

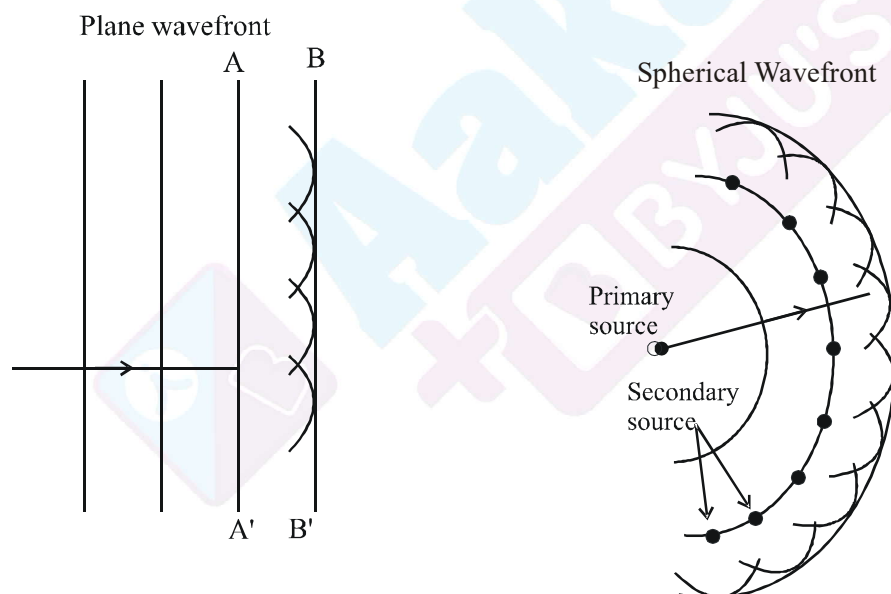


# WAVE NATURE OF LIGHT AND WAVE OPTICS

## Huygen's Wave Theory :

Huygen's in 1678 assumed that a body emits light in the form of waves.

- Each point source of light is a centre of disturbance from which waves spread in all directions. The locus of all the particles of the medium vibrating in the same phase at a given instant is called a wavefront.
- Each point on a wave front is a source of new disturbance, called secondary wavelets. These wavelets are spherical and travel with speed of light in that medium.
- The forward envelope of the secondary wavelets at any instant gives the new wavefront.
- In homogeneous medium, the wave front is always perpendicular to the direction of wave propagation.



### Coherent Sources :

Two sources will be coherent if and only if they produce waves of same frequency (and hence wavelength) and have a constant initial phase difference.

### Incoherent sources :

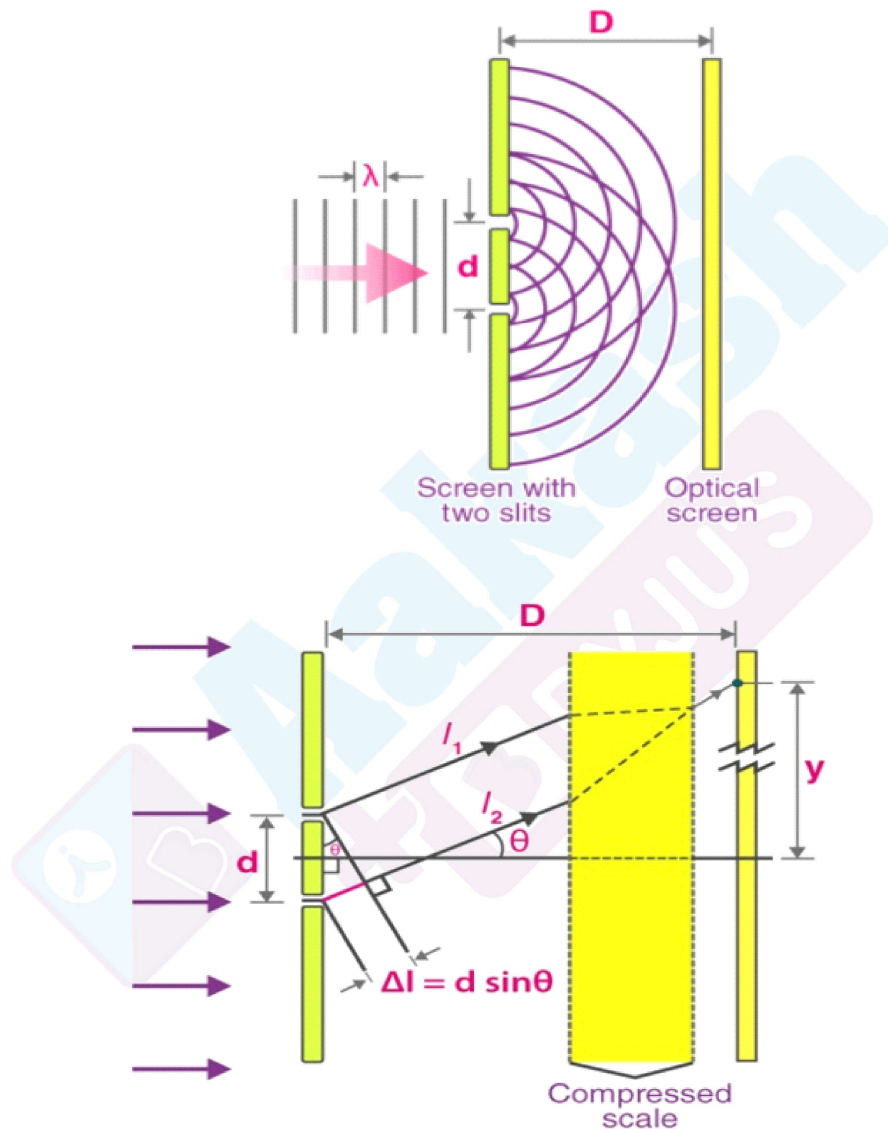
Two sources are said to be incoherent if they have different frequency and initial phase difference is not constant w.r.t. time.



## Young's Double Slit Experiment

### What is Young's Double Slit Experiment?

Young's double-slit experiment uses two coherent sources of light placed at a small distance apart, usually, only a few orders of magnitude greater than the wavelength of light is used. Young's double-slit experiment helped in understanding the wave theory of light which is explained with the help of a diagram. A screen or photodetector is placed at a large distance 'D' away from the slits as shown.



Each source can be considered as a source of coherent light waves. At any point on the screen at a distance 'y' from the centre, the waves travel distances  $l_1$  and  $l_2$  to create a path difference of  $\Delta l$  at the point. The point approximately subtends an angle of  $\theta$  at the sources (since the distance  $D$  is large there is only a very small difference of the angles subtended at sources).

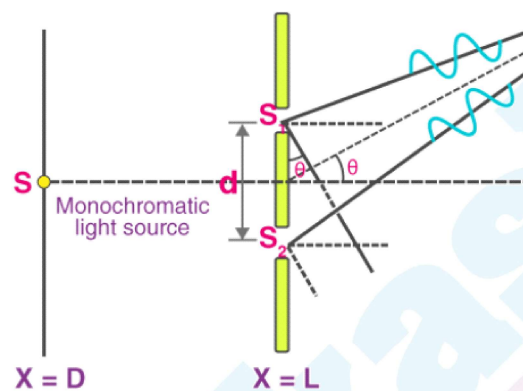


## Derivation of Young's Double Slit Experiment

Consider a monochromatic light source 'S' kept at a considerable distance from two slits  $s_1$  and  $s_2$ . S is equidistant from  $s_1$  and  $s_2$ .  $s_1$  and  $s_2$  behave as two coherent sources as both are derived from S.

The light passes through these slits and falls on a screen which is at a distance 'D' from the position of slits  $s_1$  and  $s_2$ . 'd' is the separation between two slits.

If  $s_1$  is open and  $s_2$  is closed, the screen opposite to  $s_1$  is closed, only the screen opposite to  $s_2$  is illuminated. The interference patterns appear only when both slits  $s_1$  and  $s_2$  are open.



When the slit separation (d) and the screen distance (D) are kept unchanged, to reach P the light waves from  $s_1$  and  $s_2$  must travel different distances. It implies that there is a path difference in Young's double slit experiment between the two light waves from  $s_1$  and  $s_2$ .

## Approximations in Young's double slit experiment

- **Approximation 1:**  $D \gg d$ : Since  $D \gg d$ , the two light rays are assumed to be parallel.
- **Approximation 2:**  $d/\lambda \gg 1$ : Often, d is a fraction of a millimetre and  $\lambda$  is a fraction of a micrometre for visible light.

Under these conditions  $\theta$  is small, thus we can use the approximation

$$\sin \theta = \tan \theta \approx \theta = \frac{\lambda}{d}.$$

$$\therefore \text{path difference, } \Delta x = \frac{\lambda}{d}$$

This is the path difference between two waves meeting at point on the screen. Due to this path difference in Young's double-slit experiment, some points on the screen are bright and some points are dark.

Now, we will discuss the position of these light, dark fringes and fringe width.



## Position of Fringes In Young's Double Slit Experiment

### Position of bright fringes

For maximum intensity or bright fringe to be formed at P

Path difference,  $\Delta x = n\lambda$  ( $n = 0, \pm 1, \pm 2, \dots$ )

$$\text{i.e., } \frac{x d}{D} = n\lambda$$

or

$$x = \frac{n\lambda D}{d}$$

The distance of the  $n^{\text{th}}$  bright fringe from the centre is

$$x_n = \frac{n\lambda D}{d}$$

Similarly, the distance of the  $(n-1)^{\text{th}}$  bright fringe from the centre is

$$x_{(n-1)} = (n-1) \frac{\lambda D}{d}$$

$$\text{Fringe width, } \beta = x_n - x_{(n-1)} = \frac{n\lambda D}{d} - (n-1) \frac{\lambda D}{d} = \frac{\lambda D}{d} \quad (n = 0, \pm 1, \pm 2, \dots)$$

### Position of Dark Fringes

For minimum intensity or dark fringe to be formed at P,

$$\text{Path difference, } \Delta x = (2n+1) \left( \frac{\lambda}{2} \right) \quad (n = 0, \pm 1, \pm 2, \dots)$$

$$\text{i.e., } x = (2n+1) \frac{\lambda D}{2d}$$

The distance of the  $n^{\text{th}}$  dark fringe from the centre is

$$x_n = (2n+1) \frac{\lambda D}{2d}$$

Similarly, the distance of the  $(n-1)^{\text{th}}$  dark fringe from the centre is

$$x_{(n-1)} = (2(n-1)+1) \frac{\lambda D}{2d}$$

$$\text{Fringe width, } \beta = x_n - x_{(n-1)} = (2n+1) \frac{\lambda D}{2d} - (2(n-1)+1) \frac{\lambda D}{2d} = \frac{\lambda D}{d} \quad (n = 0, \pm 1, \pm 2, \dots)$$



### Fringe Width

Distance between two adjacent bright (or dark) fringes is called the fringe width.

$$\beta = \frac{\lambda D}{d}$$

If the apparatus of Young's double slit experiment is immersed in a liquid of refractive index ( $\mu$ ), then the wavelength of light and fringe width decreases ' $\mu$ ' times.

$$\beta^1 = \frac{\beta}{\mu}$$

If white light is used in place of monochromatic light, then coloured fringes are obtained on the screen with red fringes larger in size than violet.

### Angular Width of Fringes

Let the angular position of  $n^{\text{th}}$  bright fringe is  $\theta_n$  and because of its small value

$$\tan \theta_n \approx \theta_n$$

$$\tan \theta_n = \frac{\lambda n}{d}$$

$$\Rightarrow \theta_n = \frac{n\lambda}{d}$$

Similarly, the angular position of  $(n+1)^{\text{th}}$  bright fringe is  $\theta_{n+1}$ , then

$$\theta_{n+1} = \frac{(n+1)\lambda}{d}$$

$\therefore$  Angular width of a fringe in young's double slit experiment is given by,

$$\theta = \theta_{n+1} - \theta_n = \frac{(n+1)\lambda}{d} - \frac{n\lambda}{d} = \frac{\lambda}{d}$$

$$\theta = \frac{\lambda}{d}$$

We know that

$$\beta = \frac{\lambda D}{d}$$

$$\Rightarrow \theta = \frac{\lambda}{d} = \frac{\beta}{D}$$

Angular width is independent of ' $n$ ' i.e., angular width of all fringes are same.



### Maximum Order of interference Fringes

The position of  $n^{\text{th}}$  order maxima on the screen is

$$y = \frac{n\lambda D}{d}; n = 0, \pm 1, \pm 2..$$

But 'n' values cannot take infinitely large values as it would violate 2<sup>nd</sup> approximation. i.e.,  $\theta$  is small (or)  $y \ll D$

$$\Rightarrow \frac{y}{D} = \frac{n\lambda}{d} \ll 1$$

Hence, the above formula for interference maxima is applicable when

$$n \ll \frac{d}{\lambda}$$

When 'n' value becomes comparable to  $\frac{d}{\lambda}$ ,

path difference can no longer be given by  $\frac{dy}{D}$ ,

Hence for maxima, path difference =  $n\lambda$

$$\Rightarrow d \sin \theta = n\lambda$$

$$\Rightarrow n = \frac{d \sin \theta}{\lambda}$$

$$n_{\text{max}} = \left[ \frac{d}{\lambda} \right]$$

The above represents box function or greatest integer function.

Similarly, the highest order of interference minima

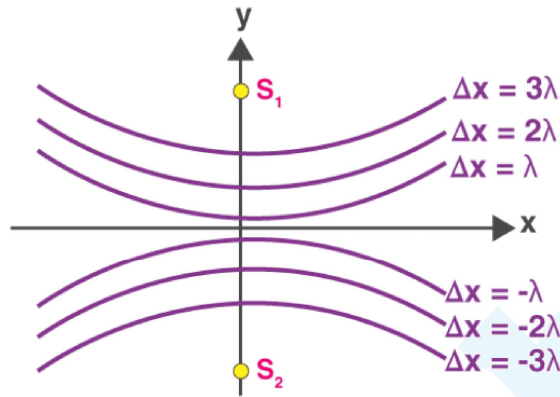
$$n_{\text{min}} = \left[ \frac{d}{\lambda} + \frac{1}{2} \right]$$



**Shape of interference Fringes in YDSE**

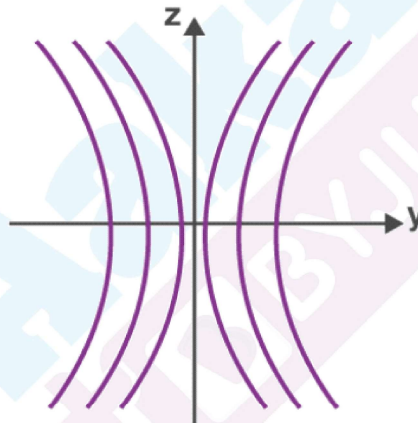
From the given YDSE diagram, the path difference from the two slits is given by,  $s_2p - s_1p \approx d \sin \theta$  (constant)

The above equation represents a hyperbola with its two foci as  $s_1$  and  $s_2$ .

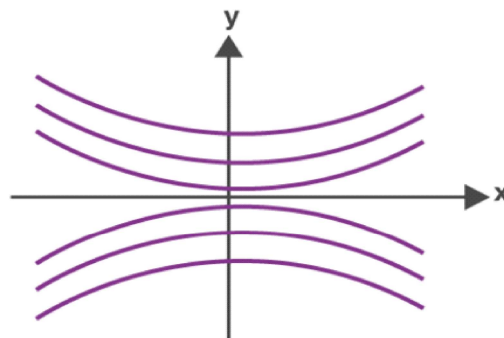


The interference pattern we get on the screen is a section of a hyperbola when we revolve hyperbola about the axis  $s_1s_2$ .

If the screen is  $yz$  plane, fringes are hyperbolic with a straight central section.



If the screen is  $xy$  plane, the fringes are hyperbolic with a straight central section.





**Intensity of Fringes In Young's Double Slit Experiment**

For two coherent sources  $s_1$  and  $s_2$ , the resultant intensity at point p is given by

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi$$

Putting  $I_1 = I_2 = I_0$  (Since,  $d \ll D$ )

$$I = I_0 + I_0 + 2\sqrt{I_0 \cdot I_0} \cos \phi$$

$$I = 2I_0 + 2(I_0) \cos \phi$$

$$I = 2I_0 (1 + \cos \phi)$$

$$I = 4I_0 \cos^2 \left( \frac{\phi}{2} \right)$$

**For maximum intensity**

$$\cos \frac{\phi}{2} = \pm 1$$

$$\frac{\phi}{2} = n\pi, n = 0, \pm 1, \pm 2, \dots$$

or  $\phi = 2n\pi$

**Phase difference**  $\phi = 2n\pi$  then,

**Path difference**  $\Delta x = \frac{\lambda}{2\pi} (2n\pi) = n\lambda$

The intensity of bright points are maximum and given by

$$I_{\max} = 4I_0$$

**For minimum Intensity**

$$\cos \frac{\phi}{2} = 0$$

$$\frac{\phi}{2} = \left( n - \frac{1}{2} \right) \pi, \text{ where } (n = \pm 1, \pm 2, \pm 3, \dots)$$

$$\phi = (2n - 1)\pi$$

**Phase difference**  $\phi = (2n - 1)\pi$

$$\frac{2\pi}{\lambda} \Delta x = (2n - 1)\pi$$

$$\Delta x = (2n - 1) \frac{\lambda}{2}$$



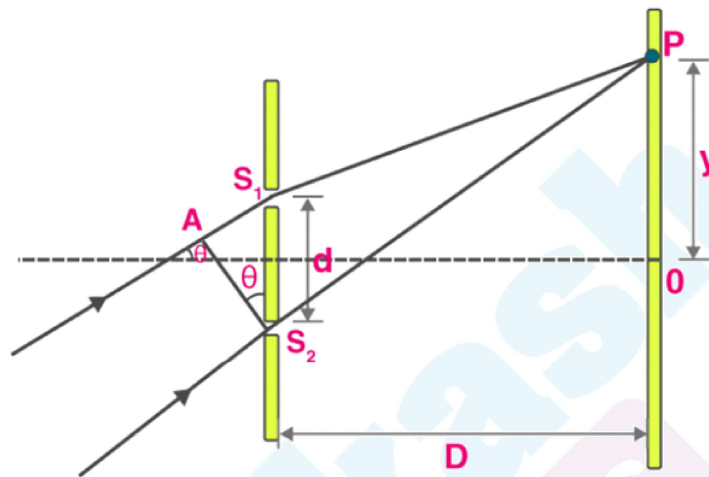


Thus, intensity of minima is given by  $I_{\min} = 0$

If  $I_1 \neq I_2, I_{\min} \neq 0$ .

### Special Cases

#### Rays Not Parallel to Principal Axis



From the above diagram,

Path difference

$$\Delta x = (AS_1 + S_1P) - S_2P$$

$$\Delta x = AS_1 - (S_2P - S_1P)$$

$$\Delta x = d \sin \theta - \frac{4d}{D}$$

For maxima  $\Delta x = n\lambda$

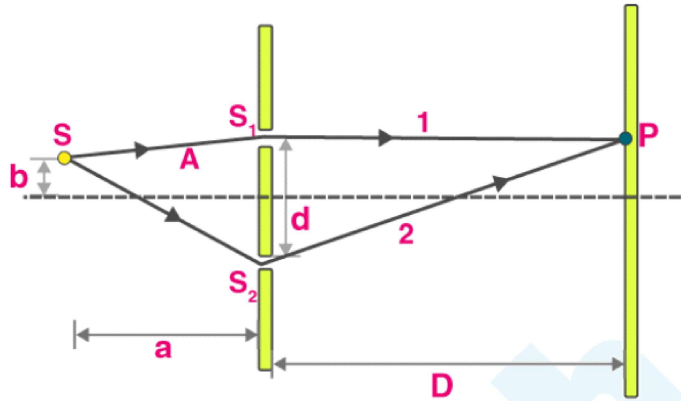
For minima  $\Delta x = (2n-1)\frac{\lambda}{2}$

Using this we can calculate different positions of maxima and minima.



**Source Placed Beyond the Central Line:**

If the source is placed a little above or below this centre line, the wave interaction with  $S_1$  and  $S_2$  has a path difference at a point P on the screen,



$$\begin{aligned} \Delta x &= (\text{distance of ray 2}) - (\text{distance of ray 1}) \\ &= (SS_2 + S_2P) - (SS_1 + S_1P) \\ &= (SS_2 + SS_1) + (S_2P - S_1P) \\ &= \frac{bd}{a} + \frac{yd}{D} \rightarrow (*) \end{aligned}$$

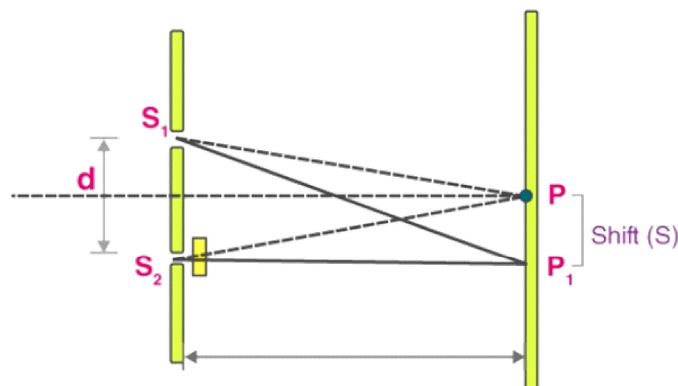
We know  $\Delta x = n\lambda$  for maximum

$$\Delta x = (2n - 1) \frac{\lambda}{2} \text{ for minimum}$$

By knowing the value of  $\Delta x$  from (\*) we can calculate different positions of maxima and minima.

**Displacement of Fringes in YDSE**

When a thin transparent plate of thickness 't' is introduced in front of one of the slits in Young's double slit experiment, the fringe pattern shifts toward the side where the plate is present.





The dotted lines denote the path of the light before introducing the transparent plate. the solid lines denote the path of the light after introducing a transparent plate.

Path difference before introducing the plate

$$\Delta x = S_1P - S_2P$$

Path difference after introducing the plate

$$\Delta x_{\text{new}} = S_1P^1 - S_2P^1$$

The path length

$$S_2P^1 = (S_2P^1 - t)_{\text{air}} + t_{\text{plate}} = (S_2P^1 - t)_{\text{air}} + (\mu t)_{\text{plate}}$$

Where,

$$' \mu t ' \text{ is optical path} = S_2P^1_{\text{air}} + (\mu - 1)t$$

Then we get

$$\begin{aligned} (\Delta x)_{\text{new}} &= S_1P^1_{\text{air}} - (S_2P^1_{\text{air}} + (\mu - 1)t) \\ &= (S_1P^1 - S_2P^1)_{\text{air}} - (\mu - 1)t \end{aligned}$$

$$(\Delta x)_{\text{new}} = d \sin \theta - (\mu - 1)t$$

$$(\Delta x)_{\text{new}} = \frac{\lambda d}{D} - (\mu - 1)t$$

Then,

$$y = \underbrace{\frac{\Delta x D}{d}}_{(1)} + \underbrace{\frac{D}{d} [(\mu - 1)t]}_{(2)}$$

Term (1) defines the position of a bright or dark fringe, the term (2) defines the shift occurred in the particular fringe due to the introduction of a transparent plate.

### Constructive and Destructive interference

For constructive interference, the path difference must be an integral multiple of the wavelength.

Thus for a bright fringe to be at 'y'.

$$n\lambda = \frac{yd}{D}$$

$$\text{or, } y = \frac{n\lambda D}{d}$$



Where,

$$n = \pm 0, 1, 2, 3, \dots$$

The 0<sup>th</sup> fringe represents the central bright fringe.

Similarly, the expression for a dark fringe in Young's double-slit experiment can be found by setting the path difference as:

$$\Delta l = (2n + 1) \frac{\lambda}{2}$$

This simplifies to

$$(2n + 1) \frac{\lambda}{2} = \frac{yd}{D}$$

$$y = (2n + 1) \frac{\lambda D}{2d}$$

The Young's double-slit experiment was a watershed moment in scientific history because it firmly established that light indeed behaved as a wave.

The double slit experiment was later conducted using electrons, and to everyone's surprise, the pattern generated was similar as expected with light. This would forever change our understanding of matter and particles, forcing us to accept that matter like light also behaves like a wave.

### KEY POINTS

- The law of conservation of energy holds good in the phenomenon of interference.
- Fringes are neither image nor shadow of slit but locus of a point which moves such a way that its path difference from the two sources remains constant.
- In YDSE the interference fringes for two coherent point sources are hyperboloids with axis  $S_1S_2$ .
- If the interference experiment is repeated with bichromatic light, the fringes of two wavelengths will be coincident for the first time when  $n(\beta)_{\text{longer}} = (n + 1)(\beta)_{\text{shorter}}$
- No interference pattern is detected when two coherent sources are infinitely close to

each other, because  $\beta \propto \frac{1}{d}$ .

- If maximum number of maximas or minimas are asked in the question, use the fact that value of  $\sin\theta$  or  $\cos\theta$  can't be greater than 1.

$$n_{\text{max}} = \frac{d}{\lambda}$$

$$\text{Total maxima} = 2n_{\text{max}} + 1$$



## DIFFRACTION

### In Fraunhofer diffraction

- ❑ For minima  $a \sin \theta_n = n\lambda$
- ❑ For maxima  $a \sin \theta_n = (2n + 1) \frac{\lambda}{2}$
- ❑ Linear width of central maxima  $W_x = \frac{2\lambda D}{a}$
- ❑ Angular width of central maxima  $W_\theta = \frac{2\lambda}{a}$
- ❑ Intensity of maxima  $I = I_0 \left[ \frac{\sin(\beta/2)}{\beta/2} \right]^2$  and  $\beta = \frac{2\pi}{\lambda} a \sin \theta$

Where,

$I_0$  = Intensity of central maxima

### Polarization

- ❑ **Brewster's law :-**

$$\mu = \tan \theta_p$$

Where,

$\theta_p \rightarrow$  polarization of Brewster's angle

Here reflecting and refracting rays are perpendicular to each other

- ❑ **Malus law :-**

$$I = I_0 \cos^2 \theta$$

Where,

$I_0 \rightarrow$  Maximum intensity of polarized light.

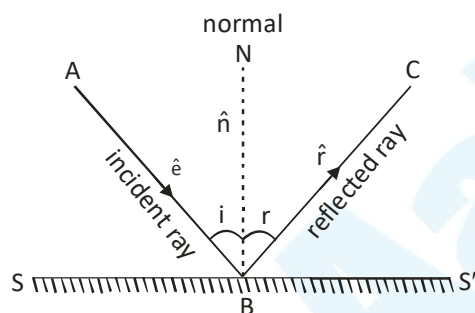


# RAY OPTICS AND OPTICAL INSTRUMENTS

## REFLECTION

### LAWS OF REFLECTION

- (i) The incident ray (AB), the reflected ray (BC) and normal (NB) to the surface (SS') of reflection at the point of incidence (B) lie in the same plane. This plane is called the plane of incidence (also plane of reflection).
- (ii) The angle of incidence (the angle between normal and the incident ray) and the angle of reflection (the angle between the reflected ray and the normal) are equal  $\angle i = \angle r$



In vector form  $\hat{r} = \hat{e} - 2(\hat{e} \cdot \hat{n})\hat{n}$

### OBJECT:

Real : Point from which rays actually diverge.

Virtual: Point towards which rays appear to converge

### IMAGE:

Image is decided by reflected or refracted rays only. The point image for a mirror is that point towards which the rays reflected from the mirror, actually converge (real image).

OR

From which the reflected rays appear to diverge (virtual image) .

### CHARACTERISTICS OF REFLECTION BY A PLANE MIRROR

- (i) The size of the image is the same as that of the object.



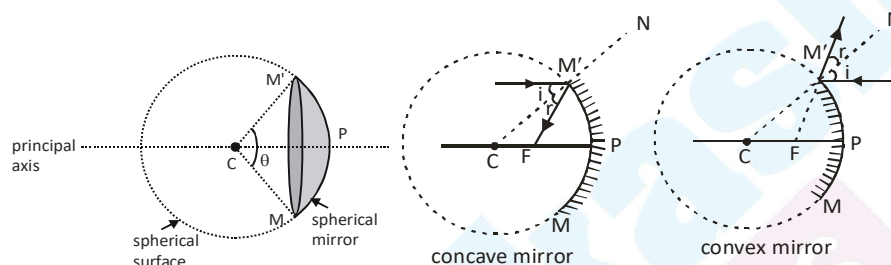
- (ii) For a real object the image is virtual and for a virtual object the image is real.
- (iii) For a fixed incident light ray, if the mirror be rotated through an angle  $\theta$  the reflected ray turns through an angle  $2\theta$  in the same sense.

### Number of images (n) in inclined mirror

Find  $\frac{360}{\theta} = m$

- If  $m$  even, then  $n = m - 1$ , for all positions of object.
- If  $m$  odd, then  $n = m$ , If object is not present on the bisector and  $n = m - 1$ , If the object is present at bisector.
- If  $m$  fraction then  $n =$  nearest even number.

### SPHERICAL MIRRORS



### PARAXIAL RAYS

A ray which makes a small angle ( $\theta$ ) to the optical axis of the system and lies close to the axis throughout the system is called paraxial rays. All formulae are valid for paraxial ray only.

### SIGN CONVENTION

- (i) We follow cartesian co-ordinate system convention according to which the pole of the mirror is the origin.
- (ii) The direction of the incident rays is considered as positive x axis. Vertically up is positive y-axis.
- (iii) All distance are measured from pole.

**Note:** According to above convention radius of curvature and focus of concave mirror is negative and of convex mirror is positive.

### MIRROR FORMULA

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$f$  = x-coordinate of focus  
 $v$  = x-coordinate of image

$u$  = x-coordinate of object  
**Note :** Valid only for paraxial rays.



**TRANSVERSE MAGNIFICATION**

$$m = \frac{h_2}{h_1} = -\frac{v}{u}$$

$h_2 = y$  co-ordinate of image  $h_1 = y$  co-ordinate of the object  
(both perpendicular to the principal axis of mirror)

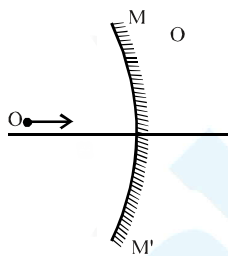
**Longitudinal magnification**

$$m_L = \frac{\text{Length of image}}{\text{Length of object}}$$

for small objects  $m_L = -m_t^2$   $m_t =$  transverse magnification

**VELOCITY OF IMAGE OF MOVING OBJECT (SPHERICAL MIRROR)**

Velocity component along axis (Longitudinal velocity)



When an object is coming from infinite towards the focus of concave mirror

$$\therefore \frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\therefore -\frac{1}{v^2} \frac{dv}{dt} - \frac{1}{u^2} \frac{du}{dt} = 0 \Rightarrow V_{IM} = -\frac{v^2}{u^2} V_{OM} = -m^2 V_{OM}$$

- $V_{IM} = \frac{dv}{dt}$  = velocity of image with respect to mirror
- $V_{OM} = \frac{du}{dt}$  = velocity of object with respect to mirror.

**NEWTON'S FORMULA**

Applicable to a pair of real object and real image position only . They are called conjugate positions or foci,  $X_1, X_2$  are the distance along the principal axis of the real object and real image respectively from the principal focus

$$X_1 X_2 = f^2$$





## OPTICAL POWER

Optical power of a mirror (in Diopters) =  $-\frac{1}{f}$

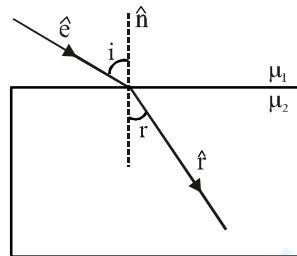
where,  $f$  = focal length (in meters) with sign .

## REFRACTION -PLANE SURFACE

### LAWS OF REFRACTION (AT ANY REFRACTING SURFACE)

#### Laws of Refraction

(i) Incident ray, refracted ray and normal always lie in the same plane



In vector form  $(\hat{e} \times \hat{n}) \cdot \hat{r} = 0$

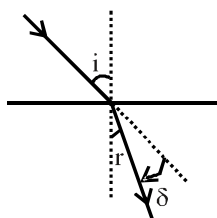
(ii) The product of refractive index and sine of angle of incidence at a point in a medium is constant.  $\mu_1 \sin i = \mu_2 \sin r$  (Snell's law)

#### Snell's law

$$\frac{\sin i}{\sin r} = \frac{n_2}{n_1} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}, \text{ In vector form } \mu_1 |\hat{e} \times \hat{n}| = \mu_2 |\hat{r} \times \hat{n}|$$

**Note** : Frequency of light does not change during refraction .

### DEVIATION OF A RAY DUE TO REFRACTION



angle of deviation,  $\delta = i - r$



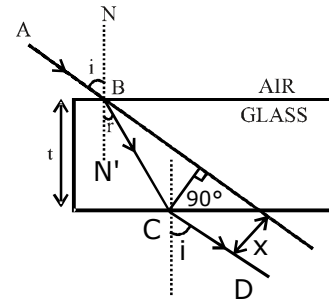
## REFRACTION THROUGH A PARALLEL SLAB

Emerged ray is parallel to the incident ray, if medium is same on both sides.

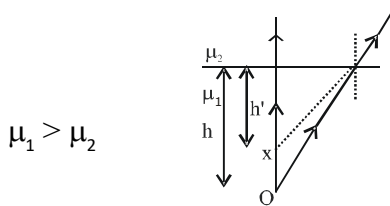
$$\text{Lateral shift } x = \frac{t \sin(i-r)}{\cos r}$$

$t$  = thickness of slab

**Note :** Emerged ray will not be parallel to the incident ray if the medium on both the sides are different.



## APPARENT DEPTH OF SUBMERGED OBJECT ( $h' < h$ )

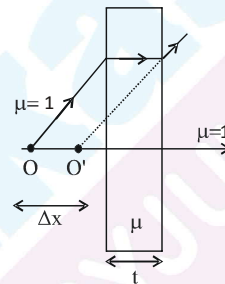


For near normal incidence  $h' = \frac{\mu_2}{\mu_1} h$

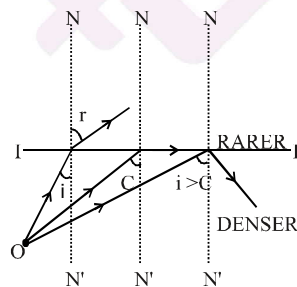
$$\Delta x = \text{Apparent shift} = t \left( 1 - \frac{1}{\mu} \right)$$

\*always in direction of incidence ray.

**Note :**  $h$  and  $h'$  are always measured from surface.



## CRITICAL ANGLE & TOTAL INTERNAL REFLECTION (TIR)



### Conditions of TIR

- Ray is going from denser to rarer medium
- Angle of incidence should be greater than the critical angle ( $i > C$ ).

$$\bullet \text{ Critical angle } C = \sin^{-1} \frac{\mu_R}{\mu_D} = \sin^{-1} \frac{V_D}{V_R} = \sin^{-1} \frac{\lambda_D}{\lambda_R}$$



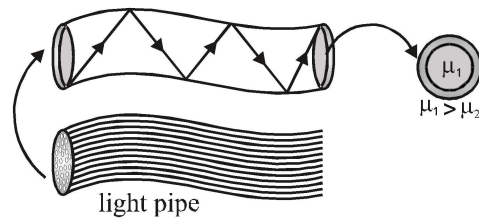
**Some Illustrations of Total Internal Reflection**

**Sparkling of diamond**

The sparkling of diamond is due to total internal reflection inside it. As refractive index for diamond is 2.5 so  $C = 24^\circ$ . Now the cutting of diamond are such that  $i > C$ . So, TIR will take place again and again inside it. The light which beams out from a few places in some specific directions makes it sparkle.

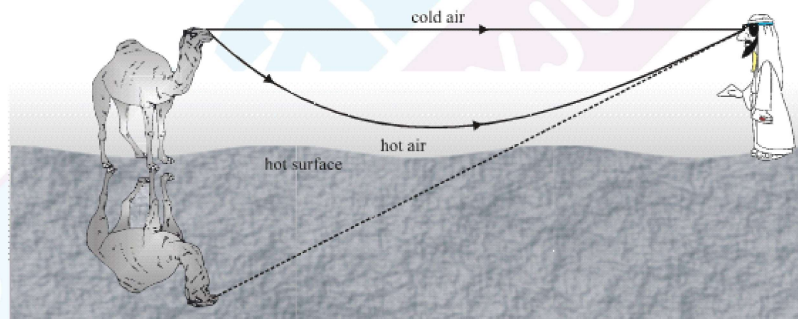
**Optical Fibre**

In it, light through multiple total internal reflections is propagated along the axis of a glass fibre of radius of few microns in which index of refraction of core is greater than that of surroundings (cladding).

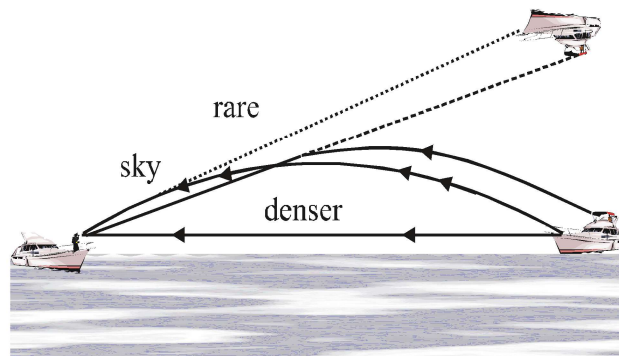


**Mirage and looming**

Mirage is caused by total internal reflection in deserts where due to heating of the earth, refractive index of air near the surface of earth becomes lesser than above it. Light from distant objects reaches the surface of earth with  $i > q_c$ , so that TIR will take place and we see the image of an object along with the object as shown in figure.



Similar to 'mirage' in deserts, in polar regions 'looming' takes place due to TIR. Here  $\mu$  decreases with height and so the image of an object is formed in air if  $(i > C)$  as shown in figure.





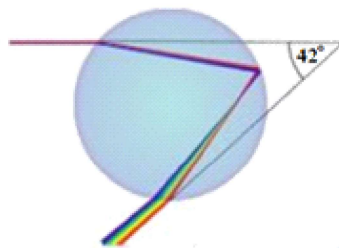
## RAINBOW FORMATION

### How rainbow is formed?

A rainbow is regarded as one of the most spectacular light shows observed on the earth. A rainbow is a multicoloured arc made due to the striking of light on water droplets. Rainbow is produced after the rain, by reflection, refraction and light dispersion process in droplets of water. All such events develop a light spectrum in the sky are called rainbow,

### The necessary conditions for the formation of the rainbow:

- Presence of raindrops.
- Sun should at your back to observe the rainbow.

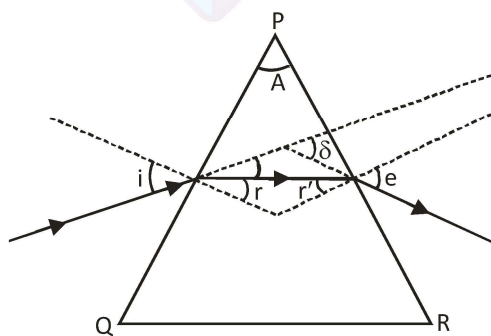


Because of the dispersion, white light separates into different colours when entering the raindrop, causing less refraction of red light than blue light.

### Formation of the rainbow

- Light rays, reach the drop near its top level. At first, there is refraction, then the dispersion of white light into colours of a different wavelength.
- The violet is the most deviated and red is the least deviated colour.
- Reaching the opposite side of the drop, each colour is refracted back into the drop due to the complete internal reflection that hits the drop surface.
- Every colour is refracted to the air again.
- We experience the rainbow when we observe between 42-40 degrees.

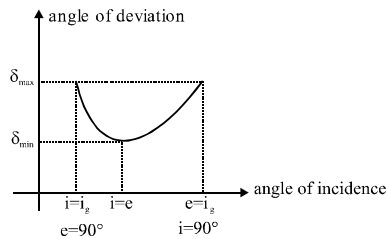
## REFRACTION THROUGH PRISM



- $\delta = (i + e) - (r + r')$
- $r + r' = A$



## Variation of $\delta$ versus $i$



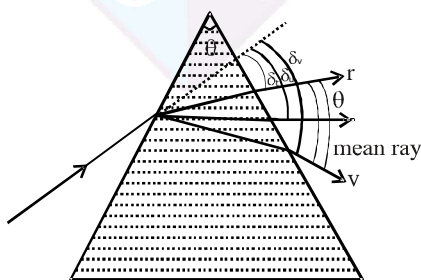
- There is one and only one angle of incidence for which the angle of deviation is minimum. When  $\delta = \delta_m$  then  $i = e$  &  $r = r'$ , the ray passes symmetrically about the prism, & then

$$n = \frac{\sin\left[\frac{A + \delta_m}{2}\right]}{\sin\left[\frac{A}{2}\right]}, \text{ where } n = \text{absolute R.I. of glass.}$$

**Note :** When the prism is dipped in a medium then  $n = \text{R.I. of glass w.r.t medium.}$

- For a thin prism ( $A \leq 10^\circ$ ) ;  $\delta = (n - 1)A$
- **Dispersion Of Light :** The angular splitting of a ray of white light into a number of components when it is refracted in a medium other than air is called **Dispersion of Light.**
- **Angle of Dispersion :** Angle between the rays of the extreme colours in the refracted (dispersed) light is called Angle of Dispersion.  $\theta = \delta_v - \delta_r$ .
- **Dispersive power ( $\omega$ )** of the medium of the material of prism

$$\omega = \frac{\text{angular dispersion}}{\text{deviation of mean ray (yellow)}}$$



$$\text{For small angled prism } (A \leq 10^\circ): \omega = \frac{\delta_v - \delta_r}{\delta_y} = \frac{n_v - n_r}{n - 1}; n = \frac{n_v + n_r}{2}$$

$n_v, n_r$  &  $n$  are R.I. of material for violet, red & yellow colours respectively.



**Combination of Two Prisms**

**(i) Achromatic Combination :** It is used for deviation without dispersion.

Condition for this is  $(n_v - n_r) A = -(n'_v - n'_r) A'$ .

$$\text{Net mean deviation} = \left[ \frac{n_v + n_r}{2} - 1 \right] A = - \left[ \frac{n'_v + n'_r}{2} - 1 \right] A' \text{ or } \omega\delta + \omega'\delta' = 0$$

where  $\omega, \omega'$  are dispersive powers for the two prisms &  $\delta, \delta'$  are the mean deviation.

**(ii) Direct Vision Combination :** It used for producing dispersion without deviation condition

$$\text{for this } \left[ \frac{n_v + n_r}{2} - 1 \right] A - \left[ \frac{n'_v + n'_r}{2} - 1 \right] A'$$

$$\text{Net angle of dispersion} = (n_v - n_r) A - (n'_v - n'_r) A'$$

**REFRACTION AT SPHERICAL SURFACE**

$$(a) \frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$v, u$  &  $R$  are to be kept with sign as

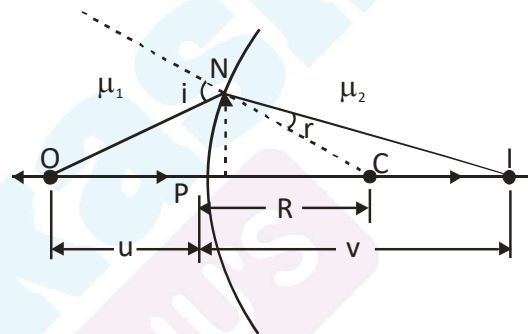
$$v = PI$$

$$u = -PO$$

$$R = PC$$

**(Note : Radius is with sign)**

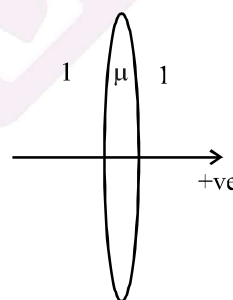
$$(b) m = \frac{\mu_1 v}{\mu_2 u}$$



**Lens Formula**

$$(a) \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$(b) \frac{1}{f} = (\mu - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right), (c) m = \frac{v}{u}$$



**Power of Lenses**

Reciprocal of focal length in meter is known as power of lens.

**SI unit :** dioptre (D)

$$\text{Power of lens : } P = \frac{1}{f(m)} = \frac{100}{f(cm)} \text{ dioptre}$$



## Combination of lenses

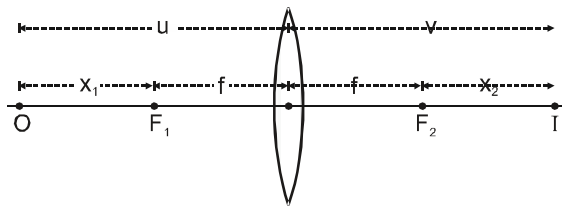
Two thin lens are placed in contact to each other

Power of combination.  $P = P_1 + P_2 \Rightarrow \frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$

Use sign convention when solving numericals



## Newton's Formula

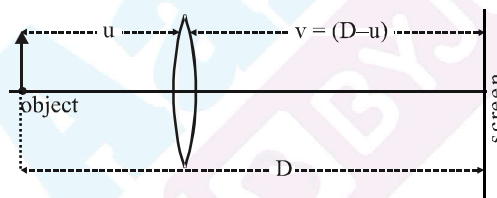


$$f = \sqrt{x_1 x_2}$$

$x_1$  = distance of object from focus,  $x_2$  = distance of image from focus.

## Displacement Method

It is used for determination of focal length of convex lens in laboratory. A thin convex lens of focal length  $f$  is placed between an object and a screen fixed at a distance  $D$  apart.



(i) For  $D < 4f$  :  $u$  will be imaginary hence physically no position of lens is possible

(ii) For  $D = 4f$  :  $u = \frac{D}{2} = 2f$  so only one position of lens is possible and since

$$v = D - u = 4f - 2f = 2f$$

(iii) For  $D > 4f$  :  $u_1 = \frac{D - \sqrt{D(D - 4f)}}{2}$  and  $u_2 = \frac{D + \sqrt{D(D - 4f)}}{2}$

So there are two positions of lens for which real image will be formed on the screen. For two positions of the lens distances of object and image are interchangeable

$$\text{so } u_1 = \frac{D - x}{2} = v_2 \text{ and } v_1 = \frac{D + x}{2} = u_2$$

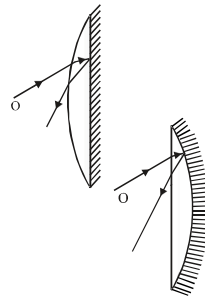


**Silvering of one surface of lens**

[use  $P_{eq} = 2P_l + P_m$ ]

□ When plane surface is silvered  $F = \frac{R}{2(\mu - 1)}$

□ When convex surface is silvered  $F = \frac{R}{2\mu}$







OPTICAL INSTRUMENTS

• **For Simple microscope**

- ❑ Magnifying power when image is formed at D :  $MP = 1 + D/f$
- ❑ When image is formed at infinity  $MP = D/f$

• **For Compound microscope :**  $MP = -\frac{v_0}{u_0} \left( \frac{D}{u_e} \right)$

- ❑ Magnifying power when final image is formed at D,  $MP = -\frac{v_0}{u_0} \left( 1 + \frac{D}{f_e} \right)$
- ❑ Tube length  $L = v_0 + |u_e|$
- ❑ When final image is formed at infinity  $MP = -\frac{v_0}{u_0} \times \frac{D}{f_e}$  and  $L = v_0 + f_e$

• **Astronomical Telescope:**  $MP = -\frac{f_0}{u_e}$

- ❑ Magnifying power when final image is formed at D:  $MP = -\frac{f_0}{f_e} \left( 1 + \frac{f_e}{D} \right)$
- ❑ Tube length :  $L = f_0 + |u_e|$
- ❑ When final image is formed at infinity :  $MP = -\frac{f_0}{f_e}$  and  $L = f_0 + f_e$

• **For terrestrial telescope :**  $MP = \frac{f_0}{f_e}$  and  $L = f_0 + f_e + 4f$

• **For Galilean telescope :**  $MP = \frac{f_0}{f_e}$  &  $L = f_0 - f_e$

• **Lens camera :** Time of exposure  $\propto \frac{1}{(\text{aperture})^2}$ , f – number =  $\frac{\text{focal length}}{\text{aperture}}$

• **For myopia or short- sightedness or near sightedness**

$$\frac{1}{\text{F.P.}} - \frac{1}{\text{object}} = \frac{1}{f} = P, \quad f = - \text{F.P.}$$

• **For long-sightedness or hypermetropia**  $\frac{1}{\text{N.P.}} - \frac{1}{\text{object}} = \frac{1}{f} = P$

• **Limit of resolution for microscope** =  $\frac{1.22\lambda}{2a \sin \theta} = \frac{1}{\text{resolving power}}$

• **Limit of resolution for telescope** =  $\frac{1.22\lambda}{a} = \frac{1}{\text{resolving power}}$



### KEY POINTS :

- For observing traffic at our back we prefer to use a convex mirror because a convex mirror has a more larger field of view than a plane mirror or concave mirror.
- A ray incident along normal to a mirror retraces its path because in reflection angle of incidence is always equal to angle of reflection.
- Images formed by mirrors do not show chromatic aberration because focal length of mirror is independent of wavelength of light and refractive index of medium.
- Light from an object falls on a concave mirror forming a real image of the object. If both the object and mirror are immersed in water, there is no change in the position of image because the formation of image by reflection does not depend on surrounding medium, there is no change in position of image provided it is also formed in water.
- The images formed by total internal reflections are much brighter than those formed by mirrors and lenses because there is no loss of intensity in total internal reflection.
- A fish inside a pond will see a person outside taller than he is actually because light bend away from the normal as it enters water from air.
- A fish in water at depth  $h$  sees the whole outside world in horizontal circle of radius.

$$r = h \tan \theta_c = \frac{h}{\sqrt{\mu^2 - 1}}$$

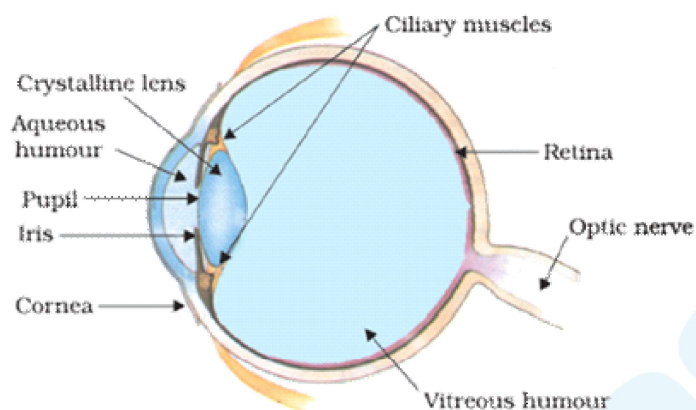
- Just before setting, the Sun may appear to be elliptical due to refraction because refraction of light ray through the atmosphere may cause different magnification in mutually perpendicular directions.
- A lens have two principal focal lengths which may differ because light can fall on either surface of the lens. The two principal focal lengths differ when medium on two sides have different refractive indices.
- A convex lens behaves as a concave lens when placed in a medium of refractive index greater than the refractive index of its material because light in that case will travel through the convex lens from denser to rarer medium. It will bend away from the-normal, i.e., the convex lens would diverge the rays.
- If lower half of a lens is covered with a black paper, the full image of the object is formed because every portion of lens forms the full image of the object but sharpness of image decrease.
- Sun glasses have zero power even though their surfaces are curved because both the surfaces of the Sun glasses are curved in the same direction with same radii.



## THE HUMAN EYE

### Structure of a Human Eye

The eye spherical in shape and has a diameter of 2.3 cm on an average. The internal structure of the eye includes. cornea, iris, pupil, lens, ciliary muscles, retina, nerve cells, optic nerve, and yellow spot, aqueous and vitreous humor, and suspensory ligament.



- **Cornea:** It is the outermost, transparent part. It provides most of the refraction of light.
- **Lens:** It is composed of a fibrous, jelly like material. Provides the focused real and inverted image of the object on the retina. This is convex lens that converges light at retina.
- **Iris:** It is a dark muscular diaphragm that controls the size of the pupil.
- **Pupil:** It is the window of the eye. It is the central aperture in iris. It regulates and controls the amount of light entering the eye.
- **Retina:** It is a delicate membrane having enormous number of light sensitive cells.
- **Far point:** The maximum distance at which object can be seen clearly is far point of the eye. For a normal adult eye, its value is infinity.

### Near point or Least distance of distinct Vision

- The minimum distance at which objects can be seen most distinctively without strain.
- For a normal adult eye, its value is 25 cm.
- Range of human vision — 25 cm to infinity.

- **Accommodation:** The ability of the eye lens to adjust its focal length is called accommodation.

Focal length can be changed with the help of ciliary muscles.

- Focal length increases when Ciliary muscles get relaxed and lens get thin.
- Focal length decreases when Ciliary muscles get contract and lens get thick.



### Myopia (Near sightedness)

- A myopic person can see nearby objects clearly but cannot see distant objects clearly.
- Image is formed in front of retina.

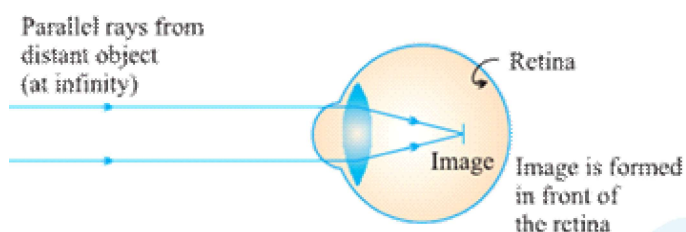
#### • Causes of Myopia

- Excessive curvature of eye lens.
- Elongation of eye ball

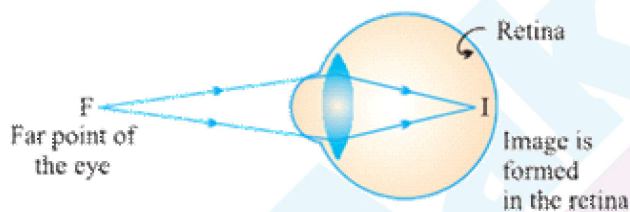
#### • Correction of Myopia

It is done by using concave lens of appropriate power.

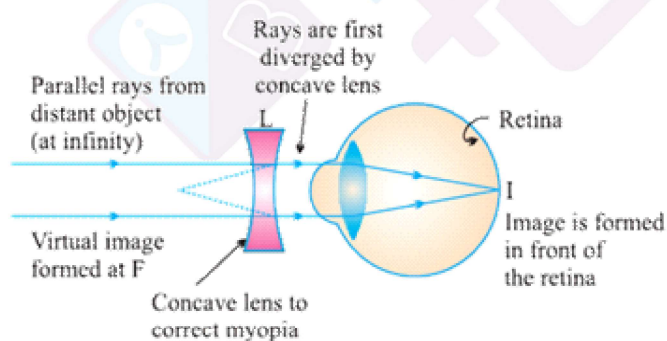
(i) In a myopic eye, image of distant object is formed in front of the retina. (and not on the retina)



(ii) The far point (F) of a myopic eye is less than infinity.



(iii) Correction of myopia. The concave lens placed in front of the eye forms a virtual image of distant object at far point (F) of the myopic eye.



### Hypermetropia (Far sightedness)

- Affected person can see far objects clearly but cannot see nearby objects clearly.
- The near point of the eye moves away.
- Image is formed behind the retina.

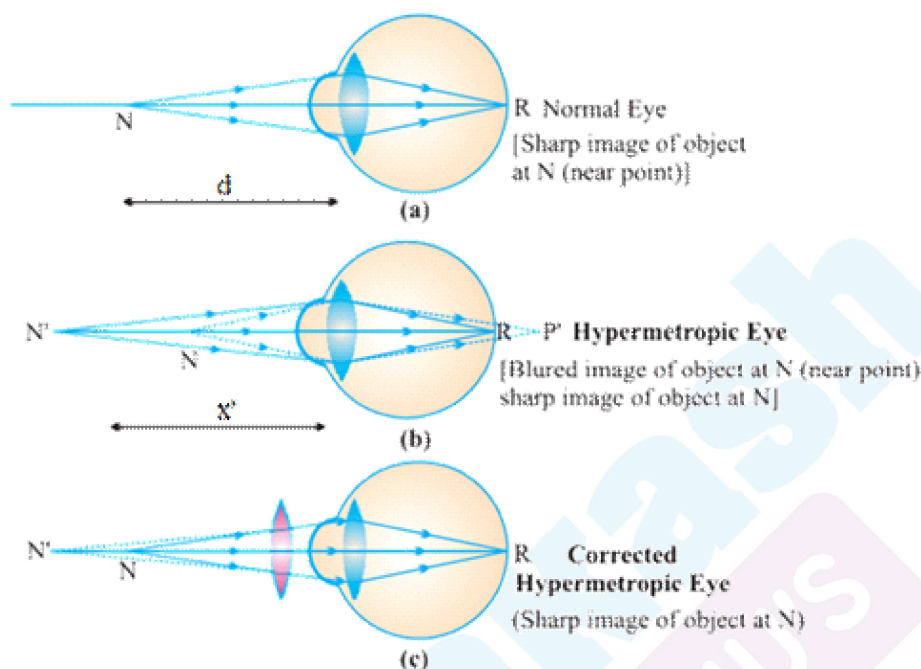


- **Causes of Hypermetropia**

- Focal length of the eye lens becomes too long.
- Eye ball becomes too small.

- **Correction of Hypermetropia**

- Use of convex lens of suitable power can correct the defect.



- **Presbyopia (Old age Hypermetropia)**

- It is the defect of vision due to which an old person cannot see the nearby objects clearly due to loss of power of accommodation of the eye.
- The near-point of the old person having presbyopia gradually recedes and becomes much more than 25 cm away.

- **Causes of Presbyopia**

- Gradual weakening of ciliary muscles.
- Diminishing flexibility of eye lens.

- **Correction of presbyopia**

- Use of convex lens of suitable power.
- Sometimes a person may suffer from both myopia and hypermetropia.
- Such people require bifocal lens for correction.

- **Advantage of the eyes in front of the face**

- It gives a wider field of view.
- It enhances the ability to detect faint objects.
- It provides three dimensional view.