JEE Main Part Test 3

Subject: Mathematics

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1. The value of $\lim\limits_{x\to 1}\left(\frac{4}{\pi}\tan^{-1}x\right)^{\frac{1}{x^2-1}}$ is equal to

   A. $\frac{1}{\pi}$

   B. $-\frac{1}{\pi}$

   C. $e^{1/\pi}$

   D. $e^{-1/\pi}$

2. Let $f(x)=x^2-6x+5$ and m is the number of points of non-derivability of $y=|f(|x|)|$. If $|f(|x|)|=k, k\in\mathbb{R}$ has at least m distinct solution(s), then the number of integral values of k is

   A. 2

   B. 3

   C. 4

   D. 5

3. Let $f(x)=\begin{cases}\dfrac{e^{\frac{-x^2}{2}}-\cos x}{x\ln(1+x)\sin x(e^x-1)}, & x\neq 0\\ k, & x=0\end{cases}$.

   If $f(x)$ is continuous at $x=0$, then k equals

   A. $\frac{1}{4}$

   B. $\frac{1}{6}$

   C. $\frac{1}{12}$

   D. $\frac{1}{8}$

4. $f(x) = (2x - 3\pi)^5 + \frac{4}{3}x + \cos x$ and g is the inverse function of f. Then $g'(2\pi)$ is equal to

   A. $\frac{7}{3}$

   B. $\frac{3}{7}$

   C. $\frac{30\pi^4 + 4}{3}$

   D. $\frac{3}{30\pi^4 + 4}$

5. The radius of a right circular cylinder increases at the rate of $0.1\ \text{cm/min}$, and the height decreases at the rate of $0.2\ \text{cm/min}$. The rate of change of the volume of the cylinder, in cm^3/min, when the radius is $2\ \text{cm}$ and the height is $3\ \text{cm}$ is
   (The negative sign(-) indicates that volume decreases)

   A. $-\frac{2\pi}{5}$

   B. $\frac{8\pi}{5}$

   C. $-\frac{3\pi}{5}$

   D. $\frac{2\pi}{5}$

6. If a variable tangent to the curve $x^2y = c^3$ makes intercepts a, b on x and $y-$ axis respectively, then the value of a^2b is

   A. $27c^3$

   B. $\frac{4}{27}c^3$

   C. $\frac{27}{4}c^3$

   D. $\frac{4}{9}c^3$

7. The absolute difference between the greatest and the least values of the function $f(x) = x(\ln x - 2)$ on $[1, e^2]$ is

   A. 2

   B. e

   C. e^2

   D. 1

8. In which of the following functions is Rolle's theorem applicable?

   A. $f(x) = \begin{cases} x, & 0 \le x < 1 \\ 0, & x = 1 \end{cases}$ on $[0, 1]$

   B. $f(x) = \begin{cases} \dfrac{\sin x}{x}, & -\pi \le x < 0 \\ 0, & x = 0 \end{cases}$ on $[-\pi, 0]$

   C. $f(x) = \dfrac{x^2 - x - 6}{x - 1}$ on $[-2, 3]$

   D. $f(x) = \begin{cases} \dfrac{x^3 - 2x^2 - 5x + 6}{x - 1}, & \text{if } x \ne 1, \\ -6, & \text{if } x = 1 \end{cases}$ on $[-2, 3]$

9. Let the function f be defined by $f(x) = x \ln x$, for all $x > 0$. Then

   A. f is increasing on $(0, e^{-1})$

   B. f is decreasing on $(0, 1)$

   C. The graph of f is concave down for all x

   D. The graph of f is concave up for all x

10. If $x = 3\cos\theta - \cos 3\theta$ and $y = 3\sin\theta - \sin 3\theta$, then $\frac{dy}{dx}$ is

A. $\tan 2\theta$

B. $\sin 2\theta$

C. $-\tan 2\theta$

D. $\cot 2\theta$

11. Let $\cos^{-1}(x) + \cos^{-1}(2x) + \cos^{-1}(3x) = \pi$, where $x > 0$. If x satisfies the cubic equation $ax^3 + bx^2 + cx - 1 = 0$, then $a + b + c$ has the value equal to

A. 24

B. 25

C. 26

D. 28

12. If a, b, c are the sides opposite to angles A, B, C of a triangle ABC, respectively and $\angle A = \frac{\pi}{3}$, $b : c = \sqrt{3} + 1 : 2$, then the value of $\angle B - \angle C$ is

A. $\frac{\pi}{12}$

B. $\frac{\pi}{6}$

C. $\frac{\pi}{4}$

D. $\frac{\pi}{2}$

13. The value of $\tan^{-1}\frac{4}{7}+\tan^{-1}\frac{4}{19}+\tan^{-1}\frac{4}{39}+\tan^{-1}\frac{4}{67}+\cdots\infty$ equals

A. $\tan^{-1}1+\tan^{-1}\frac{1}{2}$

B. $\frac{\pi}{2}-\cot^{-1}2$

C. $\frac{\pi}{2}-\cot^{-1}1$

D. $\cot^{-1}1+\tan^{-1}3$

14. The solution set of the inequality $(\cot^{-1}x)(\tan^{-1}x)+\left(2-\frac{\pi}{2}\right)\cot^{-1}x-3\tan^{-1}x-3\left(2-\frac{\pi}{2}\right)>0$, is

A. $x\in(\tan 2,\tan 3)$

B. $x\in(\cot 3,\cot 2)$

C. $x\in(-\infty,\tan 2)\cup(\tan 3,\infty)$

D. $x\in(-\infty,\cot 3)\cup(\cot 2,\infty)$

15. If two sides of a triangle are the roots of $x^2-7x+8=0$ and the angle between these sides is $\frac{\pi}{3}$, then the product of inradius and circumradius of the triangle is

A. $\frac{8}{7}$

B. $\frac{5}{3}$

C. $\frac{5\sqrt{2}}{3}$

D. 8

16. The value of $\tan^{-1}\sqrt{\frac{a(a+b+c)}{bc}}+\tan^{-1}\sqrt{\frac{b(a+b+c)}{ca}}+\tan^{-1}\sqrt{\frac{c(a+b+c)}{ab}}$, where $a,b,c>0$, is

A. $\frac{\pi}{4}$

B. $\frac{\pi}{2}$

C. π

D. 0

17. In a triangle ABC, altitudes from vertices A and B have lengths 3 and 6 respectively. Then the exhaustive set of values of the length of altitude from vertex C is

A. $(3,7)$

B. $(2,6)$

C. $(3,4)$

D. $(1,5)$

18. The value of $\lim\limits_{n\to\infty}\frac{\{x\}+\{2x\}+\ldots+\{nx\}}{n^2}$ is (where $\{x\}$ denotes the fractional part of x)

A. 0

B. 1

C. ∞

D. It does not exist

19. The derivative of $\ln\left(\frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}-x}\right)$ with respect to $\cos(\ln x)$ is

A. $\frac{2x}{\sqrt{(1+x^2)}\sin(\ln x)}$

B. $-\frac{2x}{\sqrt{(1+x^2)}\sin(\ln x)}$

C. 1

D. $-\frac{4x}{\sqrt{(1+x^2)}\sin(\ln x)}$

20. In $\triangle ABC$, sides opposite to angles A, B, C are denoted by a, b, c respectively. If $\angle A = 60^\circ, a = 5, b = 2\sqrt{3}$, then $\angle B =$

A. $\sin^{-1}\frac{3}{5}$

B. $\sin^{-1}\frac{4}{5}$

C. $180^\circ - \sin^{-1}\frac{3}{5}$

D. $180^\circ - \sin^{-1}\frac{4}{5}$

21. If $\sin^{-1}(\sin p) = 3\pi - p$ and the point of intersection of the lines $x + y = 6$ and $px - y = 3$ will have integral co-ordinates (both abscissa and ordinate), then the number of values of p is

22. Let $L_1 = \lim\limits_{x\to 0}\frac{\cos(\pi x)(e^{\lambda x}-1)}{\pi \sin x}$ and $L_2 = \lim\limits_{x\to 0}\frac{\ln(1-x)+\sin 2x}{x}$. If $L_1 = L_2$, then the value of $[\lambda]$ is
(Note: $[\lambda]$ denotes the largest integer less than or equal to λ.)

23. If the value of $\lim\limits_{x\to 0}\left(\frac{x^n \sin^n x}{x^n - \sin^n x}\right)$ is non-zero finite, then n is equal to

24. Let $f(x) = x^2 + px + 3$ and $g(x) = x + q$, where $p, q \in \mathbb{R}$. If $F(x) = \lim\limits_{n\to\infty}\frac{f(x)+x^n g(x)}{1+x^n}$ is derivable at $x = 1$, then the value of $p^2 + q^2$ is

25. The number of point(s) of non-differentiability for $f(x) = [e^x] + |x^2 - 3x + 2| \text{ in } (-1, 3)$ is (where [.] denotes greatest integer function, $e^3 = 20.1$)

26. If $f(x) = \begin{cases} a + \sin^{-1}(x + b), & x \geq 1 \\ x, & x < 1 \end{cases}$ is differentiable function, then value of $a + b$ is

27. If $f(x) = \tan^{-1} \dfrac{x}{1 + \sqrt{(1 - x^2)}} + \sin\left\{2 \tan^{-1} \sqrt{\left(\dfrac{1 - x}{1 + x}\right)}\right\}, x \in (0, 1)$, then the value of $f'\left(\dfrac{1}{2}\right)$ is

28. The normal at the point $P\left(2, \dfrac{1}{2}\right)$ on the curve $xy = 1$ meets the curve again at Q. If m is the slope of the curve at Q, then the value of $|m|$ is

29. Let $f(x) = |x^2 - 4x + 3|$ be a function defined on $x \in [0, 4]$ and α, β, γ are the abscissas of the critical points of $f(x)$. If m and M are the local and absolute maximum values of $f(x)$ respectively, then the value of $\alpha^2 + \beta^2 + \gamma^2 + m^2 + M^2$ is

30. The minimum value of α for which the equation $\dfrac{4}{\sin x} + \dfrac{1}{1 - \sin x} = \alpha$ has at least one solution in $\left(0, \dfrac{\pi}{2}\right)$ is