Subject: Mathematics

1. For each $x \in \mathbb{R}$, let $[x]$ be the greatest integer less than or equal to x. Then $\lim\limits_{x \to 0^-} \dfrac{x([x] + |x|)\ \sin[x]}{|x|}$ is equal to :

   A. $\sin 1$

   B. 0

   C. $-\sin 1$

   D. 1

2. $\lim\limits_{x \to 0} \dfrac{\int_0^{x^2} (\sin \sqrt{t}) dt}{x^3}$ is equal to :

   A. $\dfrac{2}{3}$

   B. 0

   C. $\dfrac{1}{15}$

   D. $\dfrac{3}{2}$

3. For each $t \in \mathbf{R}$, let $[t]$ be the greatest integer less than or equal to t. Then,

$$\lim_{x \to 1^+} \frac{(1 - |x| + \sin|1 - x|) \sin\left(\frac{\pi}{2}[1 - x]\right)}{|1 - x|[1 - x]}$$

   A. equals 0

   B. equals 1

   C. equals -1

   D. does not exist

4. Let $f:(-1,1) \rightarrow \mathbb{R}$ be a function defined by $f(x)=\max\left\{-|x|,\ -\sqrt{1-x^2}\right\}$. If K be the set of all points at which f is not differentiable, then K has exactly

A. one element

B. two elements

C. three elements

D. five elements

5. Let $[x]$ denote the greatest integer less than or equal to x. Then :
$\lim\limits_{x \rightarrow 0} \dfrac{\tan(\pi \sin^2 x)+(|x|-\sin(x[x]))^2}{x^2}$:

A. equals π

B. equals 0

C. equals $\pi+1$

D. does not exist

6. If $\lim\limits_{x \rightarrow 1} \dfrac{x^4-1}{x-1}=\lim\limits_{x \rightarrow k} \dfrac{x^3-k^3}{x^2-k^2}$, then k is :

A. $\dfrac{4}{3}$

B. $\dfrac{8}{3}$

C. $\dfrac{3}{8}$

D. $\dfrac{3}{2}$

7. If $f(x) = \begin{cases} \dfrac{\sin(\alpha+2)x+\sin x}{x}, & x<0 \\ b, & x=0 \\ \dfrac{(x+3x^2)^{\frac{1}{3}}-x^{\frac{1}{3}}}{x^{\frac{4}{3}}}, & x>0 \end{cases}$ is continuous at $x=0$ then $a+2b$ is equal to :

A. -2

B. 1

C. 0

D. -1

8. Let $f : \mathbb{R} \to \mathbb{R}$ be a function defined as

$$f(x) = \begin{cases} 5, & \text{if} \quad x \le 1 \\ a+bx, & \text{if} \quad 1<x<3 \\ b+5x, & \text{if} \quad 3 \le x < 5 \\ 30, & \text{if} \quad x \ge 5 \end{cases}$$

Then, which of the following is correct regarding the function f :

A. continuous if $a=-5$ and $b=10$

B. continuous if $a=5$ and $b=5$

C. continuous if $a=0$ and $b=5$

D. Not continuous for any values of a and b

9. Let the functions $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ be defined as:

$$f(x) = \begin{cases} x+2, & x<0 \\ x^2, & x \ge 0 \end{cases} \text{ and } g(x) = \begin{cases} x^3, & x<1 \\ 3x-2, & x \ge 1 \end{cases}$$

Then, the number of points in $\mathbb{R}$ where $(fog)(x)$ is non differentiable is equal to :

A. 1

B. 2

C. 3

D. 0

10. If a function $f(x)$ defined by

$$f(x)=\begin{cases} ae^{x}+be^{-x}, & -1\le x<1 \\ cx^{2}, & 1\le x\le 3 \\ ax^{2}+2cx & 3<x\le 4 \end{cases}$$

be continuous for some $a, b, c \in \mathbb{R}$ and $f'(0)+f'(2)=e$, then the value of a is :

A. $\dfrac{1}{e^{2}-3e+13}$

B. $\dfrac{e}{e^{2}-3e-13}$

C. $\dfrac{e}{e^{2}+3e+13}$

D. $\dfrac{e}{e^{2}-3e+13}$

11. $\lim\limits_{n\to\infty}\left(1+\dfrac{1+\frac{1}{2}+\;\ldots\ldots\;+\frac{1}{n}}{n^{2}}\right)^{n}$ is equal to :

A. $\dfrac{1}{2}$

B. $\dfrac{1}{e}$

C. 1

D. 0

12. The value of the limit $\lim\limits_{\theta\to 0}\dfrac{\tan\left(\pi\cos^{2}\theta\right)}{\sin\left(2\pi\sin^{2}\theta\right)}$ is equal to:

A. $-\dfrac{1}{2}$

B. $-\dfrac{1}{4}$

C. 0

D. $\dfrac{1}{4}$

13. The value of $\lim\limits_{x \to 0^+} \dfrac{\cos^{-1}(x-[x]^2)\cdot\sin^{-1}(x-[x]^2)}{x-x^3}$, where $[x]$ denotes the greatest integer $\leq x$ is:

A. 0

B. $\dfrac{\pi}{4}$

C. $\dfrac{\pi}{2}$

D. π

14. If the function $f(x) = \begin{cases} k_1(x-\pi)^2 - 1, & x \leq \pi \\ k_2 \cos x, & x > \pi \end{cases}$ is twice differentiable, then the ordered pair (k_1, k_2) is equal to:

A. $(1,1)$

B. $(1,0)$

C. $\left(\dfrac{1}{2}, -1\right)$

D. $\left(\dfrac{1}{2}, 1\right)$

15. If $\lim\limits_{x \to \infty} \left(\sqrt{x^2 - x + 1} - ax\right) = b$, then the ordered pair (a, b) is

A. $\left(1, \dfrac{1}{2}\right)$

B. $\left(-1, -\dfrac{1}{2}\right)$

C. $\left(-1, \dfrac{1}{2}\right)$

D. $\left(1, -\dfrac{1}{2}\right)$

16. Let a function $f : \mathbb{R} \to \mathbb{R}$ be defined as

$$f(x) = \begin{cases} \sin x - e^x & \text{if } x \leq 0 \\ a + [-x] & \text{if } 0 < x < 1 \\ 2x - b & \text{if } x \geq 1 \end{cases}$$

where $[x]$ is the greatest integer less than or equal to x. If f is continuous on $\mathbb{R}$, then $(a + b)$ is equal to

A. 5

B. 3

C. 4

D. 2

17. The value of $\lim\limits_{x \to 0} \left(\dfrac{x}{\sqrt[8]{1 - \sin x} - \sqrt[8]{1 + \sin x}} \right)$ is equal to

A. 4

B. -4

C. -1

D. 0

18. If α, β are the distinct roots of $x^2 + bx + c = 0$, then

$\lim\limits_{x \to \beta} \dfrac{e^{2(x^2+bx+c)} - 1 - 2(x^2 + bx + c)}{(x - \beta)^2}$ is equal to

A. $b^2 - 4c$

B. $b^2 + 4c$

C. $2(b^2 + 4c)$

D. $2(b^2 - 4c)$

19. If $f : \mathbb{R} \rightarrow \mathbb{R}$ is given by $f(x) = x + 1$, then the value of $\lim_{n \to \infty} \frac{1}{n}\left[f(0) + f\left(\frac{5}{n}\right) + f\left(\frac{10}{n}\right) + \ldots + f\left(\frac{5(n-1)}{n}\right)\right]$, is:

A. $\frac{7}{2}$

B. $\frac{3}{2}$

C. $\frac{5}{2}$

D. $\frac{1}{2}$

20. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined as

$$f(x) = \begin{cases} \frac{x^3}{(1-\cos 2x)^2} \cdot \log_e\left(\frac{1+2xe^{-2x}}{(1-xe^{-x})^2}\right) & , \quad x \neq 0 \\ \alpha & , \quad x = 0 \end{cases}$$

If f is continuous at $x = 0$, then α is equal to:

A. 0

B. 1

C. 2

D. 3

21. Let $f : \mathbb{R} \to \mathbb{R}$ be defined as

$$f(x) = \begin{cases} \dfrac{\lambda|x^2 - 5x + 6|}{\mu(5x - x^2 - 6)}, & x < 2 \\ e^{\frac{\tan(x-2)}{x - [x]}}, & x > 2 \\ \mu, & x = 2 \end{cases}$$

where $[x]$ is the greatest integer less than or equal to x. If f is continuous at $x = 2$, then $\lambda + \mu$ is equal to

A. 1

B. $e(e-2)$

C. $e(-e+1)$

D. $2e-1$

22. Let $f : \mathbb{R} \to \mathbb{R}$ be a function defined by $f(x) = max\left\{x, x^2\right\}$. Let S denote the set of all points in $\mathbb{R}$, where f is not differentiable. Then

A. $\{0, 1\}$

B. ϕ (an empty set)

C. $\{1\}$

D. $\{0\}$

23. Let $S = \left\{t \in \mathbb{R} : f(x) = |x - \pi| \cdot (e^{|x|} - 1)\sin|x| \text{ is not differentiable at t}\right\}$.
Then the set S is equal to

A. $\{0, \pi\}$

B. ϕ (an empty set)

C. $\{0\}$

D. $\{\pi\}$

24. $\lim_{x \to 0} \frac{\sin^2(\pi \cos^4 x)}{x^4}$ is equal to:

A. 4π

B. π^2

C. $4\pi^2$

D. $2\pi^2$

25. Let $f : R \to R$ be a function defined as

$$f(x) = \begin{cases} \dfrac{\sin(a+1)x + \sin 2x}{2x}, & \text{if } x < 0 \\ b, & \text{if } x = 0 \\ \dfrac{\sqrt{x + bx^3} - \sqrt{x}}{bx^{5/2}}, & \text{if } x > 0 \end{cases}$$

If f is continuous at $x = 0$, then the value of $a + b$ is equal

A. -2

B. $-\frac{5}{2}$

C. $-\frac{3}{2}$

D. -3

26. If $f(x) = \begin{cases} \dfrac{1}{|x|}; & |x| \geq 1 \\ ax^2 + b; & |x| < 1 \end{cases}$ is a differentiable at every point of the domain, then the values of a and b are respectively:

A. $\frac{5}{2}, -\frac{3}{2}$

B. $-\frac{1}{2}, \frac{3}{2}$

C. $\frac{1}{2}, \frac{1}{2}$

D. $\frac{1}{2}, -\frac{3}{2}$

27. $\lim_{x \to 0} \frac{(1-\cos 2x)(3+\cos 3x)}{x \tan 4x}$ is equal to :

A. $\frac{1}{4}$

B. $\frac{1}{2}$

C. 1

D. 2

28. If $f : \mathbb{R} \to \mathbb{R}$ is a function defined by $f(x) = [x] \cos\left(\frac{2x-1}{2}\right)\pi$, where $[x]$ denotes the greatest integer function, then f is

A. continuous for every real x.

B. discontinuous only at $x = 0$.

C. discontinuous only at non-zero integral values of x.

D. continuous only at $x = 0$.

29. If $\lim_{x \to 0} \frac{ax - (e^{4x} - 1)}{ax(e^{4x} - 1)}$ exists and is equal to b, then the value of $a - 2b$ is

30. Consider the function $f(x) = \begin{cases} \frac{P(x)}{\sin(x-2)}, & x \neq 2 \\ 7, & x = 2 \end{cases}$, where $P(x)$ is polynomial such that $P''(x)$ is always a constant and $P(3) = 9$. If $f(x)$ is continuous at $x = 2$, then $P(5)$ is equal to