ELECTROMAGNETIC INDUCTION

MAGNETIC FLUX

$\phi = \vec{B}.\vec{A} = BA\cos\theta$ for uniform $\vec{B}$

$\phi = \int \vec{B}.d\vec{A}$ for non uniform $\vec{B}$.

FARADAY'S LAWS OF ELECTROMAGNETIC INDUCTION

- Induced EMF $(\varepsilon) \propto$ Rate of change of magnetic flux $\left(\frac{d\phi}{dt}\right)$

LENZ'S LAWS

The direction of an induced emf is always such as to oppose the cause producing it.

- $(\varepsilon) = -\left(\frac{d\phi}{dt}\right)$

EMF induced in a straight conductor in uniform magnetic field:

ε = BLv sin θ

Where,

B = flux density

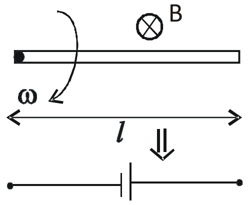
L = length of the conductor

v = velocity of the conductor

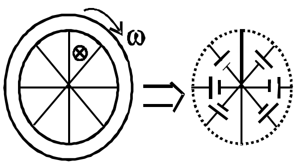
θ = angle between direction of motion of conductor & B .

EMF induced in a rod rotating perpendicular to magnetic field:

$\varepsilon = \frac{1}{2}B\omega l^2$



For a wheel rotating in a earth magnetic field effective emf induced between the periphery & centre = $\frac{1}{2}B\omega\ell^2$



COIL ROTATION IN MAGNETIC FIELD SUCH THAT AXIS OF ROTATION IS PERPENDICULAR TO THE MAGNETIC FIELD

Instantaneous induced emf.

$\varepsilon = BAN\omega\sin(\omega t)$

$\varepsilon_0 = BAN\omega$

$\varepsilon = \varepsilon_0\sin(\omega t)$

SELF INDUCTION & SELF INDUCTANCE

Coefficient of self inductance $L = \dfrac{\phi_s}{i}$ or $\phi_S = Li$

i = current in the circuit

ϕ_S = magnetic flux linked with the circuit due to the current i.

L depends only on; (i) shape of the loop & (ii) medium

self induced emf $e_s = \dfrac{d\varphi_s}{dt} = -\dfrac{d}{dt}(Li) = -L\dfrac{di}{dt}$ (if L is constant)

MUTUAL INDUCTION

Mutual inductance = $M = \dfrac{\phi_m}{I_p} = \dfrac{\text{flux linked with secondary}}{\text{current in the primary}}$

Mutually induced emf : $E_m = \dfrac{d\phi_m}{dt} = -\dfrac{d}{dt}(MI) = -M\dfrac{dI}{dt}$ (if M is constant)

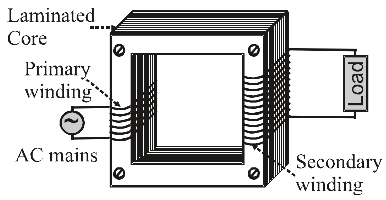
M depends on (1) geometry of loops (2) medium (3) orientation & distance loops.

- **If two coils of self inductance L_1, and L_2 are wound over each other, the mutual inductance.** $M = K\sqrt{L_1L_2}$ where K is called coupling constant.
- **For two coils wound in same direction and connected in series.** $L = L_1 + L_2 + 2M$
- **For two coils wound in opposite direction and connected in series.** $L = L_1 + L_2 - 2M$
- **For two coils in parallel** $L = \dfrac{L_1L_2 - M^2}{L_1 + L_2 \pm 2M}$

TRANSFORMER

- **Transformer**

$$\frac{E_2}{E_1} = \frac{N_2}{N_1}$$

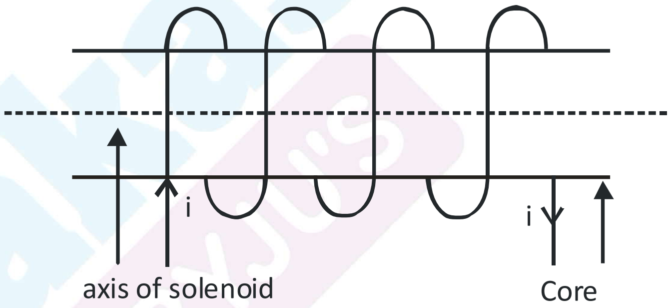


- For ideal transformer $\frac{E_2}{E_1} = \frac{I_1}{I_2} = \frac{N_2}{N_1}$

- Efficiency $\eta = \frac{P_{out}}{P_{in}} \times 100\%$

SOLENOID

$B = \mu$ ni (ideal:length >> diameter)



Where,

μ = magnetic permeability of the core material

n = number of turns in the solenoid per unit length

i = current in the solenoid

Self inductance of a solenoid $L = \mu_0 n^2 A l$

Where, A = area of cross section of solenoid .

SUPER CONDUCTION LOOP IN MAGNETIC FIELD

R = O ; $\varepsilon = 0$. Therefore ϕ_{total} = constant. Thus in a superconducting loop flux never changes. (or it opposes 100%)

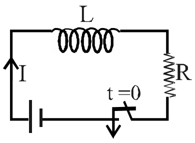
(i) Energy stored in an inductor $W = \frac{1}{2} LI^2$

(ii) Energy of interaction of two loops $U = I_1\phi_2 = I_2\phi_1 = MI_1I_2$

Where M is mutual inductance

GROWTH OF A CURRENT IN AN L- R CIRCUIT

$I = \frac{E}{R}(1 - e^{-Rt/L})$. [if initial current = 0]

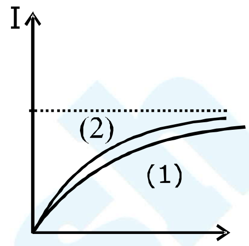


$\frac{L}{R}$ = time constant of the circuit.

$I_0 = \frac{E}{R}$

(i) L behaves as open circuit at t = 0 [if i = 0]

(ii) L behaves as short circuit at $t = \infty$ always.

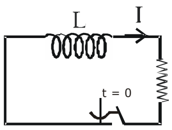


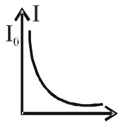
Curve (1) $\rightarrow \frac{L}{R}$ Large

Curve (2) $\rightarrow \frac{L}{R}$ Small

DECAY OF CURRENT

Initial current through the inductor = I_0; Current at any instant $i = I_0 e^{-Rt/L}$.





ALTERNATIVE CURRENT AND EM WAVES

Average value

$$I_{av} = \frac{\int_0^T I\,dt}{\int_0^T dt} = \frac{1}{T}\int_0^T I\,dt$$

RMS value

$$I_{rms} = \sqrt{\frac{\int_0^T I^2\,dt}{\int_0^T dt}}$$

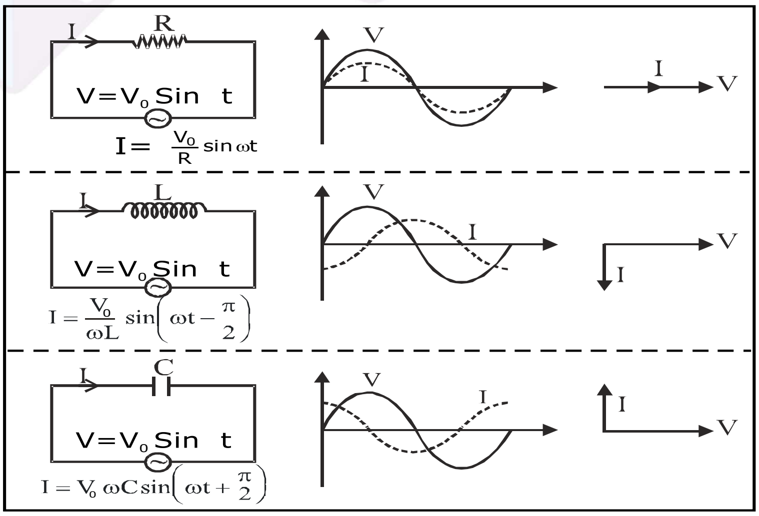
- **For sinusoidal voltage** $V = V_0 \sin \omega t$,

$$V_{av} = \frac{2V_0}{\pi} \;\&\; V_{rms} = \frac{V_0}{\sqrt{2}}$$

- **For sinusoidal current** $I = I_0 \sin(\omega t + \phi)$,

$$I_{av} = \frac{2I_0}{\pi} \;\&\; I_{rms} = \frac{I_0}{\sqrt{2}}$$

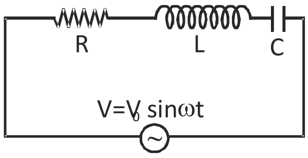
AC CIRCUITS



Impedance

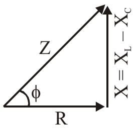
$Z = \sqrt{R^2 + X^2}$

where X = reactance



Series LCR Circuit

$Z = \sqrt{R^2 + (X_L - X_C)^2}$

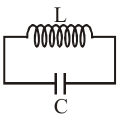


$V = \sqrt{V_R^2 + (V_L - V_C)^2}$

- **Power Factor** = $\cos\phi$ = R/Z, At resonance : $X_L = X_C \Rightarrow Z = R, V = V_R$

- **LC Oscillation**

$q = q_0 \sin(\omega t + \theta)$,



$I = I_0 \cos(\omega t + \theta)$ $\quad \boxed{I_0 = q_0\omega}$

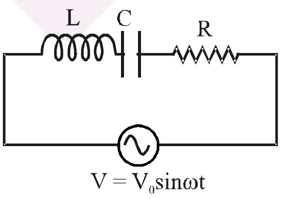
$$\text{Energy} = \frac{1}{2}LI^2 + \frac{q^2}{2C} = \frac{q_0^2}{2C} = \frac{1}{2}LI_0^2 = \text{constant}$$

Comparison with SHM $q \rightarrow x$, $I \rightarrow v$, $L \rightarrow m$, $C \rightarrow \frac{1}{K}$

Comparison of Damped Mechanical & electrical systems

(I) Series LCR circuit :

$$\frac{d^2q}{dt} + \frac{R}{L}\frac{dq}{dt} + \frac{1}{LC}q = \frac{V_0}{L}\cos\omega t$$



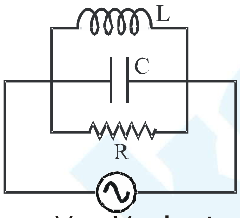
Compare with mechanical damped system equation

$$\frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}\chi = \frac{F_0}{m}\cos\omega t$$

Where b = damping coefficient.

Mechanical system	Electrical systems (series RLC)
Displacement (x)	Charge (q)
Driving force (F)	Driving voltage (V)
Kinetic energy $\left(\frac{1}{2}mv^2\right)$	Electromagnetic energy of moving Charge $\frac{1}{2}L\left(\frac{dq}{dt}\right)^2 = \frac{1}{2}Li^2$
Potential energy $\frac{1}{2}kx^2$	Energy of static Charge $\frac{q^2}{2C}$
mass (m)	L
Power $P = Fv$	Power $P = VI$
Damping (b)	Resistance (R)
Spring Constant	$1/C$

(II) Parallel LCR circuit :



$V = V_0 \sin\omega t$

$$I = I_L + I_C + I_R = \frac{\phi}{L} + \frac{d}{dt}C\left(\frac{d\phi}{dt}\right) + \frac{1}{R}\frac{d\phi}{dt} \Rightarrow \frac{d^2\phi}{dt^2} + \frac{1}{RC}\frac{d\phi}{dt} + \frac{1}{LC}\phi = \frac{V_0}{ZC}\sin\omega t$$

Displacement **(X)** $\Leftrightarrow$ Flux linkage **(ϕ)**

Velocity $\left(\frac{dx}{dt}\right)$ $\Leftrightarrow$ Voltage $\left(\frac{d\phi}{dt}\right)$

Mass **(m)** $\Leftrightarrow$ Capacitance **(C)**

Spring constant **(k)** $\Leftrightarrow$ Reciprocal Industance **(1/L)**

Driving force **(F)** $\Leftrightarrow$ Current **(i)**

Damping coefficient **(b)** $\Leftrightarrow$ Reciprocal resistance **(1/R)**

Properties of EM Waves

- The electric and magnetic fields $\vec{E}$ and $\vec{B}$ are always perpendicular to the direction in which the wave is travelling. Thus the EM wave is a transverse wave.
- EM waves carry momentum and energy.
- EM wave travel through vacuum with the speed of light c, where $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3\times 10^8$ m/s
- The instantaneous magnitude of $\vec{E}$ and $\vec{B}$ in an EM wave are related by the expression $\frac{E}{B} = c$
- The cross product $\vec{E} \times \vec{B}$ always gives the direction in which the wave travels.
- **Poynting Vector :** The rate of flow of energy crossing a unit area by electromagnetic radiation is given by poynting vector $\vec{S}$ where $\vec{S} = \frac{1}{\mu_0}(\vec{E} \times \vec{B})$
- **Displacement current :** In a region of space in which there is a changing electric field, there is a displacement current defined as $I_d = \epsilon_0 \frac{d\phi_E}{dt}$ where ε_0 is the permittivity of free space and $\phi_E = \int \vec{E}.d\vec{S}$ is the electric flux.

Maxwell's Equations

$\oint \vec{E}.d\vec{S} = \frac{q}{\varepsilon_0}$ **[Gauss law for electricity]**

$\oint \vec{B}.d\vec{S} = 0$ [Gauss law for magnetism]

$\oint \vec{E}.d\vec{\ell} = -\frac{d\phi_B}{dt}$ [Faraday's law]

$\oint \vec{B}.d\vec{\ell} = \mu_0 \left[I_c + \epsilon_0 \frac{d\phi_E}{dt} \right]$ [Ampere's law with Maxwell's correction]

KEY POINTS

- An alternating current of frequency 50Hz becomes zero, 100 times in one second because alternating current changes direction and becomes zero twice in a cycle.
- An alternating current cannot be used to conduct electrolysis because the ions due to their inertia, cannot follow the changing electric field.
- Average value of AC is always defined over half cycle because average value of AC over a complete cycle is always zero.
- AC current flows on the periphery of wire instead of flowing through total volume of wire. This is known as skin effect.