Subject: Mathematics

1. Let f and g be differentiable funcitons on R, such that fog is the identity funciton. If for some $a, b \in R, g'(a) = 5$ and $g(a) = b$, then $f'(b)$ is equal to :

   A. $\frac{2}{5}$

   B. 5

   C. 1

   D. $\frac{1}{5}$

2. Let $y = y(x)$ be a function of x satisfying $y\sqrt{1-x^2} = k - x\sqrt{1-y^2}$ where k is a constant and $y\left(\frac{1}{2}\right) = -\frac{1}{4}$. Then $\frac{dy}{dx}$at $x = \frac{1}{2}$is equal to :

   A. $-\frac{\sqrt{5}}{2}$

   B. $\frac{\sqrt{5}}{2}$

   C. $-\frac{\sqrt{5}}{4}$

   D. $\frac{2}{\sqrt{5}}$

3. If Rolle's theorem holds for the function $f(x) = 2x^3 + bx^2 + cx, x \in [-1, 1]$,at the point $x = \frac{1}{2}$, then $2b + c$ equals :

   A. 1

   B. 2

   C. -1

   D. -3

4. If f and g are differentiable functions in $[0,1]$ satisfying $f(0)=2=g(1),\ g(0)=0$ and $f(1)=6$, then for some $c \in [0,1]$:

A. $2f'(c)=g'(c)$

B. $2f'(c)=3g'(c)$

C. $f'(c)=g'(c)$

D. $f'(c)=2g'(c)$

5. The derivative of $\tan^{-1}\left(\frac{\sin x-\cos x}{\sin x+\cos x}\right)$, with respect to $\frac{x}{2}$, where $\left(x \in \left(0, \frac{\pi}{2}\right)\right)$ is:

A. 1

B. $\frac{1}{2}$

C. 2

D. $\frac{2}{3}$

6. If P is a point on the parabola $y=x^2+4$ which is closest to the straight line $y=4x-1$, then the co-ordinates of P are

A. $(-2,8)$

B. $(1,5)$

C. $(3,13)$

D. $(2,8)$

7. The maximum area (in sq. units) of a rectangle having its base on the x-axis and its other two vertices on the parabola, $y = 12 - x^2$ such that the rectangle lies inside the parabola, is:

A. 36

B. 32

C. $20\sqrt{2}$

D. $18\sqrt{3}$

8. The range of $a \in \mathbb{R}$ for which the function $f(x) = (4a-3)(x+\log_e 5) + 2(a-7)\cot\left(\frac{x}{2}\right)\sin^2\left(\frac{x}{2}\right),\ x \neq 2n\pi, n \in \mathbb{N}$ has critical points, is:

A. $\left[-\frac{4}{3}, 2\right]$

B. $(-\infty, -1]$

C. $[1, \infty)$

D. $(-3, 1)$

9. Let $P(h, k)$ be a point on the curve $y = x^2 + 7x + 2$, nearest to the line, $y = 3x - 3$. Then the equation of the normal to the curve at P is:

A. $x + 3y - 62 = 0$

B. $x - 3y - 11 = 0$

C. $x - 3y + 22 = 0$

D. $x + 3y + 26 = 0$

10. If the tangent to the curve $y = x + \sin y$ at a point (a, b) is parallel to the line joining $\left(0, \frac{3}{2}\right)$ and $\left(\frac{1}{2}, 2\right)$ then:

A. $b = \frac{\pi}{2} + a$

B. $|a + b| = 1$

C. $|b - a| = 1$

D. $b = a$

11. The shortest distance between the line $x - y = 1$ and the curve $x^2 = 2y$ is :

A. $\frac{1}{2}$

B. 0

C. $\frac{1}{2\sqrt{2}}$

D. $\frac{1}{\sqrt{2}}$

12. The function $f(x) = \frac{4x^3 - 3x^2}{6} - 2\sin x + (2 - 1)\cos x$:

A. increases in $\left[\frac{1}{2}, \infty\right)$

B. decreases $\left(-\infty, \frac{1}{2}\right]$

C. increases in $\left(-\infty, \frac{1}{2}\right]$

D. decreases $\left[\frac{1}{2}, \infty\right)$

13. Consider the function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = \begin{cases} \left(2 - \sin\left(\frac{1}{x}\right)\right)|x|, & x \neq 0 \\ 0, & x = 0 \end{cases}$. Then f is

A. monotonic on $(0, \infty)$ only

B. Not monotonic on $(-\infty, 0)$ and $(0, \infty)$

C. monotonic on $(-\infty, 0)$ only

D. monotonic on $(-\infty, 0) \cup (0, \infty)$

14. The value of $\lim\limits_{x \rightarrow 0^+} \dfrac{\cos^{-1}\left(x - [x]^2\right) \cdot \sin^{-1}\left(x - [x]^2\right)}{x - x^3}$, where $[x]$ denotes the greatest integer $\leq x$ is:

A. 0

B. $\frac{\pi}{4}$

C. $\frac{\pi}{2}$

D. π

15. The equation of the normal to the curve $y = (1 + x)^{2y} + \cos^2\left(\sin^{-1} x\right)$ at $x = 0$ is

A. $y + 4x = 2$

B. $2y + x = 4$

C. $x + 4y = 8$

D. $y = 4x + 2$

16. If $y^2 + \log_e(\cos^2 x) = y,\ x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, then

A. $|y'(0)| + |y''(0)| = 1$

B. $y''(0) = 0$

C. $|y'(0)| + |y''(0)| = 3$

D. $|y''(0)| = 2$

17. Let f be a twice differentiable function on $(1, 6)$. If $f(2) = 8, f'(2) = 5, f'(x) \geq 1$ and $f''(x) \geq 4$, for all $x \in (1, 6)$, then

A. $f(5)+f'(5) \geq 28$

B. $f'(5) + f''(5) \leq 20$

C. $f(5) \leq 10$

D. $f(5) + f'(5) \leq 26$

18. The area (in sq. units) of the largest rectangle $ABCD$ whose vertices A and B lie on the $x-$axis and vertices C and D lie on the parabola, $y = x^2 - 1$ below the $x-$axis, is

A. $\frac{2}{3\sqrt{3}}$

B. $\frac{4}{3}$

C. $\frac{1}{3\sqrt{3}}$

D. $\frac{4}{3\sqrt{3}}$

19. Let $f(x)=\cos\left(2\tan^{-1}\sin\left(\cot^{-1}\sqrt{\frac{1-x}{x}}\right)\right), 0<x<1$. Then

A. $(1-x)^2 f'(x)-2(f(x))^2=0$

B. $(1-x)^2 f'(x)+2(f(x))^2=0$

C. $(1+x)^2 f'(x)-2(f(x))^2=0$

D. $(1+x)^2 f'(x)+2(f(x))^2=0$

20. Let f be a real valued function, defined on $\mathbb{R}-\{-1,1\}$ and given by $f(x)=3\log_e\left|\frac{x-1}{x+1}\right|-\frac{2}{x-1}$. Then in which of the following intervals, function $f(x)$ is increasing ?

A. $(-\infty,-1)\cup\left(\left[\frac{1}{2},\infty\right)-\{1\}\right)$

B. $\left(-1,\frac{1}{2}\right]$

C. $(-\infty,\infty)-\{-1,1\}$

D. $\left(-\infty,\frac{1}{2}\right]-\{-1\}$

21. The maximum slope of the curve $y=\frac{1}{2}x^4-5x^3+18x^2-19x$ occurs at the point :

A. $(2,9)$

B. $(2,2)$

C. $\left(3,\frac{21}{2}\right)$

D. $(0,0)$

22. If the curves $y^2 = 6x, 9x^2 + by^2 = 16$ intersect each other at right angles, then the value of b is :

A. $\frac{9}{2}$

B. 6

C. $\frac{7}{2}$

D. 4

23. If $y(\alpha) = \sqrt{2\left(\frac{\tan\ \alpha + \cot\ \alpha}{1 + \tan^2 \alpha}\right) + \frac{1}{\sin^2 \alpha}}$ where $\alpha \in \left(\frac{3\pi}{4}, \pi\right)$, then $\frac{dy}{d\alpha}$ at $\alpha = \frac{5\pi}{6}$ is

A. $-\frac{1}{4}$

B. $\frac{4}{3}$

C. 4

D. -4

24. Let $f : \mathbf{R} \to \mathbf{R}$ be a function such that $f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3), x \in \mathbf{R}$. Then $f(2)$ equals :

A. -4

B. 30

C. 8

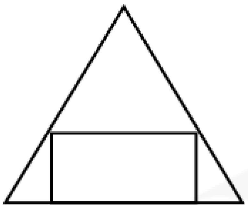
D. -2

25. Let the normal at a point P on the curve $y^2 - 3x^2 + y + 10 = 0$ intersects the y-axis at $\left(0, \frac{3}{2}\right)$. If m is the slope of the tangent at P to the curve, them $|m|$ is equal to

26. Let $f:[-1,1]\to\mathbb{R}$ be defined as $f(x)=ax^2+bx+c$ for all $x\in[-1,1]$, where $a,b,c\in\mathbb{R}$ such that $f(-1)=2, f'(-1)=1$ and for $x\in(-1,1)$ the maximum value of $f''(x)$ is $\frac{1}{2}$. If $f(x)\le\alpha, x\in[-1,1]$, then the least value of α is equal to

27. Suppose a differentiable function $f(x)$ satisfies the identity $f(x+y)=f(x)+f(y)+xy^2+x^2y$ for all real x and y. If $\lim_{x\to 0}\frac{f(x)}{x}=1$, then $f'(3)$ is equal to

28. If a rectangle is inscribed in an equilateral triangle of side length $2\sqrt{2}$ as shown in the figure, then the square of the largest area of such a rectangle is



29. Let $f(x)$ be a cubic polynomial with $f(1)=-10, f(-1)=6$, and has a local minima at $x=1$, and $f'(x)$ has a local minima at $x=-1$. Then $f(3)$ is equal to

30. If the point on the curve $y^2=6x$, nearest to the point $\left(3,\frac{3}{2}\right)$ is (α,β) then $2(\alpha+\beta)$ is equal to