Subject: Mathematics

1. Let the random variable $X$ have a binomial distribution with mean $8$ and variance $4$. If $P(X \leq 2) = \frac{k}{2^{16}}$, then $k$ is equal to:

**A.** $1$

**B.** $17$

**C.** $121$

**D.** $137$

2. Minimum number of times a fair coin must be tossed so that the probability of getting at least one head is more than $99\%$ is :

**A.** $5$

**B.** $6$

**C.** $7$

**D.** $8$

3. The probability that two randomly selected subsets of the set $\{1, 2, 3, 4, 5\}$ have exactly two elements in their intersection, is

**A.** $\frac{65}{2^7}$

**B.** $\frac{135}{2^9}$

**C.** $\frac{65}{2^8}$

**D.** $\frac{35}{2^7}$

4. Let $A$ and $B$ be two non-null events such that $A \subset B$. Then, which of the following statements is always correct?

A. $P(A|B) = 1$

B. $P(A|B) \leq P(A)$

C. $P(A|B) \geq P(A)$

D. $P(A|B) = P(B) - P(A)$

5. Let $A$ and $B$ be two events such that $P(\overline{A \cup B}) = \frac{1}{6}$, $P(A \cap B) = \frac{1}{4}$ and $P(\overline{A}) = \frac{1}{4}$, where $\overline{A}$ stands for the complement of the event $A$. Then the events $A$ and $B$ are

A. mutually exclusive and independent

B. equally likely but not independent

C. independent but not equally likely

D. independent and eqaully likely

6. A person throws two fair dice. He wins Rs. $15$ for throwing a doublet (same numbers on the two dice), wins Rs. $12$ when the throw results in the sum of $9$, and loses Rs. $6$ for any other outcome on the throw. Then the expected gain/loss (in Rs.) of the person is :

A. $2$ gain

B. $\frac{1}{2}$ loss

C. $\frac{1}{2}$ gain

D. $\frac{1}{4}$ loss

7. Box I contains $30$ cards numbered $1$ to $30$ and Box II contains $20$ cards numbered $31$ to $50$. A box is selected at random and a card is drawn from it. The number on the card is found to be a non-prime number. The probability that the card was drawn from Box I is:

**A.** $\frac{4}{17}$

**B.** $\frac{8}{17}$

**C.** $\frac{2}{5}$

**D.** $\frac{2}{3}$

8. A seven-digit number is formed using digit $3, 3, 4, 4, 4, 5, 5$. The probability, that number so formed is divisible by $2$, is:

**A.** $\frac{6}{7}$

**B.** $\frac{4}{7}$

**C.** $\frac{3}{7}$

**D.** $\frac{1}{7}$

9. The coefficients $a, b$ and $c$ of the quadratic equation, $ax^2 + bx + c = 0$ are obtained by throwing a dice three times. The probability that this equation has equal roots is :

**A.** $\frac{1}{54}$

**B.** $\frac{1}{72}$

**C.** $\frac{1}{36}$

**D.** $\frac{5}{216}$

10. A bag contains $4$ red and $6$ black balls. A ball is drawn at random from the bag, its colour is observed and this ball along with two additional balls of the same colour are returned to the bag. If now a ball is drawn at random from the bag, then the probability that this drawn ball is red, is :

**A.** $\frac{3}{4}$

**B.** $\frac{3}{10}$

**C.** $\frac{2}{5}$

**D.** $\frac{1}{5}$

11. Let $\alpha,\ \beta$ be the roots of the equation $x^2+x+1=0$. Then for $y \neq 0$ in R,

$$\begin{vmatrix} y+1 & \alpha & \beta \\ \alpha & y+\beta & 1 \\ \beta & 1 & y+\alpha \end{vmatrix}$$

is equal to:

**A.** $y(y^2-3)$

**B.** $y^3-1$

**C.** $y(y^2-1)$

**D.** $y^3$

12. Let $k$ be an integer such that the triangle with vertices $(k,-3k),(5,k)$ and $(-k,2)$ has area $28$ sq. units. Then the orthocentre of this triangle is at the point:

**A.** $\left(2,-\frac{1}{2}\right)$

**B.** $\left(1,\frac{3}{4}\right)$

**C.** $\left(1,-\frac{3}{4}\right)$

**D.** $\left(2,\frac{1}{2}\right)$

13. If the system of equations $2x+3y-z=0,\ x+ky-2z=0$ and $2x-y+z=0$ has a non-trivial solution $(x,y,z)$, then $\frac{x}{y}+\frac{y}{z}+\frac{z}{x}+k$ is equal to :

**A.** $\frac{3}{4}$

**B.** $-4$

**C.** $-\frac{1}{4}$

**D.** $\frac{1}{2}$

14. If $P=\begin{bmatrix}1 & \alpha & 3\\ 1 & 3 & 3\\ 2 & 4 & 4\end{bmatrix}$ is the adjoint of a $3\times 3$ matrix $A$ and $|A|=4$, then $\alpha$ is equal to :

**A.** $4$

**B.** $11$

**C.** $5$

**D.** $0$

15. Let $A$ and $B$ be two invertible matrices of order $3\times 3$. If $\det(ABA^T)=8$ and $\det(AB^{-1})=8$, then $\det(BA^{-1}B^T)$ is equal to :

**A.** $1$

**B.** $\frac{1}{4}$

**C.** $16$

**D.** $\frac{1}{16}$

16. If $\begin{bmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{bmatrix}$ ; then for all

$\theta \in \left(\frac{3\pi}{4}, \frac{5\pi}{4}\right)$, $\det(A)$ lies in the interval :

**A.** $\left(0, \frac{3}{2}\right]$

**B.** $\left(1, \frac{5}{2}\right]$

**C.** $\left(\frac{3}{2}, 3\right]$

**D.** $\left[\frac{5}{2}, 4\right)$

17. If $\alpha, \beta \neq 0$, and $f(n) = \alpha^n + \beta^n$ and

$\begin{vmatrix} 3 & 1+f(1) & 1+f(2) \\ 1+f(1) & 1+f(2) & 1+f(3) \\ 1+f(2) & 1+f(3) & 1+f(4) \end{vmatrix} = K(1-\alpha)^2(1-\beta)^2(\alpha-\beta)^2$, then $K$ is equal to:

**A.** $\alpha\beta$

**B.** $\frac{1}{\alpha\beta}$

**C.** $1$

**D.** $-1$

18. If the system of linear equations

$x + y + z = 5$

$x + 2y + 3z = 9$

$x + 3y + \alpha z = \beta$

has infinitely many solutions, then $\beta - \alpha$ equals:

**A.** $5$

**B.** $18$

**C.** $8$

**D.** $21$

19. Let the numbers $2, b, c$ be in an A.P. and $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & b & c \\ 4 & b^2 & c^2 \end{bmatrix}$. If $\det(A) \in [2, 16]$, then $c$ lies in the interval :

**A.** $[3, 2 + 2^{3/4}]$

**B.** $[4, 6]$

**C.** $[2, 3]$

**D.** $[2 + 2^{3/4}, 4]$

20. Let $A = \begin{bmatrix} 2 & b & 1 \\ b & b^2+1 & b \\ 1 & b & 2 \end{bmatrix}$ where $b > 0$. Then the minimum value of $\dfrac{\det(A)}{b}$ is :

**A.** $-2\sqrt{3}$

**B.** $-\sqrt{3}$

**C.** $\sqrt{3}$

**D.** $2\sqrt{3}$

21. Let $\theta \in \left(0, \dfrac{\pi}{2}\right)$. If the system of linear equations,

$(1 + \cos^2\theta)\, x + \sin^2\theta\, y + 4\sin 3\theta\, z = 0$
$\cos^2\theta\, x + (1 + \sin^2\theta)\, y + 4\sin 3\theta\, z = 0$
$\cos^2\theta\, x + \sin^2\theta\, y + (1 + 4\sin 3\theta)\, z = 0$

has a non-trivial solution, then the value of $\theta$ is

**A.** $\dfrac{7\pi}{18}$

**B.** $\dfrac{\pi}{18}$

**C.** $\dfrac{4\pi}{9}$

**D.** $\dfrac{5\pi}{18}$

22. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$. Then $A^{2025} - A^{2020}$ is equal to

**A.** $A^6 - A$

**B.** $A^5$

**C.** $A^5 - A$

**D.** $A^6$

23. Two fair dice are thrown. The numbers on them are taken as $\lambda$ and $\mu$, and a system of linear equations

$x + y + z = 5$
$x + 2y + 3z = \mu$
$x + 3y + \lambda z = 1$

is constructed. If $p$ is the probability that the system has a unique solution and $q$ is the probability that the system has no solution, then

**A.** $p = \frac{1}{6}$ and $q = \frac{1}{36}$

**B.** $p = \frac{5}{6}$ and $q = \frac{5}{36}$

**C.** $p = \frac{1}{6}$ and $q = \frac{5}{36}$

**D.** $p = \frac{5}{6}$ and $q = \frac{1}{36}$

24. Let $[\lambda]$ be the greatest integer less than or equal to $\lambda$. The set of all values of $\lambda$ for which the system of linear equations

$x + y + z = 4,\ 3x + 2y + 5z = 3,\ 9x + 4y + (28 + [\lambda])z = [\lambda]$ has a solution is

**A.** $(-\infty, -9) \cup [-8, \infty)$

**B.** $[-9, -8)$

**C.** $\mathbb{R}$

**D.** $(-\infty, -9) \cup (-9, \infty)$

25. If the system of linear equations
$2x + 2ay + az = 0$
$2x + 3by + bz = 0$
$2x + 4cy + cz = 0,$

where $a, b, c \in R$ are non-zero and distinct; has non-zero solution, then

**A.** $a + b + c = 0$

**B.** $a, b, c$ are in A.P.

**C.** $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

**D.** $a, b, c$ are in G.P.

26. If $A = \begin{bmatrix} 0 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 0 \end{bmatrix}$ and $(I_2 + A)(I_2 - A)^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, then $13(a^2 + b^2)$ is equal to

27. If the system of equations
$kx + y + 2z = 1$
$3x - y - 2z = 2$
$-2x - 2y - 4z = 3$
has intinitely many solutions, then $k$ is equal to

28. Let $A = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$, $x \in \mathbb{R}$ and $A^4 = [a_{ij}]$. If $a_{11} = 109$, then $a_{22}$ is equal to

29. If the system of linear equations
$2x + y - z = 3$
$x - y - z = \alpha$
$3x + 3y + \beta z = 3$
has infinitely many solutions, then $(\alpha + \beta - \alpha\beta)$ is equal to

30. Let $A = \{a_{ij}\}$ be a $3 \times 3$ matrix, where $a_{ij} = \begin{cases} (-1)^{j-i} & \text{if } i < j, \\ 2 & \text{if } i = j, \\ (-1)^{i+j} & \text{if } i > j, \end{cases}$

then $\det(3\,Adj(2A^{-1}))$ is equal to