potentiometer\,wire}{length\,of\,potentiometer\,wire} = I\left(\frac{R}{L}\right)$$

AMMETER

It is a modified form of suspended coil galvanometer, it is used to measure current . A shunt (small resistance) is connected in parallel with galvanometer to convert it into ammeter.

$$S = \frac{I_g R_g}{I - I_g}$$

where
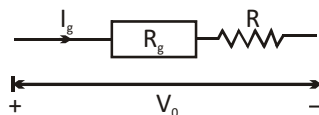
R_g = galvanometer resistance

I_g = Maximum current that can flow through the galvanometer .

I = Maximum current that can be measured using the given ammeter .

An ideal ammeter has zero resistance.

VOLTMETER

A high resistance is put in series with galvanometer. It is used to measure potential difference.

$$I_g = \frac{V_0}{R_g + R}; \; R \to \infty, \; Ideal\,voltmeter$$

ELECTRICAL POWER

The energy liberated per second in a device is called its power. The electrical power P delivered by an electrical device is given by P = VI, where V = potential difference across device & I = current. If the current enters the higher potential point of the device then

power is consumed by it (i.e. acts as load). If the current enters the lower potential point then the device supplies power (i.e. acts as source).

Power consumed by a resistor $P = I^2R = VI = \dfrac{V^2}{R}$

HEATING EFFECT Of ELECTRIC CURRENT

When a current is passed through a resistor energy is wasted in overcoming the resistances of the wire . This energy is converted into heat

$W = VIt = I^2Rt = \dfrac{V^2}{R} t$

JOULES LAW OF ELECTRICAL HEATING

The heat generated (in joules) when a current of I ampere flows through a resistance of R ohm for T second is given by :

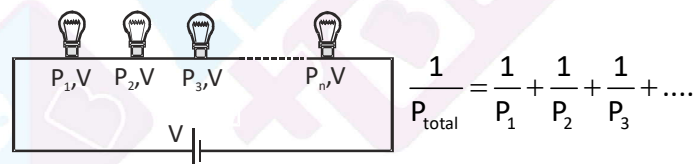$H = I^2RT$ joule $= \dfrac{I^2RT}{4.2}$ calories.

If current is variable passing through the conductor then we use it for heat produced in

resistance in time 0 to T is : $H = \displaystyle\int_0^T I^2R\,dt$
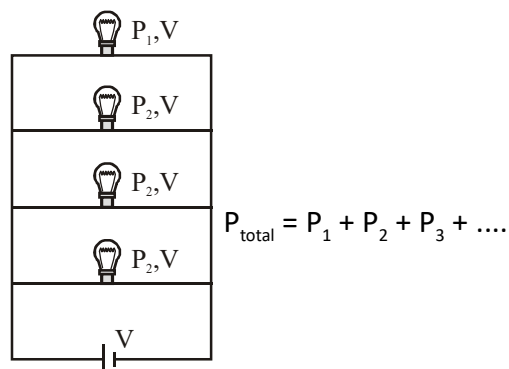
UNIT OF ELECTRICAL ENERGY CONSUMPTION

1 unit of electrical energy = kilowatt hour = 1 kWh = 3.6×10^6 joules.

(i) Series combination of Bulbs

$\dfrac{1}{P_{total}} = \dfrac{1}{P_1} + \dfrac{1}{P_2} + \dfrac{1}{P_3} + ...$

(ii) Parallel combination of Bulbs

$P_{total} = P_1 + P_2 + P_3 + ...$

KEY POINTS

(i) A current flows through a conductor only when there is an electric field within the conductor because the drift velocity of electrons is directly proportional to the applied electric field. Electric field outside the conducting wire which carries a constant current is zero because net charge on a current carrying conductor is zero.

(ii) A metal has a resistance and gets often heated by flow of current because when free electrons drift through a metal, they make occasional collisions with the lattice. These collisions are inelastic and transfer energy to the lattice as internal energy. Ohm's law holds only for small current in metallic wire, not for high currents because resistance increases with increase in temperature.

(iii) Potentiometer is an ideal instrument to measure the potential difference because potential gradient along the potentiometer wire can be made very small. An ammeter is always connected in series whereas a voltmeter is connected in parallel because an ammeter is a low-resistance galvanometer while a voltmeter is a high-resistance galvanometer. Current is passed through a metallic wire, heating it red, when cold water is poured over half of the portion, rest of the portion becomes more hot because resistance decreases due to decrease in temperature so current through wire increases.