Subject: Mathematics

1. Let S_1 be the sum of first $2n$ terms of an arithemetic progression. Let S_2 be the sum of first $4n$ terms of the same arithmetic progression. If $(S_2 - S_1)$ is 1000 then sum of the first $6n$ terms of the arithmetic progression is equal to :

A. 3000

B. 7000

C. 5000

D. 1000

$S_{4n} - S_{2n} = 1000$
$\Rightarrow \dfrac{4n}{2}(2a + (4n-1)d) - \dfrac{2n}{2}(2a + (2n-1)d) = 1000$
$\Rightarrow n(2a + (6n-1)d) = 1000$
$\Rightarrow \dfrac{6n}{2}(2a + (6n-1)d) = 3000$
$\therefore S_{6n} = 3000$

2. If sum of the first 21 terms of the series $\log_{9^{\frac{1}{2}}} x + \log_{9^{\frac{1}{3}}} x + \log_{9^{\frac{1}{4}}} x + \ldots,$ where $x > 0$ is 504, then x is equal to:

A. 81

B. 243

C. 9

D. 7

$\log_{9^{\frac{1}{2}}} x + \log_{9^{\frac{1}{3}}} x + \log_{9^{\frac{1}{4}}} x + \ldots$
$\Rightarrow \log_9 x^2 + \log_9 x^3 + \log_9 x^4 + \ldots$
$\Rightarrow \log_9 \left(x^{2+3+\cdots 21 \text{ terms}}\right) = 504$
$\Rightarrow 252 \log_9 x = 504$
$\Rightarrow x = 9^2 = 81$

3. Let S_n denote the sum of first n-terms of an arithmetic progression. If $S_{10} = 530,\ S_5 = 140$, then $S_{20} - S_6$ is equal to:

A. 1842

B. 1852

C. 1862

D. 1872

Let first term of $A.P.$ be a and common difference is d.

$\therefore S_{10} = \frac{10}{2}\{2a + 9d\} = 530$

$\therefore 2a + 9d = 106 \cdots (i)$

$S_5 = \frac{5}{2}\{2a + 4d\} = 140$

$a + 2d = 28 \cdots (ii)$

from equation (i) and $(ii), a = 8, d = 10$

$\therefore S_{20} - S_6 = \frac{20}{2}\{2 \times 8 + 19 \times 10\} - \frac{6}{2}\{2 \times 8 + 5 \times 10\}$

$= 2060 - 198 = 1862$

4. Let S_n be the sum of the first n terms of an arithmetic progression. If $S_{3n} = 3S_{2n}$, then the value of $\frac{S_{4n}}{S_{2n}}$ is

A. 4

B. 2

C. 6

D. 8

$S_{3n} = \frac{3n}{2}[2a + (3n-1)d]$
$S_{2n} = n[2a + (2n-1)d]$,
where a is the first term & d is the common difference of A.P.

$S_{3n} = 3S_{2n}$
$\Rightarrow \frac{3}{2}[2a + (3n-1)d] = 3[2a + (2n-1)d]$
$\Rightarrow 2a + (3n-1)d = 4a + 2(2n-1)d$
$\Rightarrow 2a = (3n - 1 - 4n + 2)d$
$\Rightarrow \frac{a}{d} = \frac{1-n}{2} \quad \ldots(1)$

Now, $\frac{S_{4n}}{S_{2n}} = \dfrac{\frac{4n}{2}[2a + (4n-1)d]}{\frac{2n}{2}[2a + (2n-1)d]}$

$= \dfrac{2\left[2\left(\frac{1-n}{2}\right) + (4n-1)\right]}{\left[2\left(\frac{1-n}{2}\right) + (2n-1)\right]}$

$= \dfrac{2(1-n+4n-1)}{1-n+2n-1}$

$= \dfrac{2(3n)}{n} = 6$

5. In an increasing geometric series, the sum of the second and the sixth term is $\frac{25}{2}$ and the product of the third and fifth term is 25. Then, the sum of $4^{\text{th}}, 6^{\text{th}}$ and 8^{th} terms is equal to :

A. 35 ✔

B. 30

C. 26

D. 32

Let a be the first term and r be the common ratio.

$ar + ar^5 = \frac{25}{2}$

and $ar^2 \times ar^4 = 25$

$\Rightarrow a^2r^6 = 25$

$\Rightarrow ar^3 = 5$

$\Rightarrow a = \frac{5}{r^3} \quad \ldots(1)$

Now, $\frac{5r}{r^3} + \frac{5r^5}{r^3} = \frac{25}{2} \quad$ [From (1)]

$\Rightarrow \frac{1}{r^2} + r^2 = \frac{5}{2}$

Put $r^2 = t$

$\Rightarrow \frac{t^2+1}{t} = \frac{5}{2}$

$\Rightarrow 2t^2 - 5t + 2 = 0$

$\Rightarrow (2t-1)(t-2) = 0$

$\Rightarrow t = \frac{1}{2}, 2$

$\Rightarrow r^2 = \frac{1}{2}, 2$

$\Rightarrow r = \sqrt{2}$ as the G.P. is increasing.

$\therefore ar^3 + ar^5 + ar^7$

$= ar^3(1 + r^2 + r^4)$

$= 5(1 + 2 + 4)$

$= 35$

6. $\dfrac{1}{3^2-1}+\dfrac{1}{5^2-1}+\dfrac{1}{7^2-1}+\cdots+\dfrac{1}{(201)^2-1}$ is equal to:

A. $\dfrac{101}{404}$

B. $\dfrac{101}{408}$

C. $\dfrac{99}{400}$

D. $\dfrac{25}{101}$

$$S=\sum_{r=1}^{100}\frac{1}{(2r+1)^2-1}=\sum_{r=1}^{100}\frac{1}{(2r+2)\cdot 2(r)}$$
$$\therefore S=\frac{1}{4}\sum_{r=1}^{100}\left|\frac{1}{r}-\frac{1}{r+1}\right|$$
$$S=\frac{1}{4}\left(\left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\cdots+\left(\frac{1}{100}-\frac{1}{101}\right)\right)$$
$$\therefore S=\frac{1}{4}\left|\frac{100}{101}\right|=\frac{25}{101}$$

7. If $[x]$ be the greatest integer less than or equal to x, then $\sum_{n=8}^{100}\left[\dfrac{(-1)^n n}{2}\right]$ is equal to

A. 2

B. -2

C. 0

D. 4

$$\sum_{n=8}^{100}\left[\frac{(-1)^n n}{2}\right]$$
$$=4-5+5-6+6-7+7+\ldots-50+50$$
$$=4$$

8. If $0 < \theta, \phi < \frac{\pi}{2}$, $x = \sum\limits_{n=0}^{\infty} \cos^{2n}\theta$, $y = \sum\limits_{n=0}^{\infty} \sin^{2n}\phi$ and $z = \sum\limits_{n=0}^{\infty} \cos^{2n}\theta \cdot \sin^{2n}\phi$

then :

A. $xyz = 4$

B. $xy - z = (x+y)z$

C. $xy + xy + zx = z$

D. $xy + z = (x+y)z$

$$x = 1 + \cos^2\theta + \cos^4\theta + \cdots \infty$$

$$\Rightarrow x = \frac{1}{1-\cos^2\theta} \quad \left(\text{For GP : } S_\infty = \frac{a}{1-r}\right)$$

$$\Rightarrow x = \frac{1}{\sin^2\theta} \quad \ldots(1)$$

$$y = 1 + \sin^2\phi + \sin^4\phi + \cdots \infty$$

$$\Rightarrow y = \frac{1}{1-\sin^2\phi}$$

$$\Rightarrow y = \frac{1}{\cos^2\phi} \quad \ldots(2)$$

$$z = \frac{1}{1-\cos^2\theta \cdot \sin^2\phi}$$

$$= \frac{1}{1-\left(1-\frac{1}{x}\right)\left(1-\frac{1}{y}\right)} \quad [\text{From (1) and (2)}]$$

$$z = \frac{xy}{xy-(x-1)(y-1)}$$

$$\Rightarrow xz + yz - z = xy$$
$$\Rightarrow xy + z = (x+y)z$$

9. $e^{(\cos^2 x+\cos^4 x+\cos^6 x+\cdots\infty)\log_e 2}$ satisfies the equation $t^2 - 9t + 8 = 0$, then the value of $\dfrac{2\sin x}{\sin x+\sqrt{3}\cos x}, \left(0 < x < \dfrac{\pi}{2}\right)$ is :

A. $\dfrac{3}{2}$

B. $2\sqrt{3}$

C. $\dfrac{1}{2}$

D. $\sqrt{3}$

$e^{(\cos^2 x+\cos^4+\cdots\infty)\ln 2} = 2^{\cos^2 x+\cos^4 x+\cdots\infty}$

$= 2^{\frac{\cos^2 x}{1-\cos^2 x}}$ (sum of infinite G.P.)

$= 2^{\cot^2 x}$

$t^2 - 9t + 8 = 0 \Rightarrow t = 1, 8$

$\Rightarrow 2^{\cot^2 x} = 1, 8 \Rightarrow x = 0, 3$

$0 < x < \dfrac{\pi}{2} \Rightarrow \cot x = \sqrt{3}$

Hence

$\dfrac{2\sin x}{\sin x+\sqrt{3}\cos x}$

$= \dfrac{2}{1+\sqrt{3}\cot x} = \dfrac{1}{2}$

10. If $2^{10}+2^9\cdot 3^1+2^8\cdot 3^2+\cdots+2\cdot 3^9+3^{10}=S-2^{11}$, then S is equal to:

A. 3^{11}

B. $\frac{3^{11}}{2}+2^{10}$

C. $2\cdot 3^{11}$

D. $3^{11}-2^{12}$

Let

$S'=2^{10}+2^9 3^1+2^8 3^2+\cdots+2\cdot 3^9+3^{10}$

$\frac{3\times S'}{2}=2^9\times 3^1+2^8\cdot 3^2+\cdots+3^{10}+\frac{3^{11}}{2}$

$\Rightarrow \frac{-S'}{2}=2^{10}-\frac{3^{11}}{2}$

$\Rightarrow S'=3^{11}-2^{11}$

Now, $S'=S-2^{11}$

Therefore, $S=3^{11}$

11. The maximum value of the term independent of t in the expansion of $\left(tx^{1/5}+\frac{(1-x)^{1/10}}{t}\right)^{10}$ where $x \in (0,1)$ is :

A. $\frac{10!}{\sqrt{3}(5!)^2}$

B. $\frac{2\times 10!}{3(5!)^2}$

C. $\frac{10!}{3(5!)^2}$

D. $\frac{2\times 10!}{3\sqrt{3}(5!)^2}$

$$T_{r+1} = {}^{10}C_r (tx^{1/5})^{10-r} \times \left[\frac{(1-x)^{1/10}}{t}\right]^r$$

$$= {}^{10}C_r \cdot t^{(10-2r)} \times x^{(10-r)/5} \times (1-x)^{r/10}$$

For a term to be independent of t,

$10-2r=0$

$\Rightarrow r=5$

$$T_6 = {}^{10}C_5\, x\sqrt{1-x}$$

$$\frac{dT_6}{dx} = {}^{10}C_5\left[\sqrt{1-x}-\frac{x}{2\sqrt{1-x}}\right]=0$$

$$\Rightarrow 1-x=\frac{x}{2}$$

$$\Rightarrow x=\frac{2}{3}$$

$$\therefore \max(T_6)=\frac{10!}{5!\,5!}\times\frac{2}{3\sqrt{3}}$$

12. If n is the number of irrational terms in the expansion of $\left(3^{\frac{1}{4}}+5^{\frac{1}{8}}\right)^{60}$, then $(n-1)$ is divisible by:

A. 8

B. 26

C. 7

D. 30

$T_{r+1} = {}^{60}C_r(3^{1/4})^{60-r}(5^{1/8})^r$

rational if $\dfrac{60-r}{4}, \dfrac{r}{8}$ both are whole numbers, $r \in \{0,1,2,\ldots.60\}$

$\dfrac{60-r}{4} \in W \Rightarrow r \in \{0,4,8,\ldots 60\}$

$\text{and } \dfrac{r}{8} \in W \Rightarrow r \in \{0,8,16,\ldots 56\}$

$\therefore$ Common terms $r \in \{0,8,16,\ldots.56\}$

So 8 terms are rational

Then irrational terms $= 61-8=53=n$

$\therefore n-1=52=13\times 2^2$

Factors $1,2,4,13,26,52$

13. If the fourth term in the expansion of $\left(x+x^{\log_2 x}\right)^7$ is 4480, then the value of x where $x \in N$ is equal to:

A. 4

B. 3

C. 2

D. 1

Given : $\left(x+x^{\log_2 x}\right)^7$
Now, we know
$^7C_3 x^4 \left(x^{\log_2 x}\right)^3 = 4480$

$\Rightarrow 35x^4\left(x^{\log_2 x}\right)^3 = 4480$

$\Rightarrow x^4\left(x^{\log_2 x}\right)^3 = 128$

$\Rightarrow x^{4+3\log_2 x} = 128$

Taking $\log_2$, we get
$\Rightarrow (4+3\log_2 x)\log_2 x = 7$
Let $\log_2 x = y$
$\Rightarrow 4y+3y^2 = 7$

$\Rightarrow 3y^2+4y-7=0$

$\Rightarrow (y-1)(3y+7)=0$

$\Rightarrow y = 1, -\frac{7}{3}$

$\Rightarrow \log_2 x = 1, -\frac{7}{3}$

$\Rightarrow x=2, x=2^{-7/3}$

Hence, from the given options $x=2$.

14. The coefficient of x^{256} in the expansion of $(1-x)^{101}(x^2+x+1)^{100}$ is

A. ${}^{100}C_{15}$

B. ${}^{100}C_{16}$

C. $-{}^{100}C_{16}$

D. $-{}^{100}C_{15}$

$(1-x)^{101}(x^2+x+1)^{100}$
$\Rightarrow (1-x)((1-x)(1+x+x^2))^{100}$
$\Rightarrow (1-x)(1-x^3)^{100}$
General term in $(1-x^3)^{100}$ is ${}^{100}C_r\left(-x^3\right)^r = {}^{100}C_r(-x)^{3r}$

$\therefore x^{256}$ occurs if $3r=256$ or $3r+1=256$

$r=\dfrac{256}{3}$ (not valid) $r=\dfrac{255}{3}=85$

$\therefore$ Coefficient of x^{256} is $-(-1)^{100}C_{85} = {}^{100}C_{15}$

15. If $n \geq 2$ is a positive integer, then the sum of the series
${}^{n+1}C_2 + 2\left({}^2C_2 + {}^3C_2 + {}^4C_2 + \cdots + {}^nC_2\right)$ is

A. $\dfrac{n(n+1)^2(n+2)}{12}$

B. $\dfrac{n(n-1)(2n+1)}{6}$

C. $\dfrac{n(n+1)(2n+1)}{6}$

D. $\dfrac{n(2n+1)(3n+1)}{6}$

We know ${}^2C_2 = {}^3C_3$
Let $S = {}^3C_3 + {}^3C_2 + \cdots + {}^nC_2 = {}^{n+1}C_3 \quad (\because {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r)$
$\therefore {}^{n+1}C_2 + {}^{n+1}C_3 + {}^{n+1}C_3$
$= {}^{n+2}C_3 + {}^{n+1}C_3$

$$= \frac{(n+2)!}{3!(n-1)!} + \frac{(n+1)!}{3!(n-2)!}$$

$$= \frac{(n+2)(n+1)n}{6} + \frac{(n+1)(n)(n-1)}{6}$$

$$= \frac{n(n+1)(2n+1)}{6}$$

$TRICK$: Put $n=2$ and verify the options.

16. The value of $-{}^{15}C_1 + 2 \times {}^{15}C_2 - 3 \times {}^{15}C_3 + \ldots - 15 \times {}^{15}C_{15}$ is :
$+{}^{14}C_1 + {}^{14}C_3 + {}^{14}C_5 + \ldots + {}^{14}C_{11}$

A. 2^{14}

B. $2^{13} - 13$

C. $2^{16} - 1$

D. $2^{13} - 14$ ✔

$S_1 = -{}^{15}C_1 + 2 \times {}^{15}C_2 - 3 \times {}^{15}C_3 + \ldots - 15 \times {}^{15}C_{15}$
$= \sum_{r=1}^{15} (-1)^r \times r \times {}^{15}C_r = 15 \sum_{r=1}^{15} (-1)^{r\,14}C_{r-1}$
$= 15(-{}^{14}C_0 + {}^{14}C_1 - {}^{14}C_2 + \cdots - {}^{14}C_{14}) = 15(0) = 0$

$S_2 = {}^{14}C_1 + {}^{14}C_3 + {}^{14}C_5 + \cdots + {}^{14}C_{11}$
$= ({}^{14}C_1 + {}^{14}C_3 + {}^{14}C_5 + \cdots + {}^{14}C_{11} + {}^{14}C_{13}) - {}^{14}C_{13}$
$= 2^{13} - 14$
$\therefore S_1 + S_2 = 2^{13} - 14$

17. The value of $\sum_{r=0}^{6}\left({}^6C_r \cdot {}^6C_{6-r}\right)$ is equal to:

A. 1124

B. 924

C. 1324

D. 1024

$\sum_{r=0}^{6}\left({}^6C_r \cdot {}^6C_{6-r}\right)$
$= {}^6C_0 \cdot {}^6C_6 + {}^6C_1 \cdot {}^6C_5 + \cdots + {}^6C_6 \cdot {}^6C_0$

Now, we know
$(1+x)^6 \cdot (1+x)^6$
$= \left({}^6C_0 + {}^6C_1 x + \cdots\right)\left({}^6C_0 + {}^6C_1 x + \cdots\right)$

Comparing coefficient of x^6 on both sides
${}^6C_0 \cdot {}^6C_6 + {}^6C_1 \cdot {}^6C_5 + \cdots + {}^6C_6 \cdot {}^6C_0$
$= {}^{12}C_6 = \dfrac{12 \times 11 \times 10 \times 9 \times 8 \times 7}{6 \times 5 \times 4 \times 3 \times 2}$
$= 2 \times 11 \times 2 \times 3 \times 7$
$= 924$

18. A scientific committee is to formed from 6 Indians and 8 foreigners, which includes at least 2 Indians and double the number of foreigners as Indians. Then the number of ways, the committee can be formed is:

A. 560

B. 1050

C. 1625

D. 575

The possible ways are
$(2I, 4F) + (3I, 6F) + (4I, 8F)$
$= {}^6C_2 \cdot {}^8C_4 + {}^6C_3 \cdot {}^8C_6 + {}^6C_4 \cdot {}^8C_8$
$= 15 \times 70 + 20 \times 28 + 15 \times 1$
$= 1050 + 560 + 15 = 1625$

19. Team 'A'' consists of 7 boys and n girls and Team 'B'' has 4 boys and 6 girls. If a
total of 52 single matches can be arranged between these two teams when a
boy plays against a boy and a girl plays against a girl, then n is equal to:

A. 5

B. 6

C. 2

D. 4

$7 \times 4 + 6 \times n = 52$
$\Rightarrow 6n = 24$
$\Rightarrow n = 4$

20. A natural number has prime factorization given by $n = 2^x 3^y 5^z$, where y and z are such that $y + z = 5$ and $y^{-1} + z^{-1} = \frac{5}{6}, y > z$. Then the number of odd divisors of n, including 1, is:

A. 11

B. $6x$

C. 12

D. 6

$y + z = 5 \cdots (1)$
$\frac{1}{y} + \frac{1}{z} = \frac{5}{6}$
$\Rightarrow yz = 6$
Also $(y - z)^2 = (y + z)^2 - 4yz$
$\Rightarrow y - z = \pm 1 \ldots (2)$
from (1) and (2), $y = 3$ or 2 and $z = 2$ or 3
for calculating odd divisor of $p = 2^x.3^y.5^z$
x must be zero
$P = 2^0.3^3.5^2$
$\therefore$ total odd divisors must be $(3 + 1)(2 + 1) = 12$

21. The number of seven digit integers with sum of the digits equal to 10 and formed by using the digits $1, 2$ and 3 only is :

A. 77

B. 42

C. 35

D. 82

CASE -I : $1, 1, 1, 1, 1, 2, 3$

Number of ways $= \dfrac{7!}{5!} = 42$

CASE -II : $1, 1, 1, 1, 2, 2, 2$

Number of ways $= \dfrac{7!}{4! \times 3!} = 35$

Total number of ways $= 42 + 35 = 77$

22. The sum of all the $4-$digit distinct numbers that can be formed with the digits $1, 2, 2$ and 3 is:

A. 26664

B. 122664

C. 122234

D. 22264

Digits are $1, 2, 2$ and 3.

Total distinct numbers $= \frac{4!}{2!} = 12$

There are three distinct numbers when 1 is at unit place.
There are three distinct numbers when 3 is at unit place.
There are six distinct numbers when 2 is at unit place.

So, the sum of all the $4-$digit distinct numbers is,
$= (3 + 9 + 12)(10^3 + 10^2 + 10 + 1)$
$= 26664$

Alternate Solution:

$$\begin{array}{r} 1\;2\;2\;3 \\ 1\;2\;3\;2 \\ 1\;3\;2\;2 \\ 3\;1\;2\;2 \\ 3\;2\;1\;2 \\ 3\;2\;2\;1 \\ 2\;1\;3\;2 \\ 2\;3\;1\;2 \\ 2\;2\;1\;3 \\ 2\;2\;3\;1 \\ 2\;3\;2\;1 \\ +2\;1\;2\;3 \\ \hline 2\;6\;6\;6\;4 \end{array}$$

Subject: Mathematics

1. The sum of first four terms of a geometric progression (G.P.) is $\frac{65}{12}$ and the sum of their respective reciprocals is $\frac{65}{18}$. If the product of first three terms of the G.P. is 1, and the third term is α, then 2α is

Accepted Answers

3 3.0 3.00

Solution:

Let the first four terms be a, ar, ar^2, ar^3.

$a + ar + ar^2 + ar^3 = \frac{65}{12} \quad \cdots(1)$

and $\frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} + \frac{1}{ar^3} = \frac{65}{18}$

$\Rightarrow \frac{1}{a}\left(\frac{r^3 + r^2 + r + 1}{r^3}\right) = \frac{65}{18} \quad \cdots(2)$

$\frac{(1)}{(2)}$, we get

$a^2r^3 = \frac{18}{12} = \frac{3}{2}$

Also, $a^3r^3 = 1$

$\Rightarrow a\left(\frac{3}{2}\right) = 1 \Rightarrow a = \frac{2}{3}$

$\frac{4}{9}r^3 = \frac{3}{2}$

$\Rightarrow r^3 = \frac{3^3}{2^3} \Rightarrow r = \frac{2}{3}$

$\alpha = ar^2 = \frac{2}{3} \cdot \left(\frac{3}{2}\right)^2 = \frac{3}{2}$

$\therefore 2\alpha = 3$

2. Consider an arithmetic series and a geometric series having four initial terms from the set $\{11, 8, 21, 16, 26, 32, 4\}$. If the last terms of these seris are the maximum possible four digit numbers, then the number of common terms in these two series is equal to

Accepted Answers

3 3.0 3.00

Solution:

By observation

$A.P : 11, 16, 21, 26 \cdots$

$G.P : 4, 8, 16, 32 \cdots$

So common terms are $16, 256, 4096$

3. Let $S_n(x) = \log_{a^{1/2}} x + \log_{a^{1/3}} x + \log_{a^{1/6}} x + \log_{a^{1/11}} x + \log_{a^{1/18}} x + \log_{a^{1/27}} x + \cdots$ up to n-terms, where $a > 1$. If $S_{24}(x) = 1093$ and $S_{12}(2x) = 265$, then value of a is equal to

Accepted Answers

16 16.0 16.00

Solution:

$S_n(x) = \log_{a^{1/2}} x + \log_{a^{1/3}} x + \log_{a^{1/6}} x + \log_{a^{1/11}} x + \log_{a^{1/18}} x + \log_{a^{1/27}} x + \cdots$ up to n-terms
$\Rightarrow S_n(x) = 2\log_a x + 3\log_a x + 6\log_a x + 11\log_a x + \cdots$
$\Rightarrow S_n(x) = (\log_a x)(2 + 3 + 6 + 11 + \cdots)$

$S_r = 2 + 3 + 6 + 11 + \cdots$
General term, $T_r = r^2 - 2r + 3$

$$S_n(x) = \sum_{r=1}^{n} (\log_a x)(r^2 - 2r + 3)$$

$$\Rightarrow S_{24}(x) = (\log_a x) \sum_{r=1}^{24} (r^2 - 2r + 3)$$

$\Rightarrow 1093 = 4372 \log_a x$

$$\Rightarrow \log_a x = \frac{1}{4}$$

$\Rightarrow x = a^{1/4} \quad \cdots (1)$

$$S_{12}(2x) = \log_a(2x) \sum_{r=1}^{12} (r^2 - 2r + 3)$$

$\Rightarrow 265 = 530 \log_a(2x)$

$$\Rightarrow \log_a(2x) = \frac{1}{2}$$

$\Rightarrow 2x = a^{1/2} \quad \cdots (2)$

After solving (1) and (2), we get
$a^{1/4} = 2$
$\Rightarrow a = 16$

4. Let n be a positive integer. Let $A=\sum_{k=0}^{n}(-1)^{k}\,{}^{n}C_{k}\left[\left(\frac{1}{2}\right)^{k}+\left(\frac{3}{4}\right)^{k}+\left(\frac{7}{8}\right)^{k}+\left(\frac{15}{16}\right)^{k}+\left(\frac{31}{32}\right)^{k}\right]$. If $63A=1-\frac{1}{2^{30}}$, then n is equal to

Accepted Answers

6 6.0 6.00

Solution:

$$A=\sum_{k=0}^{n}(-1)^{k}\,{}^{n}C_{k}\left[\left(\frac{1}{2}\right)^{k}+\left(\frac{3}{4}\right)^{k}+\left(\frac{7}{8}\right)^{k}+\left(\frac{15}{16}\right)^{k}+\left(\frac{31}{32}\right)^{k}\right]$$

$$\Rightarrow A=\left(1-\frac{1}{2}\right)^{n}+\left(1-\frac{3}{4}\right)^{n}+\left(1-\frac{7}{8}\right)^{n}+\left(1-\frac{15}{16}\right)^{n}+\left(1-\frac{31}{32}\right)^{n}$$

$$\Rightarrow A=\left(\frac{1}{2}\right)^{n}+\left(\frac{1}{4}\right)^{n}+\left(\frac{1}{8}\right)^{n}+\left(\frac{1}{16}\right)^{n}+\left(\frac{1}{32}\right)^{n}$$

$$=\frac{1}{2^{n}}+\frac{1}{2^{2n}}+\frac{1}{2^{3n}}+\frac{1}{2^{4n}}+\frac{1}{2^{5n}}$$

$$=\frac{1}{2^{n}}\left[\frac{1-\left(\frac{1}{2^{n}}\right)^{5}}{1-\frac{1}{2^{n}}}\right]$$

$$A=\frac{2^{5n}-1}{2^{5n}(2^{n}-1)}$$

$$63A=\frac{63\left(2^{5n}-1\right)}{2^{5n}\left(2^{n}-1\right)}$$

$$=\frac{63}{2^{n}-1}\left(1-\frac{1}{2^{5n}}\right)=1-\frac{1}{2^{30}}$$

$$\Rightarrow n=6$$

5. If the remainder when x is divided by 4 is 3, then the remainder when $(2020+x)^{2022}$ is divided by 8 is

Accepted Answers

1 1.0 1.00

Solution:

Let $x=4k+3$
$(2020+x)^{2022}$
$=(2020+4k+3)^{2022}$
$=(2024+4k-1)^{2022}$
$=(4A-1)^{2022}$
$={}^{2022}C_{0}(4A)^{2022}(-1)^{0}+{}^{2022}C_{1}(4A)^{2021}(-1)^{1}+.....+{}^{2022}C_{2021}(4A)^{1}(-1)^{2021}+{}^{2022}C_{2022}(4A)^{0}(-1)^{2022}$
Which will be of the form $8\lambda+1$
So, Remainder is 1.

6. The ratio of the coefficient of the middle term in the expansion of $(1+x)^{20}$ and the sum of the coefficients of two middle terms in expansion of $(1+x)^{19}$ is

Accepted Answers

1 1.0 1.00 01

Solution:

Coefficient of middle term in $(1+x)^{20} = {}^{20}C_{10}$
Sum of coefficient of two middle terms in $(1+x)^{19} = {}^{19}C_9 + {}^{19}C_{10}$

$$= {}^{20}C_{10} \ \left(\because\ {}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r\right)$$

Hence, required ratio is 1

7. The students $S_1, S_2, \ldots, S_{10}$ are to be divided into 3 groups A, B and C such that each group has at least one student and the group C has at most 3 students. Then the total number of possibilities of forming such groups is

Accepted Answers

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Solution:

$C \to 1 \qquad 9\begin{bmatrix} A \\ B \end{bmatrix}$

$C \to 2 \qquad 8\begin{bmatrix} A \\ B \end{bmatrix}$

$C \to 3 \qquad 7\begin{bmatrix} A \\ B \end{bmatrix}$

Number of ways

$$= {}^{10}C_1[2^9-2] + {}^{10}C_2[2^8-2] + {}^{10}C_3[2^7-2]$$
$$= 2^7[{}^{10}C_1 \times 4 + {}^{10}C_2 \times 2 + {}^{10}C_3] - 20 - 90 - 240$$
$$= 128[40+90+120] - 350$$
$$= (128 \times 250) - 350$$
$$= 10(3165)$$
$$= 31650$$

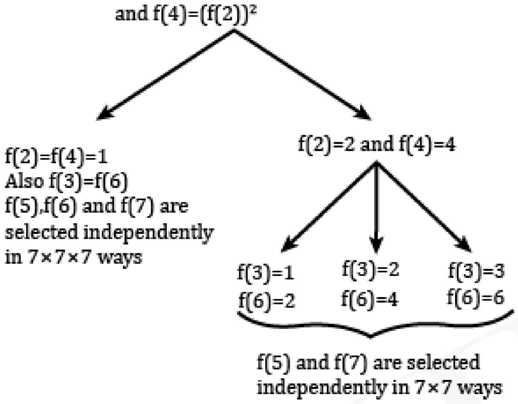
8. Let $S=\{1,2,3,4,5,6,7\}$. Then the number of possible functions $f: S \rightarrow S$ such that $f(m \cdot n)=f(m) \cdot f(n)$ for every $m, n \in S$ and $m \cdot n \in S$ is equal to

Accepted Answers

490 490.0 490.00

Solution:

Given $f(m \cdot n)=f(m) \cdot f(n)$
Clearly, $f(1)=1$



Total number of ways $=7^3+3 \cdot 7^2=490$