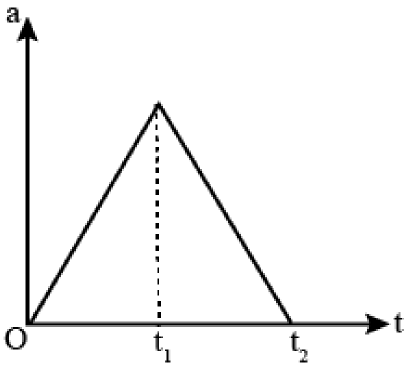
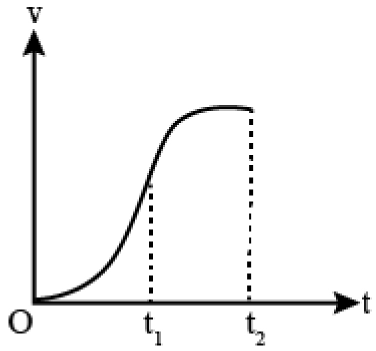
JEE Main Part Test 1

Subject: Physics Class: Standard XII

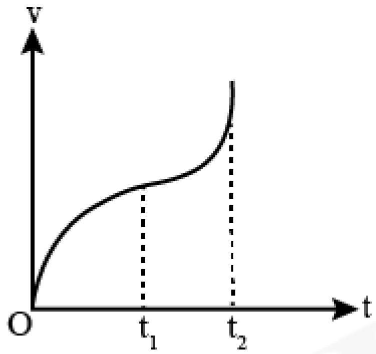
1. The acceleration-time graph of a particle is shown in figure. The respective $v\!-\!t$ graph of the particle is



A.



B.



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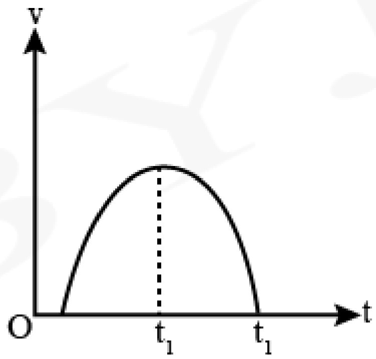

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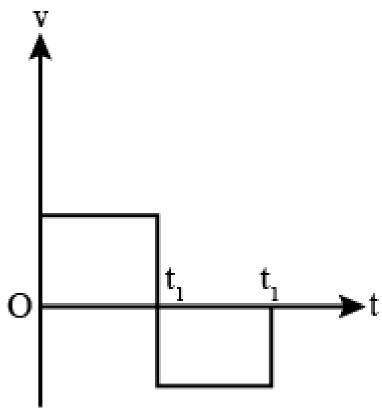


As per given graph, from 0 to t_1, acceleration is increasing linearly with time

i.e., $a \propto t$

$$\Rightarrow a = kt = \frac{dv}{dt} = kt$$

$$\therefore \; v = \frac{kt^2}{2}$$

Hence, $v - t$ graph should be parabolic upwards.

Similarly, from t_1 to t_2 acceleration is decreasing linearly with time.

Hence, $v - t$ graph should be parabolic downwards.

Hence, option (A) is the correct answer.

2. A particle is projected at an angle with the horizontal such that it follows a trajectory given by the equation $y = 6x – 2x^2$. Find the maximum height attained by it.

A. $2.5\ m$

B. $5.2\ m$

C. $4.5\ m$

D. $6\ m$

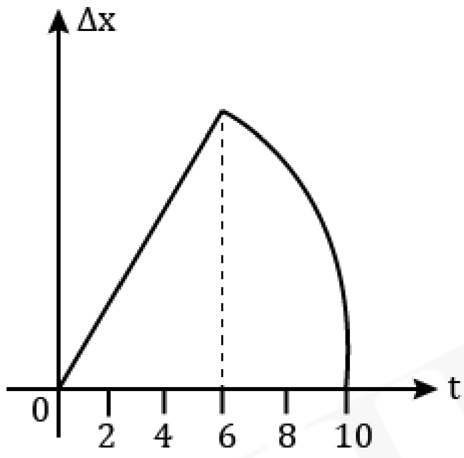
Trajectory of a projectile is given by the eqn
$y = x\tan\theta\left(1-\frac{x}{R}\right)$
If $y = ax - bx^2$, then $\tan\theta = a$
and $b = \frac{\tan\theta}{R} \Rightarrow R = \frac{a}{b}$
For maximum height H, we have
maximum height, $H = \frac{u^2\sin^2\theta}{2g}$ and Range $R = \frac{u^2\sin 2\theta}{g}$

So, we have the ratio $\frac{H}{R} = \frac{\frac{u^2\sin^2\theta}{2g}}{\frac{u^2\sin 2\theta}{g}}$

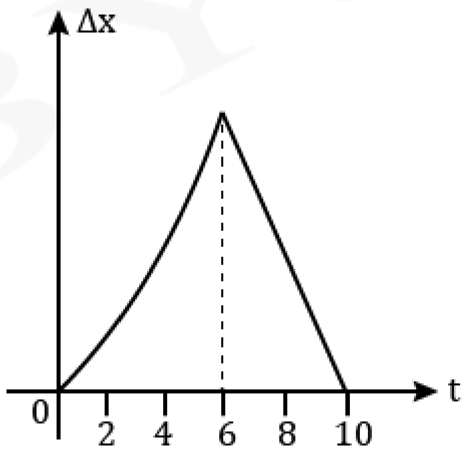
$\Rightarrow \frac{4H}{R} = \tan\theta$
Since, $y = 6x – 2x^2$
$\Rightarrow H = \frac{1}{4}R\tan\theta = \frac{a^2}{4b} = \frac{6^2}{4\times 2} = 4.5\ m$

3. Two stones are thrown up simultaneously with initial speeds u_1 and $u_2 (u_2 > u_1)$. They hit the ground after $6\ \mathrm{s}$ and $10\ \mathrm{s}$ respectively. Which graph correctly represents the time variation of, $\Delta x = (x_2 - x_1)$, the relative position of the second stone with respect to the first upto $t = 10\ \mathrm{s}$? Assume that the stones do not rebound after hitting the ground.

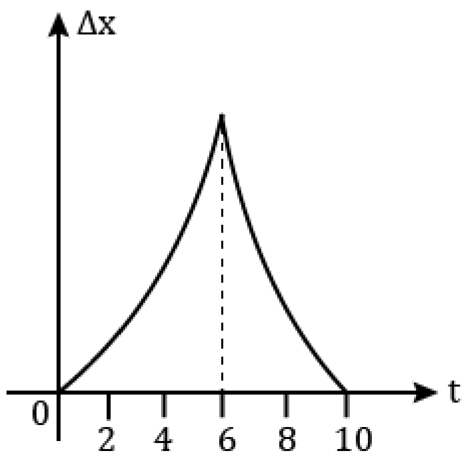
A.



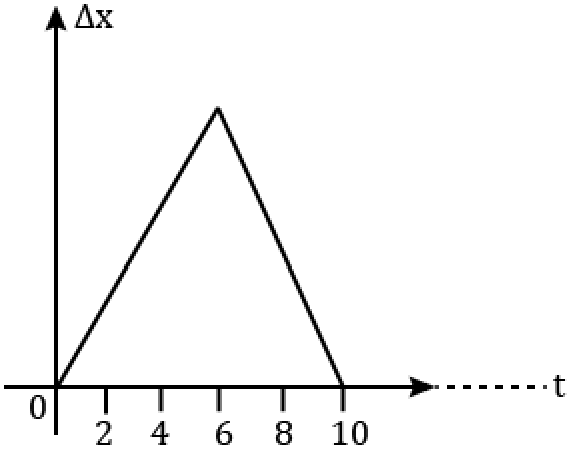
B.



C.



D.



Upto $6\ \mathrm{s}$
$u_{rel} = u_2 - u_1$
$a_{rel} = g - g = 0$
Using second equation of motion, we get
$S_{rel} = u_{rel}t + \frac{1}{2}a_{rel}t^2$
$\Delta x = (u_2 - u_1) \times t$
Initially relative acceleration between them is zero, so distance between them will increase linearly.
Hence, Δx increases linearly with t upto $6\ \mathrm{s}$.
From $t = 6\ \mathrm{s}$ to $t = 10\ \mathrm{s}$,
First stone is at rest just after 6 s
Now after that
$a_{rel} = g$
$\Delta x = (u_2 - u_1)t - \frac{1}{2}gt^2$ (in the downward journey Δx decreases parabolically)
Hence, the correct answer is option (a)

4. In a lift moving up with an acceleration of $5\ \mathrm{ms}^{-2}$, a ball is dropped from a height of $1.25\ \mathrm{m}$. The time taken by the ball to reach the floor of the lift is approximately $(g = 10\ \mathrm{ms}^{-2})$

A. 0.3 second

B. 0.2 second

C. 0.16 second

D. 0.4 second

Taking upward y-axis as positive
Relative velocity of the lift with respect to ball $u_{lb} = 0\ \mathrm{m/s}$
Distance travelled by the lift w.r.t ball $= S_{lb} = 1.25\ \mathrm{m}$
Acceleration of the lift $= a = 5\ \mathrm{m/s}^2$
Acceleration due to gravity $= g = -10\ \mathrm{m/s}^2$ (because a and g are in opposite direction)
Acceleration of the lift w.r.t ball $= a_{lb} = a-(-g) = 5-(-10) = 15\ \mathrm{m/s}^2$
Using second equation of motion, we get

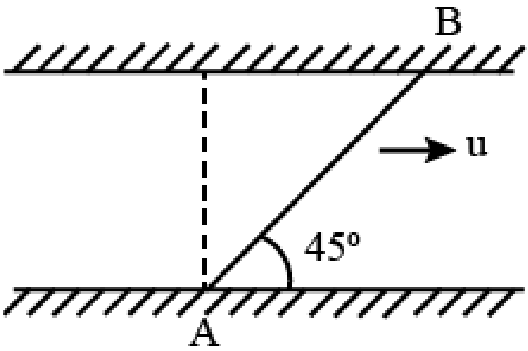
$$S_{lb} = u_{lb}.t + \frac{1}{2}a_{lb}t^2$$

$$\Rightarrow 1.25 = \frac{1}{2} \times 15 \times t^2$$

$$t = \sqrt{\frac{2 \times 1.25\ \mathrm{m}}{(5+10)\ \mathrm{ms}^{-2}}} \approx 0.4\ \mathrm{s}$$

Hence, the ball will take nearly 0.4 seconds to touch the floor of the lift.

5. A man wants to reach point B on the opposite bank of a river flowing at a speed u as shown. What minimum velocity relative to water should the man have so that he can reach directly to point B?



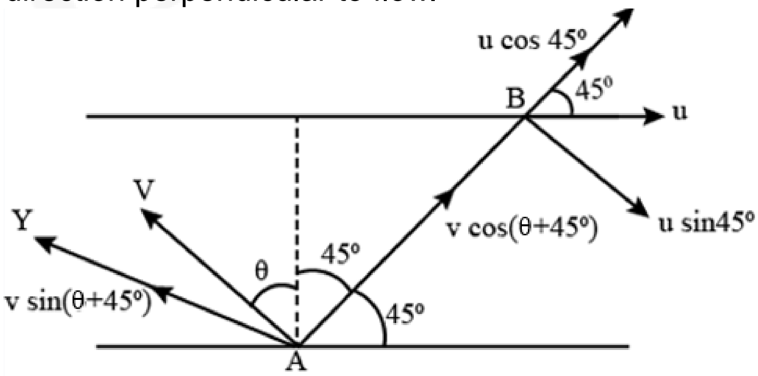
A. $\frac{u}{\sqrt{2}}$, in the upstream at an angle 45° with the vertical

B. $\sqrt{2}u$, in the upstream at an angle 45° with the vertical

C. $\frac{u}{\sqrt{2}}$, in the downstream at an angle 45° with the vertical

D. $\sqrt{2}u$, in the downstream at an angle 45° with the vertical

Let v be the velocity of man w.r.t flow in a direction at an angle θ with the direction perpendicular to flow.



For the man to go along AB,

Along y-axis, $v \sin(\theta + 45^\circ) = u \sin 45^\circ$

for him to go along AB, then

$$v = \frac{u \sin 45^\circ}{\sin(\theta + 45^\circ)}$$

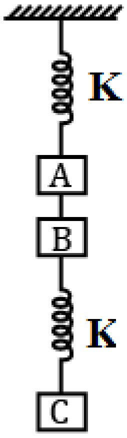
For v to be minimum, $\sin(\theta + 45^\circ)$ should be maximum

$\Rightarrow \theta + 45^\circ = 90^\circ$

$\Rightarrow \theta = 45^\circ$

$\Rightarrow V_{min} = \frac{u}{\sqrt{2}}$, in the upstream at an angle 45° to the vertical.

6. The system shown is in equilibrium. Find the accelerations of the blocks A, B and C just after the spring between B and C is cut. All blocks are of equal masses 'm' each and springs are of equal stiffness. (Assume springs to be ideal and take downward acceleration to be positive).



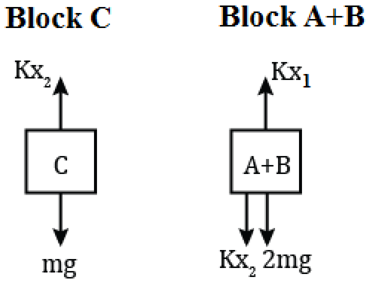
A. $+g, +g, -g$

B. $-\frac{g}{2}, -\frac{g}{2}, +g$

C. $+g, -\frac{g}{2}, -g$

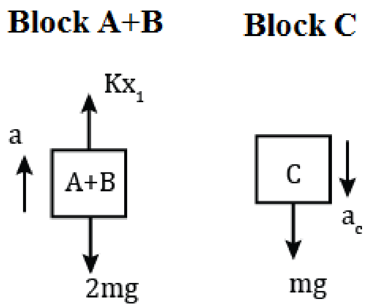
D. $0, -\frac{g}{2}, -g$

Since A and B are connected by inextensible string, therefore both will experience same acceleration. Hence they will be considered as single system.
Let the extension in the string between ceiling and block A be x_1 and that for spring between B and C be x_2
Before cutting



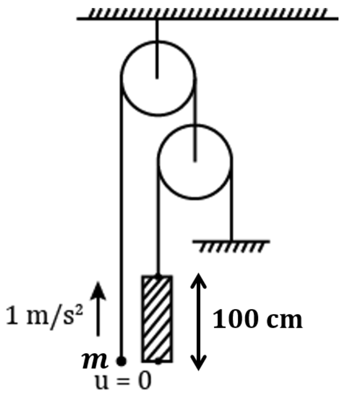
For block C,
$Kx_2 = mg$
For block A and B,
$Kx_1 = kx_2 + 2mg$
Since $Kx_2 = mg$
$\Rightarrow Kx_1 = 3mg$

After Cutting, spring force $kx_2 = 0$



Let a be the acceleration of block A and B.
For block A and B,
$Kx_1 - 2mg = 2ma$
Since $Kx_1 = 3mg$
$\Rightarrow 3mg - 2mg = 2ma$
$a = \frac{g}{2}$(Upwards) $\Rightarrow a_B = a_C = -\frac{g}{2}$
Let the acceleration for block C be a_c, then from the free body diagram,
$mg = ma_c$
$a_c = g$ (Downwards)

7. In the arrangement shown in the figure, the mass 'm' is going upward with an acceleration of $1\ m/s^2$. The length of the rod is $100\ cm$. Pulley is frictionless and threads are massless . If the mass is set at the level of the lower end of the rod and released, how much time it will take for ball to reach the level of the upper end?



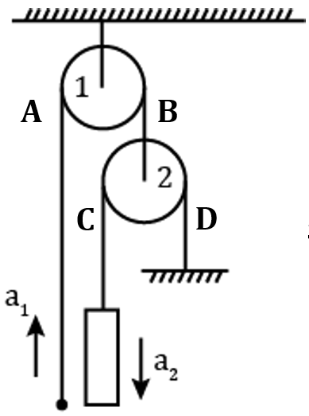
A. $\sqrt{\frac{1}{3}}\ s$

B. $\sqrt{\frac{2}{3}}\ s$

C. $\frac{2}{3}\ s$

D. $\frac{1}{3}\ s$

Let acceleration of the mass be a_1 and for the rod be a_2
Given that $a_1 = 1\ m/s^2$



Since pulley 1 is fixed,

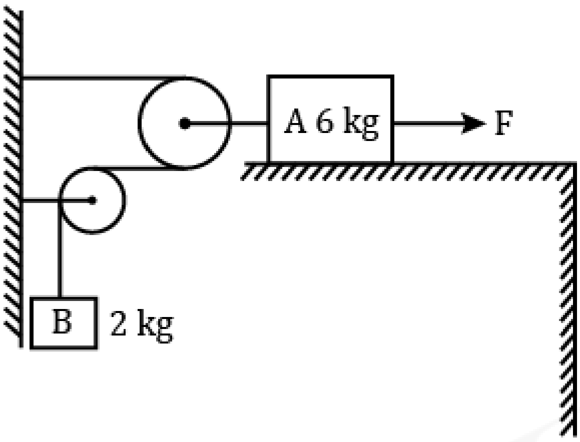
$a_A = a_B = a_1$

Considering pulley 2,
$\dfrac{a_C + a_D}{2} = a_B = a_1$
Since D end is fixed, $a_D = 0$ and $a_C = a_2$
$\Rightarrow a_1 = \dfrac{a_2 + 0}{2} = \dfrac{a_2}{2}$
$\Rightarrow a_2 = 2a_1 = 2\ m/s^2$

Now for the rod w.r.t the mass m, we know
Relative initial velocity $u_{rel} = 0$
Relative acceleration $a_{rel} = a_2 - a_1$
$= 2 - (-1) = 3\ m/s^2$ ($\because$ in opposite directions)
Relative displacement $S_{rel} = 100\ cm = 1\ m$ (length of the rod)

Using 2^{nd} equation of motion,
$S_{rel} = u_{rel}t + \frac{1}{2}a_{rel}t^2$
$\Rightarrow 1 = \dfrac{1}{2} \times (3) \times t^2$
$\Rightarrow t = \sqrt{\dfrac{2}{3}}\, s$

8. The system starts from rest and block A attains a velocity of $5\ \mathrm{m/s}$ after it has moved $5\ \mathrm{m}$ towards the right. Assuming the arrangement to be frictionless everywhere and pulley & strings to be light, find the value of the constant force F applied on block A. (Take $g = 10\ m/s^2$)



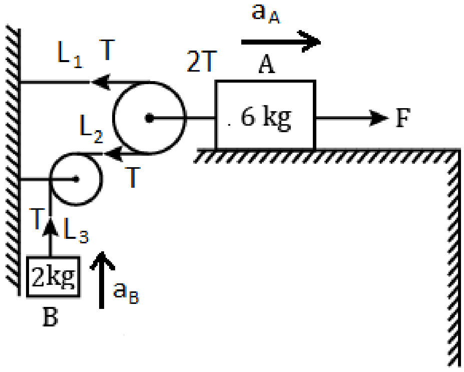
A. $50\ \mathrm{N}$

B. $75\ \mathrm{N}$

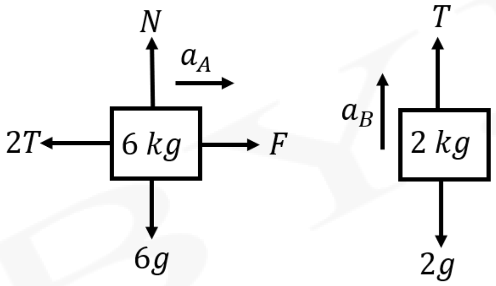
C. $100\ \mathrm{N}$

D. $96\ \mathrm{N}$

The given system can be reframed as shown

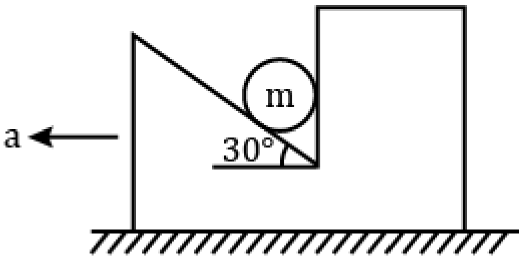


From the constrained condition, using method of intercept we get
$\ddot{L}_1 + \ddot{L}_2 + \ddot{L}_3 = 0$
$\Rightarrow\ a_A + a_A + 0 - a_B = 0$
$\Rightarrow\ a_B = 2a_A$...........(1)
Since A starts from rest, using 3rd equation of motion, we get
$a_A = \dfrac{v^2}{2s} = \dfrac{5^2}{2\times 5} = 2.5\ m/s^2$
$\Rightarrow\ a_B = 5\ m/s^2$



From the FBDs of the masses $6\ kg$ and $2\ kg$, we have
$T - 2g = 2a_B \Rightarrow\ T = 2g + 2a_B = 20 + 2\times 5 = 30\ N$
$F - 2T = 6a_A \Rightarrow\ F = 2T + 6a_A = 2\times 30 + 6\times 2.5 = 75\ N$
Thus, the value of F is $75\ N$

9. In the figure shown, if a ball of mass m is at rest relative to the wedge moving to the left with an acceleration $a = g\sqrt{3}$, find the force exerted by the vertical face of the wedge on mass m.



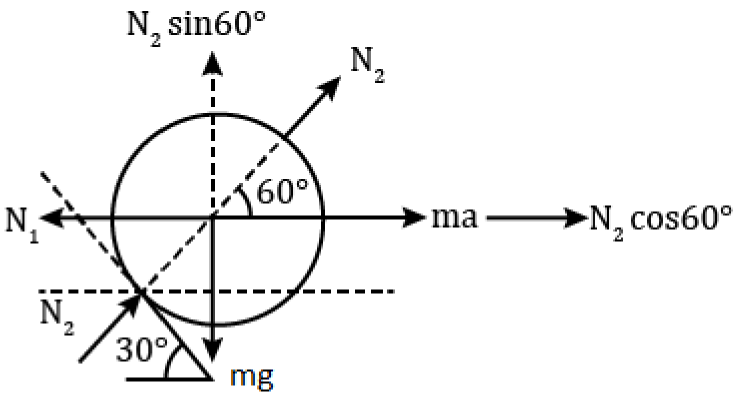
A. $\frac{2mg}{\sqrt{3}}$

B. $mg\sqrt{3}$

C. $\frac{4mg}{\sqrt{3}}$

D. $\frac{\sqrt{3}mg}{4}$

Using pseudo force concept
In the frame of reference of the wedge, the mass m has no acceleration.
The force exerted by verticle face on m is the normal force N_1
From FBD of the sphere we have

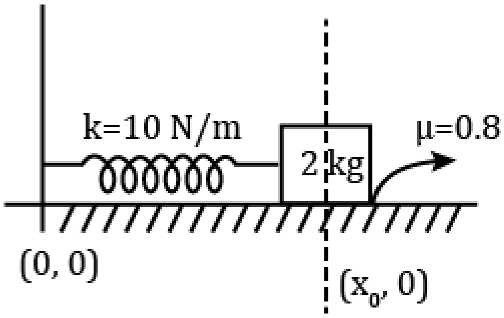


$\therefore\ N_1 = ma + N_2 \cos 60^\circ$
and, $N_2 \sin 60^\circ = mg$

$\Rightarrow N_2 = mg\frac{2}{\sqrt{3}}$

$\Rightarrow N_1 = mg\sqrt{3} + \frac{2mg}{\sqrt{3}}\left(\frac{1}{2}\right) = \frac{4mg}{\sqrt{3}}$

10. A block of mass $2\ \mathrm{kg}$ is attached to a spring of force constant $k = 10\ \mathrm{N/m}$ as shown in the figure. Find the range in which the block can be kept without slipping when the block is pulled or pushed towards the spring (spring is elongated or compressed). Take $g = 10\ \mathrm{m/s^2}$.



A. $3.2\ \mathrm{m}$

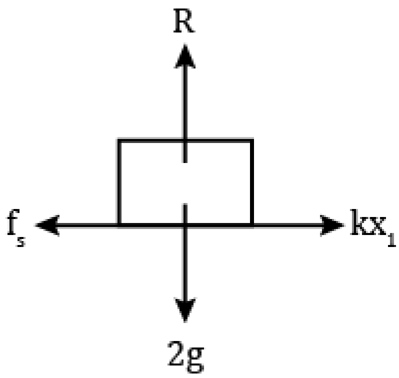
B. $1.6\ \mathrm{m}$

C. $2.4\ \mathrm{m}$

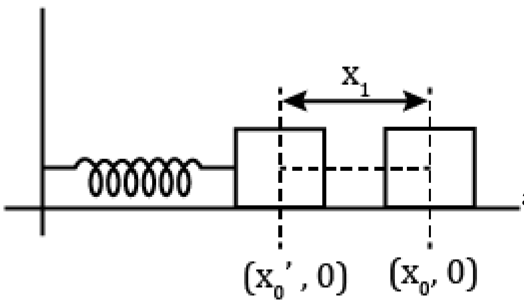
D. $4.8\ \mathrm{m}$

When the spring is compressed:

By say x_1, from FBD we get (here, R is the normal force)



Where x_1 is compression

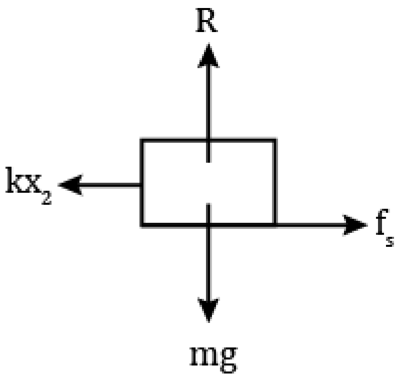


$x_1 = |x_0 - x_0'|$

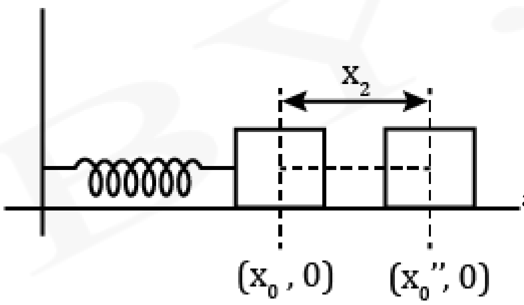
To keep the block without slipping i.e., keeping it in equilibrium using

$\sum F_x = 0$
$\Rightarrow \; f_s = kx_1$
and, $\sum F_y = 0$
$\Rightarrow \; R = 2g$
$\because f_s = \mu_s R = 0.8 \times 2 \times 10 = 16 \text{ N}$
$x_1 = \dfrac{f_s}{k} = \dfrac{16}{10} = 1.6 \text{ m}$

When the spring is elongated:
By say x_2, from FBD we get



Where x_2 is elongation

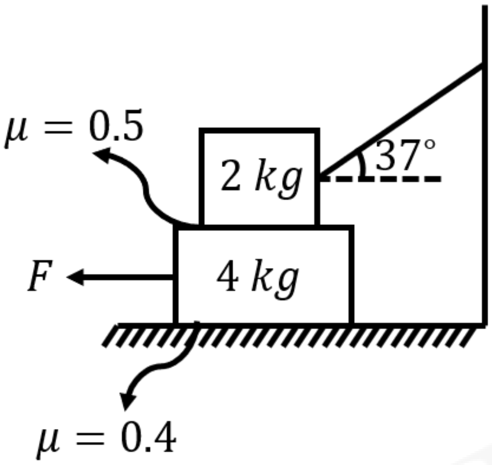


$x_2 = |x_0'' - x_0|$

At equilibrium,
$\sum F_x = 0$
$\Rightarrow \; kx_2 = f_s$
and, $\sum F_y = 0$
$\Rightarrow \; R = mg$
$\because f_s = \mu_s R = 0.8 \times 2g = 16 \; N$
$x_2 = \dfrac{f_s}{k} = \dfrac{16}{10} = 1.6 \text{ m}$
$\therefore$
Range in which block can be kept without slipping is
$x_1 + x_2 = 1.6 + 1.6 = 3.2 \text{ m}$

JEE Main Part Test 1

11. In the figure shown below, a horizontal force F is applied on $4\ \mathrm{kg}$ block towards left. If the coefficient of friction between the surfaces are 0.5 and 0.4 as shown in the figure. The value of Tension in the rope and force required just to slide the $4\ \mathrm{kg}$ block under $2\ \mathrm{kg}$ block is
(rope is massless and inextensible)



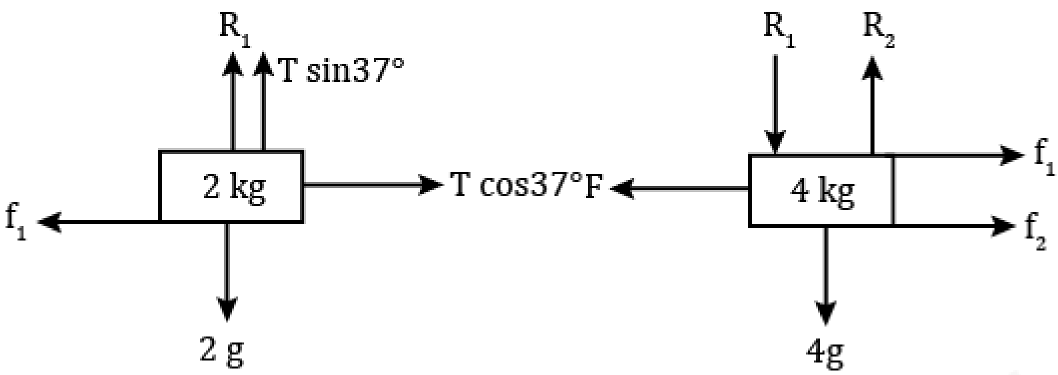
A. $T = 90.9\ \mathrm{N}$ and $F = 29\ \mathrm{N}$

B. $T = 9.09\ \mathrm{N}$ and $F = 14.5\ \mathrm{N}$

C. $T = 9.09\ \mathrm{N}$ and $F = 29\ \mathrm{N}$

D. $T = 90.9\ \mathrm{N}$ and $F = 14.5\ \mathrm{N}$

The FBDs of the blocks are as shown below
R_1 and R_2 are the normal forces acting on the respective blocks.



The condition is just to slide, it means in the question it is talking about limiting condition.
So, from FBD of $2\ \mathrm{kg}$ we have,
$R_1 + T\sin 37^\circ = 2g = 20$
$R_1 = 20 - \frac{3T}{5}$ ... (1)
$f_1 = T\cos 37^\circ$
$\Rightarrow\ f_1 = \frac{4T}{5}$ ... (2)
$\because\ f_1 = \mu R_1$
$\Rightarrow\ f_1 = 0.5\left(20 - \frac{3T}{5}\right) = 10 - \frac{3T}{10}$ ... (3)

From (3) and (2) we get
$\frac{4T}{5} = 10 - \frac{3T}{10}$
$\frac{4T}{5} + \frac{3T}{10} = 10$
$\frac{11T}{10} = 10 \Rightarrow T = \frac{100}{11}\mathrm{N}$ ... (4)

From (4) in (1) we get
$R_1 = 20 - \frac{3\times 100}{5\times 11}$
$= 20 - 5.454$
$R_1 = 14.546\ \mathrm{N}$

Now, from the FBD of $4\ \mathrm{kg}$ we get,
At equilibrium,
$R_2 = R_1 + 4g = 14.546 + 40 = 54.546\ \mathrm{N}$
$F = f_1 + f_2$
$\because f_2 = \mu_2 R_2 = 0.4\times 54.546 = 21.8\ \mathrm{N}$
and from equation (3)
$f_1 = 10 - \frac{3\times 100}{10\times 11} = 7.27$
$\Rightarrow F = 21.8 + 7.3 = 29.1\ \mathrm{N}$

12. A block of mass m is placed on a surface with a vertical cross section given by $y = \frac{x^3}{6}$. If the coefficient of friction is 0.5, the maximum height above the ground at which the block can be placed without slipping is

A. $\frac{1}{6}m$

B. $\frac{2}{3}m$

C. $\frac{1}{3}m$

D. $\frac{1}{2}m$

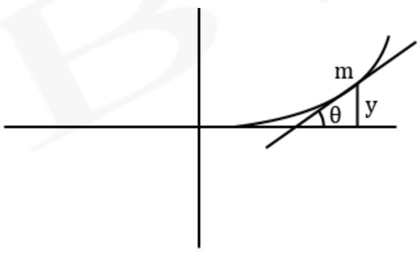
At equilibrium for limiting value of friction, $\mu = \tan\theta$

From the question,

$y = \frac{x^3}{6}$

$\Rightarrow \tan\theta = \frac{dy}{dx} = \frac{x^2}{2}$

$\therefore \mu = \frac{x^2}{2}$



Given $\mu = 0.5$

$\Rightarrow 0.5 = \frac{x^2}{2}$

$\Rightarrow x = 1$

Therefore, maximum vertical height for no slipping $y = \frac{x^3}{6} = \frac{1}{6}m$

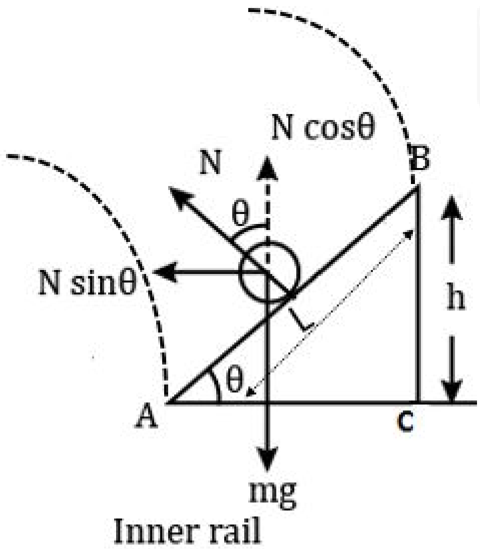
13. At certain place on railway track, the radius of curvature of railway track is $200 \mathrm{~m}$. If the distance between the rails is $1.6 \mathrm{~m}$, and the outer rail is raised by $0.08 \mathrm{~m}$ above the inner rail, find the speed of train for which there is no side pressure of the rails.
(Take $g=10 \mathrm{~m} / \mathrm{s}^{2}$)

A. $5 \mathrm{~m} / \mathrm{s}$

B. $10 \mathrm{~m} / \mathrm{s}$

C. $15 \mathrm{~m} / \mathrm{s}$

D. $20 \mathrm{~m} / \mathrm{s}$



$\Rightarrow$ For equilibrium in vertical direction,
$N \cos \theta=m g$...........(1)

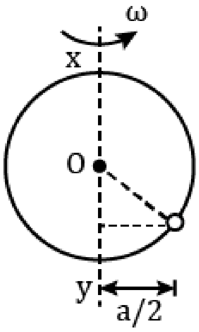
$\Rightarrow$ Dynamics equation towards centre of circular curve,
$N \sin \theta=\frac{m v^{2}}{r} \Rightarrow \tan \theta=\frac{v^{2}}{r g}$.........(2)

Also,
Distance between rails is $A B=1.6\, m$ and height between inner and outer rails, $h=0.08\, m$

$\therefore \sin \theta=\frac{h}{L}=\frac{0.08}{1.6}=\frac{1}{20}$

As $'\theta'$ is small; $\sin \theta \simeq \tan \theta$
Now putting value of $\tan \theta$ in equation (2):
$\Rightarrow \frac{1}{20}=\frac{v^{2}}{r g} \Rightarrow v=\sqrt{\frac{200 \times 10}{20}}=10 \mathrm{~m} / \mathrm{s}$

14. A small ring P is threaded on a smooth wire bent in the form of a circle of radius a and center O. The wire is rotating with constant angular speed ω about a vertical diameter xy, while the ring remains at rest relative to the wire at a distance $\frac{a}{2}$ from xy. Then ω^2 is equal to

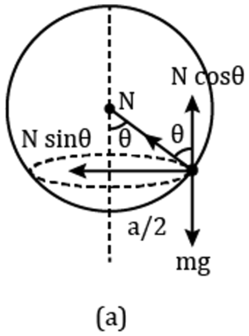


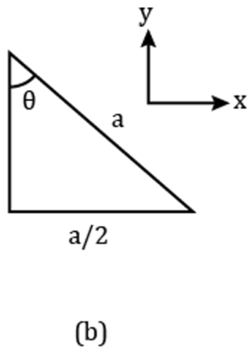
A. $\frac{2g}{a}$

B. $\frac{g}{2a}$

C. $\frac{2g}{a\sqrt{3}}$

D. $\frac{g\sqrt{3}}{2a}$





From FBD:

$\sum F_y = 0 \Rightarrow N\cos\theta = mg$... (i)

$\sum F_x = 0 \Rightarrow N\sin\theta = \dfrac{m\omega^2 a}{2}$... (ii)

Dividing Eq. (ii) $\div$ (i), $\tan\theta = \dfrac{\omega^2 a}{2g} \Rightarrow \omega^2 = \dfrac{2g\tan\theta}{a}$

Now, $\sin\theta = \dfrac{a/2}{a} = \dfrac{1}{2}$ or $\theta = 30^\circ$

$\Rightarrow \tan\theta = \dfrac{1}{\sqrt{3}}$

($\because \sin\theta = \dfrac{\text{opp}}{\text{hyp}}$ from the triangle)

i.e $\omega^2 = \dfrac{2g}{\sqrt{3}a}$

15.

A force F acting on a body depends on its displacement x as $F \propto x^{\frac{-1}{3}}$. The power delivered by F will depend on displacement as

A. $x^{\frac{2}{3}}$

B. $x^{\frac{5}{3}}$

C. $x^{\frac{1}{2}}$

D. x^{0}

JEE Main Part Test 1

Given that force F acting on a body depends on its displacement x as

$$F \propto x^{\frac{-1}{3}}$$

$$\Rightarrow F = kx^{\frac{-1}{3}} \ldots (i) \text{ (as } F = ma)$$

$$\Rightarrow a = \frac{k}{m} x^{\frac{-1}{3}}$$

We know that acceleration (a) as a function of (x) is

$$a = v\frac{dv}{dx} = \frac{k}{m} x^{\frac{-1}{3}}$$

$$\Rightarrow vdv = \frac{k}{m} x^{\frac{-1}{3}} dx$$

Integrating on both sides,

$$\int_{v_0}^{v} vdv = \frac{k}{m} \int_{x_0}^{x} x^{\frac{-1}{3}} dx$$

$$\frac{v^2 - v_0^2}{2} = \frac{3k}{2m}\left(x^{\frac{2}{3}}\right)$$

$$\Rightarrow v^2 \propto x^{\frac{2}{3}}$$

$$\Rightarrow v \propto x^{\frac{1}{3}}$$

$\Rightarrow v = cx^{\frac{1}{3}} \ldots (ii)$, here let c be the constant term.

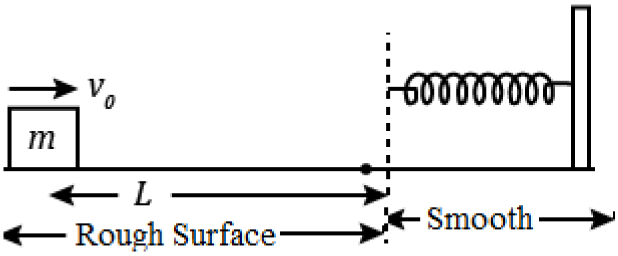
We also know that power (P) = $F.v$, by (i) and (ii)

$$P = kx^{\frac{-1}{3}} \times cx^{\frac{1}{3}}$$

$$\Rightarrow P \propto x^0$$

Hence option D is the correct answer.

16. A block of mass m starts moving with an initial velocity v_0 at a distance L towards a stationary spring of stiffness K attached to the wall as shown in the figure.



Find the distance travelled by block on smooth surface before coming to rest.
Given that surface is rough only for distance L and friction coefficient μ is such that $\left(\frac{1}{2}mv_0^2 > \mu mgL\right)$.

A. $v_0\sqrt{\frac{m}{K}}$

B. $\sqrt{\frac{\frac{1}{2}mv_0 - \mu mL}{K}}$

C. $\sqrt{\frac{\frac{1}{2}mv_0 - \mu mL}{2K}}$

D. $\sqrt{\frac{mv_0^2 - 2\mu mgL}{K}}$

Let block covers distance$(L + X_0)$ before coming to rest.
Work done by all forces $= \Delta KE$
$(W)_{\text{friction}}$ + $(W)_{\text{spring}} = \Delta KE$

$$-\mu mgL - \frac{1}{2}Kx_0^2 = -\frac{1}{2}mv_0^2$$

$$\mu mgL + \frac{1}{2}Kx_0^2 = \frac{1}{2}mv_0^2$$
$$\frac{1}{2}Kx_0^2 = \frac{1}{2}mv_0^2 - \mu mgL$$
$$x_0 = \sqrt{\frac{mv_0^2 - 2\mu mgL}{K}}$$

Hence option D is the correct answer

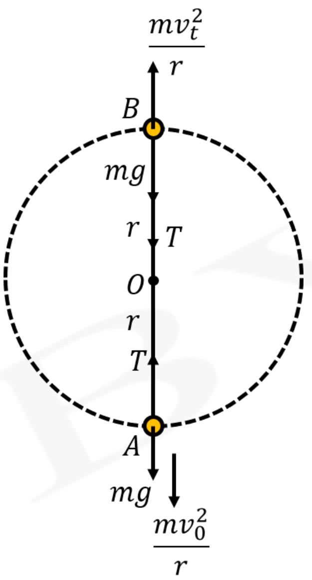
17. A stone of mass $1\ \mathrm{kg}$ tied to a light string of length $\frac{10}{3}\mathrm{m}$ is whirled in a vertical circle. If the ratio of the maximum tension to minimum tension is 4 and $g = 10\ \mathrm{m/s^2}$, then the speed of the stone at the highest point of the circle is

A. $0\ \mathrm{m/s}$

B. $20\ \mathrm{m/s}$

C. $10\ \mathrm{m/s}$

D. $5\ \mathrm{m/s}$



Let v_o & v_t be the velocities at the bottom and top of the circle.
Tension will be maximum at the bottom of the circle.

$\Rightarrow T_{max} = \frac{mv_o^2}{r} + mg$ ------(1)

Tension will be minimum at the top of the circle.

$\Rightarrow T_{min} = \frac{mv_t^2}{r} - mg$ -------(2)

$\frac{T_{max}}{T_{min}} = 4$ (given)

$\Rightarrow \frac{\frac{mv_o^2}{r} + mg}{\frac{mv_t^2}{r} - mg} = 4$ -----(3)

Applying conservation of energy at points A & B, taking A as a reference point

$KE_A + PE_A = PE_B + KE_B$

$\frac{1}{2}mv_o^2 + 0 = mg(2r) + \frac{1}{2}mv_t^2$

$\frac{v_o^2 - v_t^2}{2} = 2gr$ --------(4)

From equation (3)

$\frac{mv_o^2}{r} + mg = \frac{4mv_t^2}{r} - 4mg$

$\Rightarrow 5mg = \frac{4mv_t^2}{r} - \frac{mv_o^2}{r}$

$4v_t^2 - v_o^2 = 5gr$ ----------(5)

From equation (4)

$v_o^2 - v_t^2 = 4gr - - - - - - - - - (6)$

By solving Equation (5) & (6)

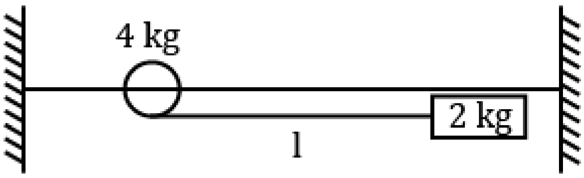
$3v_t^2 = 9gr$

$v_t^2 = 3gr$

$v_t = \sqrt{3gr}$

$v_t = \sqrt{3 \times 10 \times \frac{10}{3}} = 10 \text{ m/s}$

18. As shown in figure, a particle of mass $2\ \mathrm{kg}$ is attached to a bead. The bead can slide on a smooth straight wire. Length of the string which connect the particle and the bead is l. Initially, the particle is held in contact with the wire with the string taut as shown in figure, and then it is let to fall. If the bead has a mass $4\ \mathrm{kg}$, then, when the string makes an angle $\theta = 60^\circ$ with the wire, find the distance it slides upto this instant.



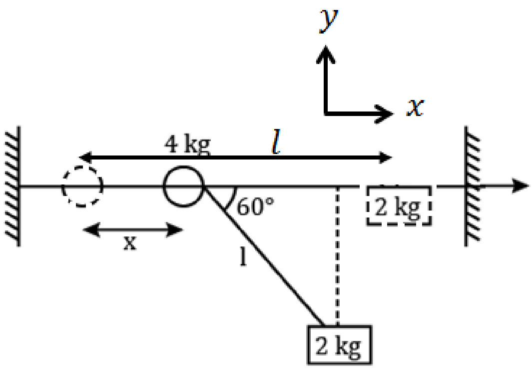
A. $\frac{l}{6}$ towards left

B. $\frac{l}{6}$ towards right

C. $\frac{l}{2}$ towards left

D. $\frac{l}{2}$ towards right

When string make an angle 60° with the wire:



Let the bead travel x distance as shown above.
Then, the distance covered by the $2\ \mathrm{kg}$ particle in x direction is
$= [-(l - l\cos\theta) + x]$
and the distance covered by the bead $= x$ (towards right)
As we know, there is no external force acting on the system. So, x_{com} will not change. It will be zero.

$$x_{com} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = 0$$

(Assuming motion towards right to be positive.)

$$x_{com} = \frac{4 \times (x) + 2(l\cos 60^{\circ} - l + x)}{4 + 2}$$

$\Rightarrow 4x - 2l + 2l\cos 60^{\circ} + 2x = 0$
$\Rightarrow 6x - l = 0$
$\Rightarrow x = \dfrac{l}{6}$
+ve sign means the bead will move towards right.

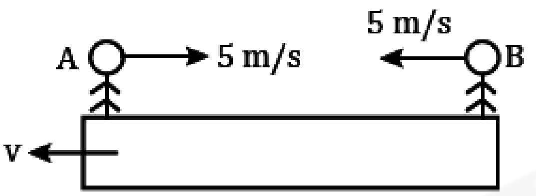
19. Two boys A and B of mass $60\ \mathrm{kg}$ and $40\ \mathrm{kg}$ are standing on a platform. Both start walking towards each other with the same velocity $5\ \mathrm{m/s}$. Find the velocity of the platform if mass of the platform is $100\ \mathrm{kg}$.

A. $0.5\ \mathrm{m/s}$

B. $2\ \mathrm{m/s}$

C. $1\ \mathrm{m/s}$

D. $5\ \mathrm{m/s}$



Given, mass of $A, m_1 = 60\ \mathrm{kg}$
mass of $B, m_2 = 40\ \mathrm{kg}$
and mass of platform, $m_3 = 100\ \mathrm{kg}$

Let velocity of platform w.r.t ground be v towards left, then velocity of boy A w.r.t ground is $(5 - v)$ towards right.
Similarly, velocity of boy B w.r.t ground will is $(5 + v)$ towards left.
i.e $v_1 = 5 - v, v_2 = -(5 + v)$ and $v_3 = -v$

Using momentum conservation, (treating rightwards direction as +ve)
$m_1 v_1 + m_2 v_2 + m_3 v_3 = 0$
$60(5 - v) - 40(5 + v) - 100v = 0$
$60 \times 5 - 60v - 40 \times 5 - 40v - 100v = 0$
$300 - 200 = 200v$
$\Rightarrow v = \dfrac{100}{200} = 0.5\ \mathrm{m/s}$ (towards left)

20. A solid cylinder is released from rest from the top of an inclined plane of inclination θ and length $'l'$. If the cylinder rolls without slipping, then find it's speed when it reaches the bottom of inclined plane.

A. $\sqrt{\dfrac{4gl\sin\theta}{3}}$

B. $\sqrt{\dfrac{3gl\sin\theta}{2}}$

C. $\sqrt{\dfrac{4gl}{3\sin\theta}}$

D. $\sqrt{\dfrac{4g\sin\theta}{3l}}$

In case of pure rolling, total mechanical energy remains conserved i.e $W_f = 0$,
$v_{\mathrm{CM}} = r\omega$ for pure rolling
Since solid cyliinder is starting from rest, $\omega_i = 0,\ v_i = 0$

$$KE_{Trans} = \frac{1}{2}mv_{\mathrm{CM}}^2$$

$$KE_{Rot} = \frac{1}{2}I_{CM}\omega^2$$

$\Rightarrow$ Total mechanical energy at initial position:

$$E_1 = PE + KE_{Trans} + KE_{Rot}$$

$$\therefore E_1 = mgh + 0 + 0 = mgh$$

Taking reference of $PE = 0$ at final position i.e at bottom of inclined plane
$\Rightarrow$Total mechanical energy at final position:

$$E_2 = PE + KE_{Trans} + KE_{Rot}$$

$$E_2 = 0 + \frac{1}{2}mv_{\mathrm{CM}}^2 + \frac{1}{2}I_{CM}\omega^2$$

$\because \dfrac{v_{\mathrm{CM}}}{r} = \omega$ and $I_{CM} = \dfrac{mr^2}{2}$

$$\Rightarrow E_2 = \frac{1}{2}mv_{\mathrm{CM}}^2 + \frac{1}{2}\left(\frac{mr^2}{2}\right)\left(\frac{v_{\mathrm{CM}}^2}{r^2}\right)$$

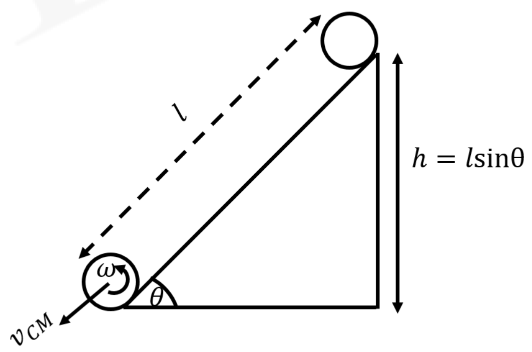
$$\therefore E_2 = \frac{3}{4}mv_{\mathrm{CM}}^2$$

From conservation of total mechanical energy,

$$\Rightarrow E_1 = E_2$$

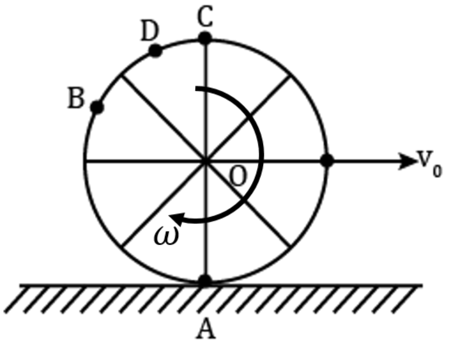
$$mgh = \frac{3}{4}mv_{\mathrm{CM}}^2$$

$$v_{\mathrm{CM}} = \sqrt{\frac{4}{3}gh}$$



$$\therefore v_{\mathrm{CM}} = \sqrt{\frac{4gl\sin\theta}{3}}$$

21. Consider a bicycle tyre rolling without slipping on a smooth horizontal surface with a linear velocity v_0 as shown in figure. Then, which of the following statement is incorrect?



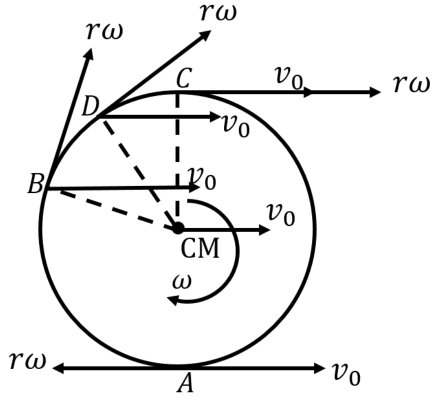
A. Speed of point A is zero

B. Speed of point B, C and D are equal to v_0

C. Speed of point $B >$ speed of point O

D. Speed of point $C = 2v_0$

Let the radius of bicycle wheel be r
For rolling without slipping $v_0 = r\omega$
$\Rightarrow$ Net velocity of any point on the bicycle wheel will be a vector sum of translational velocity and tangential velocity due to rotational motion, at that point.

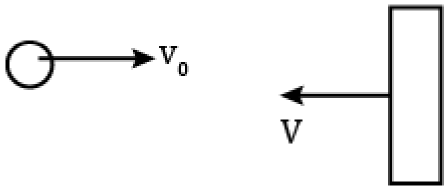


We can see that at point B and D, $\Rightarrow \vec{v}_B, \vec{v}_D$ will be greater in magnitude compared to $\vec{v}_0$, since taking resultant of velocities, $\theta < 90^\circ$, $\cos\theta$ is $+ve$ hence component vectors will add up together at points B and D to yield a higher velocity than $\vec{v}_0$

$\Rightarrow$At point C net speed is :
$|\vec{v}_C| = v_0 + r\omega = 2v_0$
$\Rightarrow$At point A net speed is :
$|\vec{v}_A| = v_0 - r\omega = v_0 - v_0 = 0 \because v_0 = r\omega$

$\therefore$ Option (b) is incorrect

22. A prticle of mass m moves with velocity $v_0 = 20\ \mathrm{m/s}$ towards a large wall that is moving with velocity $v = 5\ \mathrm{m/s}$ towards the particle as shown. If the particle collides with the wall elastically, then find the speed of the particle just after collision. (Assume collision with the wall is elastic)



A. $30\ \mathrm{m/s}$

B. $20\ \mathrm{m/s}$

C. $25\ \mathrm{m/s}$

D. $22\ \mathrm{m/s}$

As we know

$$e = \frac{v_{2f} - v_{1f}}{v_{1i} - v_{2i}}$$

(Take positive motion towards right)

Here, $v_{1i=}$ Speed of particle before collision $= v_0$

$v_{2i} =$ Speed of wall before collision $= -V$

$v_{2f} =$ Speed of wall after collision $= -V$

$v_{1f} =$ Speed of particle after collision $= -v'$

Assume after the collision, particle will move in leftward direction.

Hence, $1 = \dfrac{-v - (-v')}{v_0 - (-v)}$

$v_o + v = -v + v'$

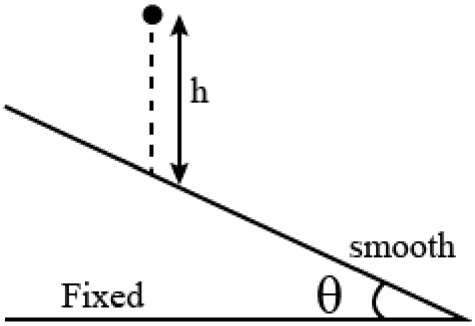
$v' = 2v + v_0$

$v' = 2 \times 5 + 20$

$v' = 30\ \mathrm{m/s}$

Therefore, speed of the particle after collision is $v' = 30\ \mathrm{m/s}$ towards left.

23. A ball of mass m strikes the fixed inclined plane after falling through a height h. If it rebounds elastically, the impulse imparted on the ball is



A. $2m\sqrt{2gh}\cos\theta$

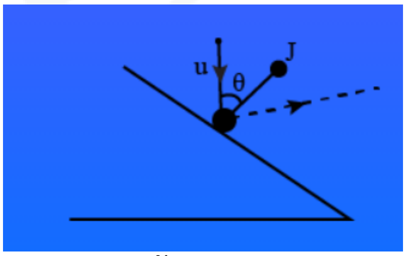
B. $2m\sqrt{gh}\cos\theta$

C. $2m\sqrt{2gh}\sin\theta$

D. $2m\sqrt{2gh}$

Speed of ball just before the collision $u = \sqrt{2gh}$
Collision is elastic hence, $e = 1$

$$e = \left(\frac{\text{Velocity of seperation}}{\text{Velocity of approach}}\right)_{\text{along line of impact}}$$



$\Rightarrow \ e = \dfrac{v}{u\cos\theta}$

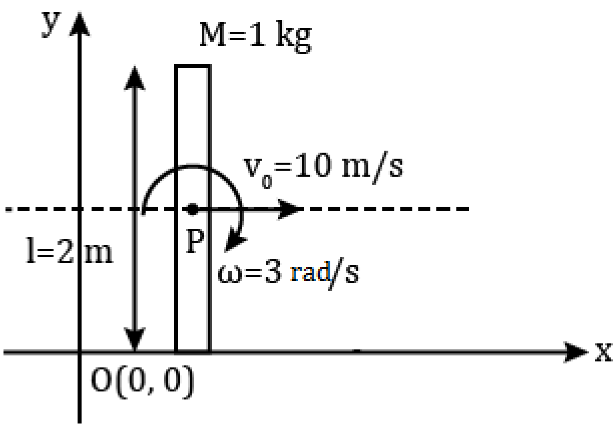
$\Rightarrow \ v = eu\cos\theta = u\cos\theta \quad \because e = 1$

Here, v is the velocity component of ball just after collision along line of impact.
Impulse $J = \Delta P = mv - (-m\ u\cos\ \theta)$

$\Rightarrow \ \Delta P = mu\cos\theta + mu\cos\theta$

$\Rightarrow \ J = \Delta P = 2m\cos\theta\sqrt{2gh}$

24. A rod of mass $1 \mathrm{~kg}$ and length $2 \mathrm{~m}$ is performing combined translational and rotational motion as shown in figure. Find the magnitude of total angular momentum about the origin.



A. $10 \mathrm{~kgm}^2/\mathrm{s}$

B. $1 \mathrm{~kgm}^2/\mathrm{s}$

C. $11 \mathrm{~kgm}^2/\mathrm{s}$

D. $20 \mathrm{~kgm}^2/\mathrm{s}$

For combined rotation and translation motion,

Total angular momentum $\vec{L} = \vec{L}_{\text{translational}} + \vec{L}_{\text{rotational}}$

$\vec{L} = M(\vec{r}_0 \times \vec{v}_0) + \vec{L}_{\text{COM}}$

Both angular momentum due to translation and rotation, have clockwise sense of rotation about origin. Hence both will add up

$\therefore L = M(rv_0 \sin\theta) + I_{\text{COM}}.\omega$

i.e $L = Mv_0 r_\perp + \dfrac{ML^2}{12}.\omega \ldots (i)$

Here $r_\perp$ is the perpendicular distance of velocity vector from origin.

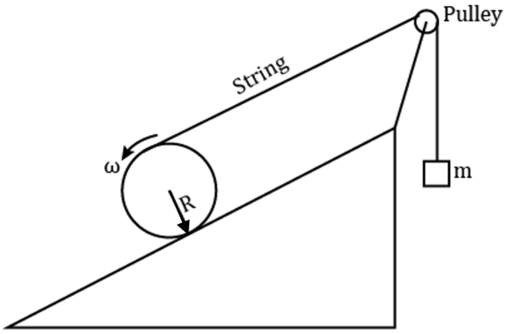
$r_\perp = \dfrac{l}{2} = \dfrac{2}{2} = 1 \mathrm{~m}$

Putting in Eq (i), the magnitude of total angular momentum is,

$L = (1 \times 10 \times 1) + \left(\dfrac{1 \times 2^2}{12}\right) \times 3$

$\therefore L = 11 \mathrm{~kgm}^2/\mathrm{s}$

25. A uniform disc of mass M and radius R is attached to a block of mass m by means of a light string and a light pulley fixed at the top of an inclined plane of inclination θ. The string is wrapped around the disc. The disc rolls down the incline. If $M = 6\ \mathrm{m}$ and $\theta = 30^{\circ}$, the acceleration of the centre of mass of the disc is: (Assume no slipping at any contact point)



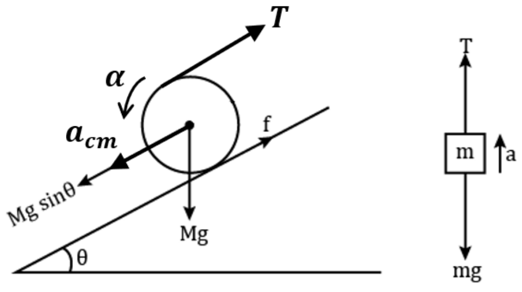
A. $\frac{g}{6}$

B. $\frac{g}{13}$

C. $\frac{g}{16}$

D. g

Figure (a) and (b) shows the free body diagrams of the disc and the block.



(a) (b)

Disc is rolling without sliding. Hence, $a_{cm} = \alpha R$
Here, $T =$ tension in the string
and $f =$ frictional force between the disc and the incline

From $\sum F_x = Ma$ (along the inclined plane),
For the disc: $Mg\sin\theta - T - f = Ma_{cm} \quad \ldots(1)$
For block: $T - mg = ma \quad \ldots(2)$

Acceleration of the point on the periphery of the disc where string is attached $= a_{cm} + \alpha R = 2a_{cm}$
[$\because$ for pure rolling $a_{cm} = \alpha R$]
From the string constraint, $a = 2a_{cm}$. Therefore
$T - mg = 2ma_{cm} \quad \ldots(3)$

Net torque on the disc is
$fR - TR = I\alpha$

$$\Rightarrow fR - TR = \frac{MR^2}{2} \times \frac{a_{cm}}{R}$$

$$\Rightarrow f = T + \frac{1}{2}Ma_{cm} \quad \ldots(4)$$

Putting (4) in (1) we get

$$Mg\sin\theta - 2T = \frac{3}{2}Ma_{cm} \quad .....(5)$$

Using (3) and (5) we get

$$Mg\sin\theta - 2(mg + 2ma_{cm}) = \frac{3}{2}Ma_{cm}$$

$$\Rightarrow a_{cm} = \left[\frac{(M\sin\theta - 2m)}{\left(\frac{3}{2}M + 4m\right)}\right] g \ldots(6)$$

Putting $M = 6m$ and $\theta = 30^\circ$ in (6),
we get $a_{cm} = \dfrac{g}{13}$
So the correct choice is (b).

26. At some instant $\vec{v} = 4\hat{i} - 3\hat{j}\ \mathrm{m/s}$ and $\vec{a} = 2\hat{i} + \hat{j}\ \mathrm{m/s^2}$. Find the radius of curvature at that instant.

A. $12.5\ \mathrm{m}$

B. $25\ \mathrm{m}$

C. $15\ \mathrm{m}$

D. $20\ \mathrm{m}$

Given, $\vec{v} = 4\hat{i} - 3\hat{j}$ and $\vec{a} = 2\hat{i} + \hat{j}$

We have, $|\vec{v}| = \sqrt{4^2 + (-3)^2} = 5\ \mathrm{m/s}$

and, $|\vec{a}| = \sqrt{2^2 + 1^2} = \sqrt{5}\ \mathrm{m/s^2}$

Also, we know that the component of acceleration parallel to the velocity is given as

$$a_{\|} = \frac{\vec{a} \cdot \vec{v}}{|\vec{v}|} = \frac{(2\hat{i} + \hat{j}) \cdot (4\hat{i} - 3\hat{j})}{5} = \frac{8 - 3}{5} = 1\ \mathrm{m/s^2}$$

Let us say that the component of acceleration perpendicular to the velocity be a_t

So, we have

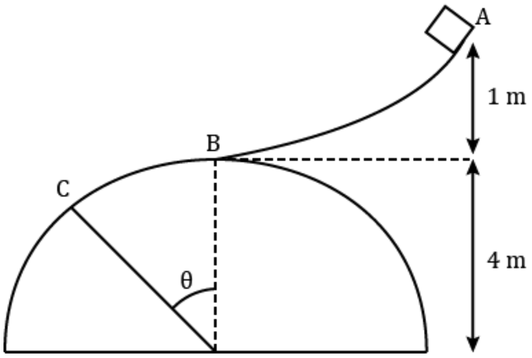
$$|\vec{a}|^2 = a_t^2 + a_{\|}^2$$

$$\Rightarrow\ (\sqrt{5})^2 = a_t^2 + 1^2 \Rightarrow\ a_t = 2\ \mathrm{m/s^2}$$

Thus, radius of curvature at that instant is given by

$$R_c = \frac{|\vec{v}|^2}{a_t} = \frac{5^2}{2} = 12.5\ \mathrm{m}$$

27. An object at rest at point A slides down on a smooth surface ending at point B at a fixed hemisphere as shown in the figure. Determine the angle θ at which the object will leave the hemisphere.

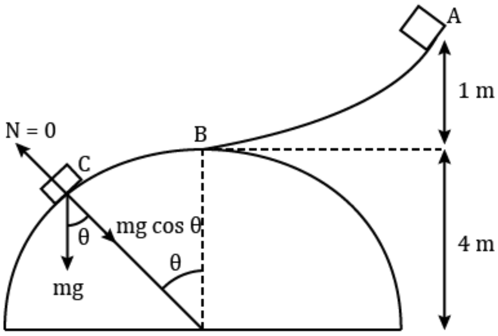


A. $\cos^{-1}\left(\frac{4}{7}\right)$

B. $\cos^{-1}\left(\frac{5}{6}\right)$

C. $\cos^{-1}\left(\frac{1}{2}\right)$

D. $\cos^{-1}\left(\frac{1}{3}\right)$



When the block leaves the surface then the normal force between the block and the surface becomes zero.

$N=0$

At point C,

$\Rightarrow \mathrm{mg}\ \cos\theta=\frac{mv^2}{R}+N$

$\Rightarrow \mathrm{mg}\ \cos\ \theta=\frac{mv^2}{R}$

[as $N=0$ at C]

$\Rightarrow v^2=Rg\cos\theta$ ....(1)

By using law of conservation of mechanical energy,

Loss in Potential energy $=$ Gain in Kinetic energy

$$mg(R+h)-mgR\cos\theta=\frac{mv^2}{2}-\frac{mv_0^2}{2}$$

$$\Rightarrow g(R+h)-gR\cos\theta=\frac{Rg\cos\theta}{2}-\frac{v_0^2}{2}$$

[from (1)]

$$\Rightarrow \frac{3gR\cos\theta}{2}=g(R+h)+\frac{v_0^2}{2}$$

$$\Rightarrow \cos\theta=\frac{2}{3gR}\left(g(R+h)+\frac{v_0^2}{2}\right)$$

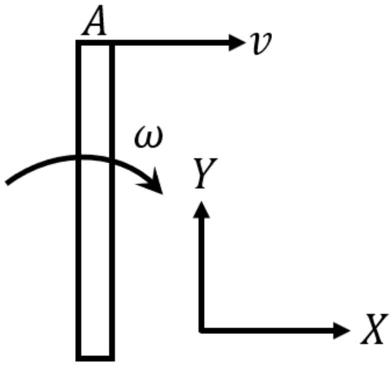
$$\Rightarrow \cos\theta=\frac{2}{3\times10\times4}\left(10\times(4+1)+\frac{0^2}{2}\right)$$

$$\Rightarrow \cos\theta=\frac{5}{6}$$

$$\Rightarrow \theta=\cos^{-1}\left(\frac{5}{6}\right)$$

Hence option (b) is correct.

28. Find the instantaneous axis of rotation of a rod of length l from the end A when it moves with a velocity $\overrightarrow{v_A} = v\hat{i}$ and the rod rotates with an angular velocity $\overrightarrow{\omega} = -\frac{v}{2l}\hat{k}$, shown in the figure.



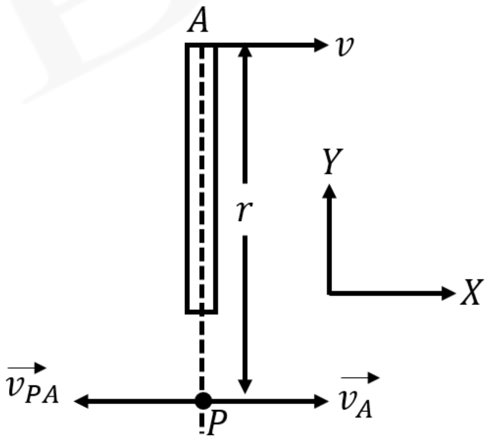
A. $\frac{l}{2}$

B. l

C. $2l$

D. $\sqrt{2}l$

Let us choose the point P as instantaneous center of rotation in the extended rod at a distance r from point A as shown below.



We can say ICR is point of zero velocity.

So, we can write

$\overrightarrow{v_P} = \overrightarrow{v_{PA}} + \overrightarrow{v_A}$

Here, $\overrightarrow{v_P} = 0, \overrightarrow{v_{PA}} = -\omega r\hat{i}, \overrightarrow{v_A} = v\hat{i}$

$\Rightarrow \quad -\omega r\hat{i} + v\hat{i} = 0$

$\Rightarrow \ r = \frac{v}{\omega} = \frac{v}{\frac{v}{2l}} = 2l$

Hence, ICR will be located at a distance $2l$ from A.

JEE Main Part Test 1

29. A physical quantity $'y'$ is represented by the formula $y = m^2 r^{-4} g^x l^{-3/2}$. If the relative errors found in y, m, r, l and g are $18, 1, 0.5, 4$ and p respectively, then which of the following combination satisfy value of x and p

A. 5 and 2

B. 4 and 3

C. $\frac{16}{3}$ and $\frac{3}{2}$ ✔

D. 8 and 2

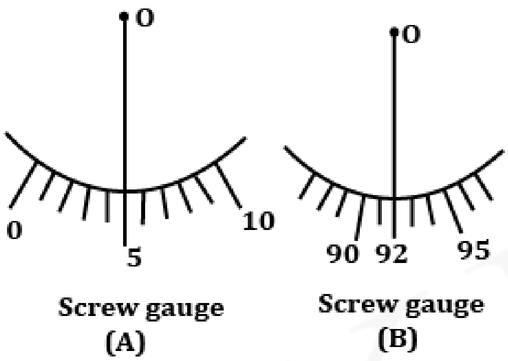
Given,
$y = m^2 r^{-4} g^x l^{-3/2}$

As we know that, relative errors can be written as
$$\frac{\Delta y}{y} = 2\frac{\triangle m}{m} + 4\frac{\triangle r}{r} + x\frac{\triangle g}{g} + \frac{3}{2}\frac{\triangle l}{l}$$

On multiplying each term by 100, each term will represent percentage error.
So, putting the given values in the above equation we get,
$$18 = 2(1) + 4(0.5) + xp + \frac{3}{2}(4)$$
$\Rightarrow \ 18 = 10 + xp$
$\Rightarrow \ 8 = xp$

By trial and error, only option (C) is valid for this scenario. Thus,
$$x = \frac{16}{3}, p = \frac{3}{2}$$

30. Student A and Student B used two screw gauges of equal pitch and 100 circular divisions each, to measure the radius of a given wire. The actual value of the radius of the wire is 0.322 cm. The absolute value of the difference between the final circular scale readings observed by the students A and B is.
[Figure shows position of reference $'O'$ when jaws of screw gauge are closed]
Given pitch = 0.1 cm.



A. 31

B. 13

C. 21

D. 15

The least count of each screw gauge is,

$$LC = \frac{\text{pitch}}{\text{total divisions on circular scale}}$$

$$\therefore\ LC = \frac{0.1}{100} = 0.001\text{ cm}$$

For A, the error is on the positive side, so,

$$\text{Actual value} = \text{Reading} - \text{Error}$$

$$\Rightarrow \text{Reading} = \text{Actual value} + \text{Error}$$

$$MSR + CSR = 0.322 + 5 \times 0.001$$
$$0.300 + CSR = 0.327$$
$$CSR = 0.027\text{ cm}$$

For B the error is on the negative side, so,

$$\text{Actual value} = \text{Reading} + \text{Error}$$

$$\Rightarrow \text{Reading} = \text{Actual value} - \text{Error}$$

$$0.300 + CSR = 0.322 - 0.008$$
$$CSR = 0.014\text{ cm}$$

Difference in $CSR = 0.027 - 0.014 = 0.013\text{ cm}$

Division on circular scale $= \dfrac{0.013}{0.001} = 13$