Subject: Mathematics

1. Let S_1 be the sum of first $2n$ terms of an arithemetic progression. Let S_2 be the sum of first $4n$ terms of the same arithmetic progression. If $(S_2 - S_1)$ is 1000 then sum of the first $6n$ terms of the arithmetic progression is equal to :

   A. 3000

   B. 7000

   C. 5000

   D. 1000

2. If sum of the first 21 terms of the series $\log_{9^{\frac{1}{2}}} x + \log_{9^{\frac{1}{3}}} x + \log_{9^{\frac{1}{4}}} x + \ldots,$ where $x > 0$ is 504, then x is equal to:

   A. 81

   B. 243

   C. 9

   D. 7

3. Let S_n denote the sum of first n-terms of an arithmetic progression. If $S_{10} = 530,\ S_5 = 140$, then $S_{20} - S_6$ is equal to:

   A. 1842

   B. 1852

   C. 1862

   D. 1872

4. Let S_n be the sum of the first n terms of an arithmetic progression. If $S_{3n} = 3S_{2n}$, then the value of $\dfrac{S_{4n}}{S_{2n}}$ is

A. 4

B. 2

C. 6

D. 8

5. In an increasing geometric series, the sum of the second and the sixth term is $\dfrac{25}{2}$ and the product of the third and fifth term is 25. Then, the sum of $4^{\text{th}}, 6^{\text{th}}$ and 8^{th} terms is equal to :

A. 35

B. 30

C. 26

D. 32

6. $\dfrac{1}{3^2-1} + \dfrac{1}{5^2-1} + \dfrac{1}{7^2-1} + \cdots + \dfrac{1}{(201)^2-1}$ is equal to:

A. $\dfrac{101}{404}$

B. $\dfrac{101}{408}$

C. $\dfrac{99}{400}$

D. $\dfrac{25}{101}$

7. If $[x]$ be the greatest integer less than or equal to x, then $\sum_{n=8}^{100}\left[\frac{(-1)^n n}{2}\right]$ is equal to

A. 2

B. -2

C. 0

D. 4

8. If $0<\theta,\phi<\frac{\pi}{2}$, $x=\sum_{n=0}^{\infty}\cos^{2n}\theta$, $y=\sum_{n=0}^{\infty}\sin^{2n}\phi$ and $z=\sum_{n=0}^{\infty}\cos^{2n}\theta\cdot\sin^{2n}\phi$ then :

A. $xyz=4$

B. $xy-z=(x+y)z$

C. $xy+xy+zx=z$

D. $xy+z=(x+y)z$

9. $e^{(\cos^2 x+\cos^4 x+\cos^6 x+\cdots\infty)\log_e 2}$ satisfies the equation $t^2-9t+8=0$, then the value of $\frac{2\sin x}{\sin x+\sqrt{3}\cos x}$, $\left(0<x<\frac{\pi}{2}\right)$ is :

A. $\frac{3}{2}$

B. $2\sqrt{3}$

C. $\frac{1}{2}$

D. $\sqrt{3}$

10. If $2^{10}+2^9\cdot 3^1+2^8\cdot 3^2+\cdots+2\cdot 3^9+3^{10}=S-2^{11}$, then S is equal to:

A. 3^{11}

B. $\frac{3^{11}}{2}+2^{10}$

C. $2\cdot 3^{11}$

D. $3^{11}-2^{12}$

11. The maximum value of the term independent of t in the expansion of $\left(tx^{1/5}+\frac{(1-x)^{1/10}}{t}\right)^{10}$ where $x\in(0,1)$ is :

A. $\frac{10!}{\sqrt{3}(5!)^2}$

B. $\frac{2\times 10!}{3(5!)^2}$

C. $\frac{10!}{3(5!)^2}$

D. $\frac{2\times 10!}{3\sqrt{3}(5!)^2}$

12. If n is the number of irrational terms in the expansion of $\left(3^{\frac{1}{4}}+5^{\frac{1}{8}}\right)^{60}$, then $(n-1)$ is divisible by:

A. 8

B. 26

C. 7

D. 30

13. If the fourth term in the expansion of $\left(x+x^{\log_2 x}\right)^7$ is 4480, then the value of x where $x \in N$ is equal to:

A. 4

B. 3

C. 2

D. 1

14. The coefficient of x^{256} in the expansion of $(1-x)^{101}(x^2+x+1)^{100}$ is

A. ${}^{100}C_{15}$

B. ${}^{100}C_{16}$

C. $-\,{}^{100}C_{16}$

D. $-\,{}^{100}C_{15}$

15. If $n \geq 2$ is a positive integer, then the sum of the series ${}^{n+1}C_2 + 2\left({}^2C_2 + {}^3C_2 + {}^4C_2 + \cdots + {}^nC_2\right)$ is

A. $\dfrac{n(n+1)^2(n+2)}{12}$

B. $\dfrac{n(n-1)(2n+1)}{6}$

C. $\dfrac{n(n+1)(2n+1)}{6}$

D. $\dfrac{n(2n+1)(3n+1)}{6}$

16. The value of $-{}^{15}C_1 + 2 \times {}^{15}C_2 - 3 \times {}^{15}C_3 + \ldots - 15 \times {}^{15}C_{15}$ is :
$+\,{}^{14}C_1 + {}^{14}C_3 + {}^{14}C_5 + \ldots + {}^{14}C_{11}$

A. 2^{14}

B. $2^{13}-13$

C. $2^{16}-1$

D. $2^{13}-14$

17. The value of $\sum_{r=0}^{6}\left({}^{6}C_r \cdot {}^{6}C_{6-r}\right)$ is equal to:

A. 1124

B. 924

C. 1324

D. 1024

18. A scientific committee is to formed from 6 Indians and 8 foreigners, which includes at least 2 Indians and double the number of foreigners as Indians. Then the number of ways, the committee can be formed is:

A. 560

B. 1050

C. 1625

D. 575

19. Team 'A' consists of 7 boys and n girls and Team 'B' has 4 boys and 6 girls. If a
total of 52 single matches can be arranged between these two teams when a
boy plays against a boy and a girl plays against a girl, then n is equal to:

A. 5

B. 6

C. 2

D. 4

20. A natural number has prime factorization given by $n = 2^x 3^y 5^z$, where y and z are such that $y + z = 5$ and $y^{-1} + z^{-1} = \frac{5}{6}, y > z$. Then the number of odd divisors of n, including 1, is:

A. 11

B. $6x$

C. 12

D. 6

21. The number of seven digit integers with sum of the digits equal to 10 and formed by using the digits $1, 2$ and 3 only is :

A. 77

B. 42

C. 35

D. 82

22. The sum of all the $4-$digit distinct numbers that can be formed with the digits $1, 2, 2$ and 3 is:

A. 26664

B. 122664

C. 122234

D. 22264

Subject: Mathematics

1. The sum of first four terms of a geometric progression (G.P.) is $\frac{65}{12}$ and the sum of their respective reciprocals is $\frac{65}{18}$. If the product of first three terms of the G.P. is 1, and the third term is α, then 2α is

2. Consider an arithmetic series and a geometric series having four initial terms from the set $\{11, 8, 21, 16, 26, 32, 4\}$. If the last terms of these seris are the maximum possible four digit numbers, then the number of common terms in these two series is equal to

3. Let $S_n(x) = \log_{a^{1/2}} x + \log_{a^{1/3}} x + \log_{a^{1/6}} x + \log_{a^{1/11}} x + \log_{a^{1/18}} x + \log_{a^{1/27}} x + \cdots$ up to n-terms, where $a > 1$. If $S_{24}(x) = 1093$ and $S_{12}(2x) = 265$, then value of a is equal to

4. Let n be a positive integer. Let $A = \sum_{k=0}^{n} (-1)^k \, {}^nC_k \left[\left(\frac{1}{2}\right)^k + \left(\frac{3}{4}\right)^k + \left(\frac{7}{8}\right)^k + \left(\frac{15}{16}\right)^k + \left(\frac{31}{32}\right)^k \right]$. If $63A = 1 - \frac{1}{2^{30}}$, then n is equal to

5. If the remainder when x is divided by 4 is 3, then the remainder when $(2020 + x)^{2022}$ is divided by 8 is

6. The ratio of the coefficient of the middle term in the expansion of $(1 + x)^{20}$ and the sum of the coefficients of two middle terms in expansion of $(1 + x)^{19}$ is

7. The students $S_1, S_2, \ldots, S_{10}$ are to be divided into 3 groups A, B and C such that each group has at least one student and the group C has at most 3 students. Then the total number of possibilities of forming such groups is

8. Let $S = \{1, 2, 3, 4, 5, 6, 7\}$. Then the number of possible functions $f : S \to S$ such that $f(m \cdot n) = f(m) \cdot f(n)$ for every $m, n \in S$ and $m \cdot n \in S$ is equal to