Mechanical properties of solids and fluid mechanics

1. A steel wire having a radius of $2.0\ mm$, carrying a load of $4\ kg$, is hanging from a ceiling. Given that $g = 3.1\pi\ ms^{-2}$, what will be the tensile stress that would be developed in the wire?

A. $4.8 \times 10^6\ Nm^{-2}$

B. $5.2 \times 10^6\ Nm^{-2}$

C. $6.2 \times 10^6\ Nm^{-2}$

D. $3.1 \times 10^6\ Nm^{-2}$ ✔

Tensile stress in wire will be

$$= \frac{\text{Tensile force}}{\text{Cross-sectional Area}}$$

$$= \frac{mg}{\pi R^2} = \frac{4 \times 3.1\pi}{\pi \times 4 \times 10^{-6}} Nm^{-2} = 3.1 \times 10^6\ Nm^{-2}.$$

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2. A man grows into a giant such that his linear dimensions increase by a factor of 9. Assuming that his density remains same, the stress in the leg will change by a factor of

A. $\frac{1}{81}$

B. 9

C. $\frac{1}{9}$

D. 81

$$Stress = \frac{Force}{Area}$$

Let linear dimensions be l, b, h.

$$S = \frac{Force}{l \times b} = \frac{\rho V a}{l \times b}$$

Now, dimensions increase by factor of g.

$$S' = \frac{Force}{9l \times 9b} = \frac{ma}{81lb}$$

$$S' = \frac{\rho V' a}{81 \times lb} = \frac{\rho(9^3 V)a}{81 \times lb}$$

$$\therefore \frac{S'}{S} = \frac{9^3 V}{81 \times lb} \times \frac{lb}{V}$$

$$S' = 9S$$

Hence, stress in leg will change by a factor of 9

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3. A pendulum made of a uniform wire of cross sectional area A has time period T. When an additional mass M is added to its bob, the time period changes to T_M. If the Young's modulus of the material of the wire is Y, then $1/Y$ is equal to

A. $\left[\left(\frac{T_M}{T}\right) - 1\right]\frac{Mg}{A}$

B. $\left[1 - \left(\frac{T_M}{T}\right)^2\right]\frac{A}{Mg}$

C. $\left[1 - \left(\frac{T}{T_M}\right)^2\right]\frac{A}{Mg}$

D. $\left[\left(\frac{T_M}{T}\right)^2 - 1\right]\frac{A}{Mg}$

Initial time period $T = 2\pi\sqrt{\frac{L}{g}} - - - (1)$

When the additional mass M is added to the bob,

$T_M = 2\pi\sqrt{\frac{l+\Delta l}{g}}$

Δl is the extension in the wire due to mass M.

$\Delta l = \frac{Mgl}{AY}$

$\Rightarrow = 2\pi\sqrt{\frac{l+\frac{Mgl}{AY}}{g}} - - - (2)$

$(2)^2 \div (1)^2$

$\left(\frac{T_M}{T}\right)^2 = 1 + \frac{Mg}{AY}$

$\Rightarrow \frac{1}{Y} = \frac{A}{Mg}\left[\left(\frac{T_M}{T}\right)^2 - 1\right]$

4. Speed of transverse wave on a straight wire (mass $6.0\ g$, length $60\ cm$ and area of cross-section $1.0\ mm^2$) is $90\ m/s$. If the Young's modulus of wire is $16 \times 10^{11}\ Nm^{-2}$, the extension of wire over its natural length is

A. $0.03\ mm$

B. $0.02\ mm$

C. $0.04\ mm$

D. $0.01\ mm$

Given, $M = 6\ grams = 6 \times 10^{-3}\ kg$

$$L = 60\ cm = 0.6\ m$$
$$A = 1\ mm^2 = 1 \times 10^{-6}\ m^2$$
$$\text{Using the relation, } v^2 = \frac{T}{\mu}$$
$$\Rightarrow T = \mu v^2 = v^2 \times \frac{M}{L}$$
$$\text{As Young's modulus, } Y = \frac{stress}{strain}$$
$$strain = \frac{stress}{Y} = \frac{T}{AY}$$
$$strain = \frac{\Delta L}{L} = \frac{v^2 \frac{M}{L}}{AY} = v^2 \frac{M}{AYL}$$
$$\Rightarrow \Delta L = \frac{v^2 M}{AY}$$
$$\Delta L = \frac{8100 \times 6 \times 10^{-3}}{1 \times 10^{-6} \times 16 \times 10^{11}}$$
$$\Delta L = 0.03\ mm$$

5. The normal density of a material is ρ and its bulk modulus of elasticity is K. The magnitude of increase in density of material, when a pressure P is applied uniformly on all sides, will be:

A. $\frac{\rho K}{P}$

B. $\frac{K}{\rho P}$

C. $\frac{PK}{\rho}$

D. $\frac{\rho P}{K}$

Bulk modulus $K = \frac{-P}{\frac{\Delta V}{V}}$

We know, $\rho = \frac{M}{V}$

So, $\frac{-\Delta\rho}{\rho} = \frac{\Delta V}{V}$

$$K = \frac{-P}{\left(-\frac{\Delta\rho}{\rho}\right)} = \frac{\rho P}{\Delta\rho}$$

$$\Delta\rho = \frac{\rho P}{K}$$

6. An object is located at $2\ \mathrm{km}$ beneath the surface of the water. If the fractional compression $\frac{\Delta V}{V}$ is 1.36%, the ratio of hydraulic stress to the corresponding hydraulic strain will be:
[Given : density of water is $1000\ \mathrm{kg\ m^{-3}}$ and $g = 9.8\ \mathrm{m\ s^{-2}}$]

A. $2.26 \times 10^9\ \mathrm{Nm^{-2}}$

B. $1.96 \times 10^7\ \mathrm{Nm^{-2}}$

C. $1.44 \times 10^7\ \mathrm{Nm^{-2}}$

D. $1.44 \times 10^9\ \mathrm{Nm^{-2}}$

Bulk modulus,

$$B = \frac{\Delta p}{\frac{\Delta V}{V}}$$

$$B = \frac{\rho g h}{\frac{\Delta V}{V}} = \frac{1000 \times 9.8 \times 2 \times 10^3}{\frac{1.36}{100}}$$

$$B = 1.44 \times 10^9\ \mathrm{Nm^{-2}}$$

7. A raindrop with radius $R = 0.2\ \mathrm{mm}$ falls from a cloud at a height $h = 2000\ \mathrm{m}$ above the ground. Assume that the drop is spherical throughout its fall and the force of buoyance may be neglected, then the terminal speed attained by the raindrop is :
[Density of water $\rho_w = 1000\ \mathrm{kg\ m^{-3}}$ and density of air
$\rho_a = 1.2\ \mathrm{kg\ m^{-3}}, g = 10\ \mathrm{m/s^2}$
Coefficient of viscosity of air$= 1.8 \times 10^{-5}\ \mathrm{Nsm^{-2}}$

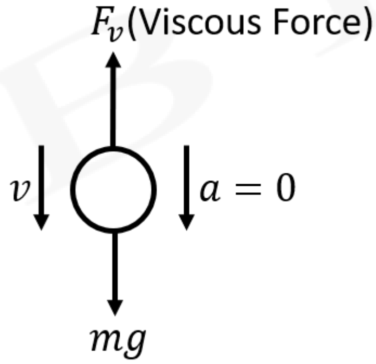
A. $250.6\ \mathrm{ms^{-1}}$

B. $43.56\ \mathrm{ms^{-1}}$

C. $4.94\ \mathrm{ms^{-1}}$

D. $14.4\ \mathrm{ms^{-1}}$

When the drop will attain terminal speed (v), the velocity will be constant, thus $a = 0$
$\Rightarrow\ F_{net} = 0$



From the FBD, we have
$mg = F_v = 6\pi\eta R v$

$$\Rightarrow\ v = \frac{mg}{6\pi\eta R v}$$

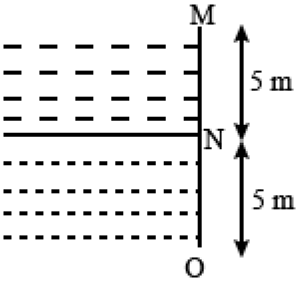
$$\Rightarrow\ v = \frac{\rho_w \frac{4\pi}{3} R^3 g}{6\pi\eta R}$$

$$= \frac{2\rho_w R^2 g}{9\eta}$$

$$= \frac{2 \times 1000 \times (0.2 \times 10^{-3})^2 \times 10}{9 \times 1.8 \times 10^{-5}}$$

$$= \frac{400}{81}\ \mathrm{m/s}$$

$$= 4.94\ \mathrm{m/s}$$

8. 

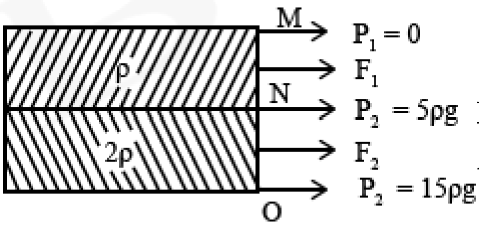
Two liquids of densities ρ_1 and ρ_2 $(\rho_2 = 2\rho_1)$ are filled up behind a square wall of side $10\ \mathrm{m}$ as shown in figure. Each liquid has a height of $5\ \mathrm{m}$. The ratio of the forces due to these liquids exerted on upper part MN to that at the lower part NO is (Assume that the liquids are not mixing)

A. $1/3$

B. $2/3$

C. $1/2$

D. $1/4$



Let $P_1,\ P_2$ and P_3 be the pressure at points $M,\ N$ and O respectively.

Pressure is given by $\mathrm{P} = \rho g h$

Now, $P_1 = 0(\because\ h = 0)$

$P_2 = \rho g(5)$

$P_3 = \rho g(5) + 2\rho g(5) = 15\ \rho g$

Force on upper part, $F_1 = \dfrac{(P_1 + P_2)}{2} A$

Force on lower part, $F_2 = \dfrac{(P_2 + P_3)}{2} A$

$\therefore\ \dfrac{F_1}{F_2} = \dfrac{5\rho g}{20\rho g} = \dfrac{5}{20} = \dfrac{1}{4}$

Mechanical properties of solids and fluid mechanics

9. An ideal fluid flows (laminar flow) through a pipe of non-uniform diameter. The maximum and minimum diameters of the pipes are 6.4 cm and 4.8 cm, respectively. The ratio of the minimum and maximum velocities of fluid in this pipe is:

A. $\frac{9}{16}$ ✔

B. $\frac{\sqrt{3}}{2}$

C. $\frac{3}{4}$

D. $\frac{81}{256}$

From the equation of the continuity,

$A_1 v_1 = A_2 v_2$

Here, v_1 and v_2 are the velocities at the areas of cross-section A_1 and A_2 of the pipe.

As, $v \propto \frac{1}{A}$

v will be minimum at A_{max} and vice versa.

$\therefore v_{min} A_{max} = v_{max} A_{min}$

$$\Rightarrow \frac{v_{min}}{v_{max}} = \frac{A_{min}}{A_{max}}$$

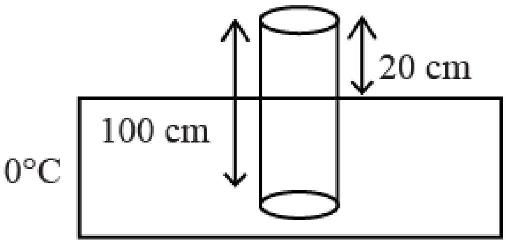
$$\Rightarrow \frac{v_{min}}{v_{max}} = \frac{\pi\left(\frac{4.8}{2}\right)^2}{\pi\left(\frac{6.4}{2}\right)^2} = \frac{9}{16}$$

Hence, option (A) is correct.

Mechanical properties of solids and fluid mechanics

10. A leakproof cylinder of length $1\ \mathrm{m}$, made of a metal which has very low coefficient of expansion, is floating vertically in water at $0^{\circ}\mathrm{C}$ such that its height above the water surface is $20\ \mathrm{cm}$. When the temperature of water is increased to $4^{\circ}\mathrm{C}$, the height of the cylinder above the water surface becomes $21\ \mathrm{cm}$. The density of water at $T=4^{\circ}C$, relative to the density at $T=0^{\circ}C$ is close to:

   - ✗ **A.** 1.26
   - ✗ **B.** 1.04
   - ✓ **C.** 1.01
   - ✗ **D.** 1.03



Here,
$h_1 = 20 \text{ cm} \ : \ \ h_2 = 100 \text{ cm}$

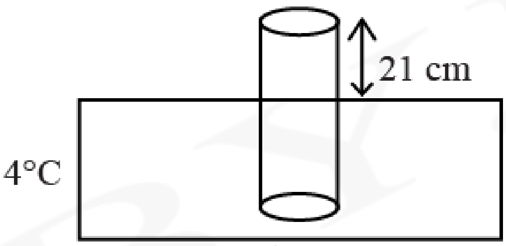
When cylinder is floating in water at $0^{\circ}C$:

The buoyant force is given by,

$$F_b = A(h_2 - h_1)\rho_0 g$$

$$\Rightarrow F_b = A(100 - 20)\rho_0 g$$

$$\Rightarrow F_b = A(80)\rho_0 g$$



When cylinder is floating in water at $4^{\circ}C$:

The buoyant force is given by,

$$F_b = A(h_2 - h_1)\rho_4 g$$

$$\Rightarrow F_b = A(100 - 21)\rho_4 g$$

$$\Rightarrow F_b = A(79)\rho_4 g$$

As the buoyant force has to counter the weight (mg) in both the cases, the buoyant force is equal in both the cases.

$$\Rightarrow A(79)\rho_4 g = A(80)\rho_0 g$$

$$\therefore \frac{\rho_4}{\rho_0} = \frac{80}{79} = 1.01$$

Hence, option (C) is correct.

11. A hollow spherical shell at outer radius R floats just submerged under the water surface. The inner radius of the shell is r. If the specific gravity of the shell material is $\frac{27}{8}$ w.r.t water, the value of r is

A. $\frac{8}{9}R$

B. $\frac{4}{9}R$

C. $\frac{2}{3}R$

D. $\frac{1}{3}R$

In equilibrium,
$mg = F_B$
Here
$F_B = V\rho_0 g = \frac{4}{3}\pi R^3 \rho_w g$ and $m = \rho_0 g V_{net} = \frac{4}{3}\pi(R^3 - r^3)\rho_0 g$

$$\frac{4}{3}\pi(R^3 - r^3)\rho_0 g = \frac{4}{3}\pi R^3 \rho_w g$$

Given, relative density, $\frac{\rho_0}{\rho_w} = \frac{27}{8}$

$$\Rightarrow \left[1 - \left(\frac{r}{R}\right)^3\right] \frac{27}{8}\rho_w = \rho_w$$

$$\Rightarrow 1 - \frac{r^3}{R^3} = \frac{8}{27}$$

$$\Rightarrow \frac{r^3}{R^3} = 1 - \frac{8}{27} = \frac{19}{27}$$

$$\therefore r = 0.89R = \frac{8}{9}R$$

Mechanical properties of solids and fluid mechanics

12. In an experiment to verify Stokes law, a small spherical ball of radius r and density ρ falls under gravity through a distance h in air before entering a tank of water. If the terminal velocity of the ball inside water is same as its velocity just before entering the water surface, then the value of h is proportional to
(ignore viscosity of air)

- A. r^4 ✔
- B. r ✘
- C. r^3 ✘
- D. r^2 ✘

Using,
$v^2 - u^2 = 2gh$

$\Rightarrow v^2 - 0^2 = 2gh$
$\Rightarrow v = \sqrt{2gh}$

Termianl velocity,

$$V_T = \frac{2\, r^2(\rho-\sigma)g}{9\qquad \eta}$$

After falling through h the velocity should be equal to terminal velocity

$$\therefore \sqrt{2gh} = \frac{2\, r^2(\rho-\sigma)g}{9\qquad \eta}$$
$$\Rightarrow 2gh = \frac{4\ \ r^4g^2(\rho-\sigma)^2}{81\qquad \eta^2}$$
$$\Rightarrow h = \frac{2r^4g(\rho-\sigma)^2}{81\eta^2}$$
$$\Rightarrow h\ \propto\ r^4$$

Mechanical properties of solids and fluid mechanics

13. An Object of mass m is suspended at the end of a massless wire of length L and area of cross-section A, Young modulus of the material of the wire is Y. If the mass is pulled down Slightly, its frequency of oscillation along the vertical direction is:

A. $f=\frac{1}{2\pi}\sqrt{\frac{mL}{YA}}$

B. $f=\frac{1}{2\pi}\sqrt{\frac{YA}{mL}}$

C. $f=\frac{1}{2\pi}\sqrt{\frac{mA}{YL}}$

D. $f=\frac{1}{2\pi}\sqrt{\frac{YL}{mA}}$

An elastic wire can be treated as a spring and its spring constant is given as

$$k=\frac{YA}{L} \quad \left[\because Y=\frac{\left(\frac{F}{A}\right)}{\left(\frac{\Delta l}{l_0}\right)}\right]$$

Thus, frequency of oscillation will be

$$f=\frac{1}{2\pi}\sqrt{\frac{k}{m}}=\frac{1}{2\pi}\sqrt{\frac{YA}{mL}}$$

Hence, (B) is the right option.

14. An air bubble of radius 1 cm in water has an upward acceleration 9.8 cm/s^2. The density of water is 1 g/cm^3 and water offers negligible drag force on the bubble. The mass of the bubble is $(g = 980 \text{ cm/s}^2)$

A. 4.51 gm

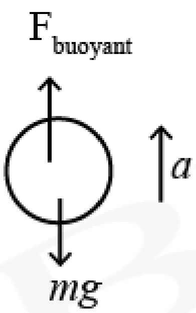
B. 3.15 gm

C. 4.15 gm

D. 1.52 gm

Given, $r = 1 \text{ cm} \;\; ; \;\; a = 9.8 \text{ cm s}^{-2}$

$\rho_w = 1 \text{ g cm}^{-3} \;\; ; \;\; g = 980 \text{ cm s}^{-2}$



Volume, $V = \dfrac{4\pi r^3}{3} = \dfrac{4\pi}{3} \times (1)^3 = 4.19 \text{ cm}^3$

$F_{buoyant} - mg = ma$

$$\Rightarrow m = \frac{F_{buoyant}}{g + a}$$

$$\therefore \;\; m = \frac{V\rho_\omega g}{g + a} = \frac{V\rho_\omega}{1 + \dfrac{a}{g}}$$

$$= \frac{(4.19) \times 1}{1 + \dfrac{9.8}{980}} = \frac{4.19}{1.01} \approx 4.15 \text{ g}$$

Hence, (C) is the correct answer.

Mechanical properties of solids and fluid mechanics

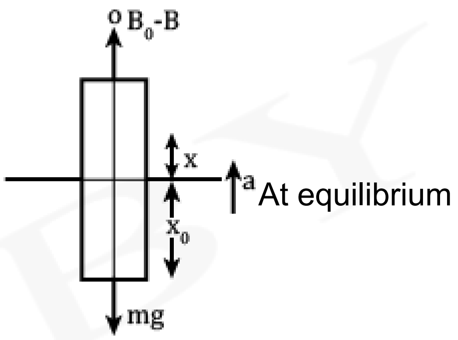
15. A cylindrical plastic bottle of negligible mass is filled with 310 ml of water and left floating in a pond with still water. If pressed downward slightly and released, it starts performing simple harmonic motion at angular frequency ω. If the radius of the bottle is 2.5 cm then ω is close to: (density of water 10^3 kg/m^3)

A. 3.75 rad s^{-1}

B. 1.25 rad s^{-1}

C. 2.50 rad s^{-1}

D. $2.50\sqrt{10} \text{ rad s}^{-1}$



$B_0 = mg$
Extra boyant force $= \rho A x g$
From NLM,
$mg - B_0 - B = ma$
$\therefore B = ma = -\rho A x g = -(\pi r^2 \rho g)x$
$$a = -\frac{(\pi r^2 \rho g)x}{m}$$

Using,
$a = -\omega^2 x$
$$\Rightarrow \omega = \sqrt{\frac{\pi r^2 \rho g}{m}}$$
$\Rightarrow \omega \approx 7.9 \text{ rad s}^{-1}$.

16. Two identical cylindrical vessels are kept on the ground and each contain the same liquid of density d. The area of the base of both vessels is S but the height of liquid in one vessel is x_1 and in the other, x_2. When both cylinders are connected through a pipe of negligible volume, very close to the bottom, the liquid flows from one vessel to the other until it comes to equilibrium at a new height. The change in energy of the system in the process is:

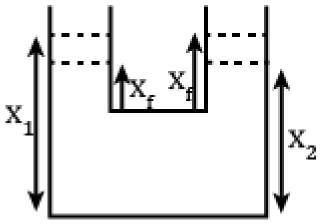
A. $gdS(x_2^2 + x_1^2)$

B. $gdS(x_2 + x_1)^2$

C. $\frac{3}{4}gdS(x_2 - x_1)^2$

D. $\frac{1}{4}gdS(x_2 - x_1)^2$

Mechanical properties of solids and fluid mechanics



Initial potential energy,

$$U_i = (\rho S x_1) g. \frac{x_1}{2} + (\rho S x_2) g. \frac{x_2}{2}$$

$$= \frac{\rho S g x_1^2}{2} + \frac{\rho S g x_2^2}{2}$$

Final potential energy,

$$U_f = (\rho S x_f) g. \frac{x_f}{2} \times 2 = \frac{\rho S g x_f^2}{2}$$

By volume conservation,
$S x_1 + S x_2 = S(2x_f)$

$$x_f = \frac{x_1 + x_2}{2}$$

When the valve is opened, change in potential energy will occur till the water level become same, on both the sides.

Now, $\Delta U = U_i - U_f$

$$\Delta U = \rho S g \left[\left(\frac{x_1^2}{2} + \frac{x_2^2}{2} \right) - x_f^2 \right]$$

$$= \rho S g \left[\frac{x_1^2}{2} + \frac{x_2^2}{2} - \left(\frac{x_1 + x_2}{2} \right)^2 \right]$$

$$= \frac{\rho S g}{2} \left[\frac{x_1^2}{2} + \frac{x_2^2}{2} - x_1 x_2 \right]$$

$$= \frac{\rho S g}{4} (x_1 - x_2)^2$$

Hence, (D) is the correct answer.

Mechanical properties of solids and fluid mechanics

17. A cube of metal is subjected to a hydrostatic pressure of $4\ \mathrm{GPa}$. The percentage change in the length of the side of the cube is close to :(Given bulk modulus of metal, $B = 8 \times 10^{10}\ \mathrm{Pa}$)

- A. 5
- B. 0.6
- C. 20
- D. 1.67

Bulk modulus, $B = \dfrac{P}{\frac{dV}{V}}$

$$\Rightarrow \frac{dV}{V} = \frac{P}{B} \quad \ldots\ldots (i)$$

If the side of cube is L, then,
$V = L^3$
Differentiating both sides, we get, $dV = 3L^2\, dL$

$$\therefore \frac{dV}{V} = \frac{3dL}{L} = \frac{P}{B}$$

$$\Rightarrow \frac{dL}{L} = \frac{1}{3} \times \frac{P}{B}$$

$$= \frac{4 \times 10^9}{3 \times 8 \times 10^{10}} = \frac{1}{60}$$

$$\Rightarrow \frac{dL}{L} \times 100 = \frac{1}{60} \times 100 = 1.67\%.$$

Hence, (D) is the correct answer.

Mechanical properties of solids and fluid mechanics

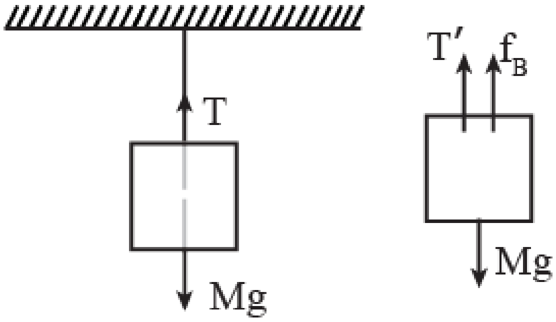
18. A load of mass M is suspended from a steel wire of length 2 and radius $1.0 \mathrm{~mm}$ in Searle's apparatus experiment. The increase in length produced in the wire is $4.0 \mathrm{~mm}$. Now the load is fully immersed in a liquid of relative density 2. The relative density of the material of load is 8. The new value of increase in length of the steel wire is:

✔ **A.** $3.0 \mathrm{~mm}$

✖ **B.** $4.0 \mathrm{~mm}$

✖ **C.** $5.0 \mathrm{~mm}$

✖ **D.** Zero



Using $\frac{F}{A} = Y \cdot \frac{\Delta l}{l}$

$\Delta l \propto F - - - (1)$

When the load is suspended,
$T = Mg$

When the load is fully immersed in the liquid,
$T' = Mg - f_B = Mg - \frac{M}{\rho_o} \cdot \rho_l \cdot g$

$\Rightarrow T' = \left(1 - \frac{\rho_l}{\rho_o}\right) Mg$

$\Rightarrow T' = \left(1 - \frac{2}{8}\right) Mg$

$\Rightarrow T' = \frac{3}{4} Mg$

From equation (1)

$\frac{\Delta l'}{\Delta l} = \frac{T'}{T} = \frac{3}{4}$

$[\text{Given:} \Delta l = 4 \text{ mm}]$

$\therefore \Delta l' = \frac{3}{4} \cdot \Delta l = \frac{3}{4} \times 4 = 3 \text{ mm}$

Hence, option (A) is correct.

19. The top of a water tank is open to air and its water level is maintained. It is giving out $0.74 \mathrm{~m}^3$ water per minute through a circular opening of $2 \mathrm{~cm}$ radius in its wall. The depth of the centre of the opening from the level of water in the tank is close to

A. $6.0 \mathrm{~m}$

B. $4.8 \mathrm{~m}$

C. $9.6 \mathrm{~m}$

D. $2.9 \mathrm{~m}$

Volumetric flow rate through circular opening is given by-
$Q = Av$
[where $Q =$ water flow rate, $A =$ area of opening and $v =$ velocity through opening]

$$\Rightarrow \frac{0.74}{60} = \pi r^2 v = (\pi \times 4 \times 10^{-4}) \times \sqrt{2gh}$$

$$\Rightarrow \sqrt{2gh} = \frac{74 \times 100}{240\pi} \Rightarrow \sqrt{2gh} = \frac{740}{24\pi}$$

$$\Rightarrow 2gh = \frac{740 \times 740}{24 \times 24 \times 10} \; (\because \pi^2 = 10)$$

$$\Rightarrow h = \frac{74 \times 74}{2 \times 24 \times 24} \approx 4.8\, m$$

i.e., The depth of the centre of the opening from the level of water in the tank is close to $4.8 \mathrm{~m}$

20. A wooden block floating in a bucket of water has $\frac{4}{5}$ of its volume submerged. When a certain amount of an oil poured into the bucket, it is found that the block is just under the oil surface, with half of its volume underwater and half in oil. The density of oil relative to that of water is-

A. 0.5

B. 0.8

C. 0.6 ✔

D. 0.7

When no oil poured :

$$Mg = \left(\frac{4V}{5}\right)\rho_w g$$

$$\rho_b V g = \left(\frac{4V}{5}\right)\rho_w g$$

$$\rho_b = \frac{4\rho_w}{5}$$

After pouring oil into the water :

$$Mg = F_{b_1} + F_{b_2}$$

$$(\rho_b V)g = \left(\frac{V}{2}\right)\rho_{\text{ oil}} g + \left(\frac{V}{2}\right)\rho_w g$$

$$\Rightarrow \quad \rho_{\text{ oil}} = \frac{3}{5}\rho_{\text{water}}$$

$$\therefore \quad \frac{\rho_{\text{ oil}}}{\rho_{\text{water}}} = 0.6$$

Hence, (C) is the correct answer.

Mechanical properties of solids and fluid mechanics

21. In an environment, brass and steel wires of length $1 \mathrm{~m}$ each with areas of cross section $1 \mathrm{~mm}^2$ are used. The wires are connected in series and one end of the combined wire is connected to a rigid support and other end is subjected to elongation. The stress required to produce a net elongation of $0.2 \mathrm{~mm}$ is

[Given the Young's modulus for steel and brass are respectively $120 \times 10^9 \mathrm{~N/m}^2$and $60 \times 10^9 \mathrm{~N/m}^2$]

A. $1.2 \times 10^6 \mathrm{~N/m}^2$

B. $4.0 \times 10^6 \mathrm{~N/m}^2$

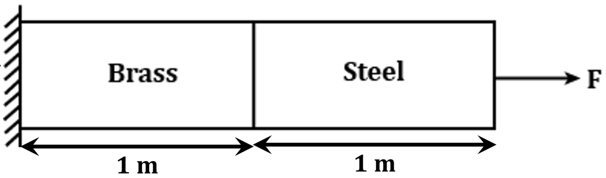
C. $1.8 \times 10^6 \mathrm{~N/m}^2$

D. $0.2 \times 10^6 \mathrm{~N/m}^2$

E. None of the above

Mechanical properties of solids and fluid mechanics

Given:
The Young's modulus for steel $Y_s = 120 \times 10^9\ \mathrm{N/m^2},\ l_s = 1\ \mathrm{m}, A_s = 1\ \mathrm{mm^2}$
The Young's modulus for brass $Y_b = 60 \times 10^9\ \mathrm{N/m^2},\ l_b = 1\ \mathrm{m}, A_b = 1\ \mathrm{mm^2}]$

$\Delta l_{net} = 0.2\ \mathrm{mm}$ 

Young's Modulus is given as

$$Y = \frac{\text{Stress}}{\left(\frac{\Delta l}{L}\right)}$$

Let σ be the stress

$$Y = \frac{\sigma}{\left(\frac{\Delta l}{l}\right)} \Rightarrow \Delta l = \frac{\sigma l}{Y}$$

As per the question
Total elongation $\Delta l_{\text{net}} = 0.2\ \mathrm{mm}$

$$\Rightarrow \frac{\sigma L_1}{Y_1} + \frac{\sigma L_2}{Y_2} = 0.2\ \mathrm{mm}$$

$$\sigma\left[\frac{1}{Y_1} + \frac{1}{Y_2}\right] = 0.2\ \mathrm{mm} \qquad [\because L_1 = L_2 = 1\ \mathrm{m}]$$

$$\sigma = 0.2\ \mathrm{mm}\left(\frac{Y_1 Y_2}{Y_1 + Y_2}\right) = 0.2 \times 10^{-3} \times \left(\frac{120 \times 60}{180}\right) \times 10^9$$

$$\sigma = 8 \times 10^6 \frac{\mathrm{N}}{\mathrm{m}^2}$$

22. A cubical block of side 0.5 m floats on water with 30% of its volume under water. What is the maximum weight that can be put on the block without fully submerging it under water? [Take, density of water $= 10^3 \text{ kg/m}^3$]

A. 46.3 kg

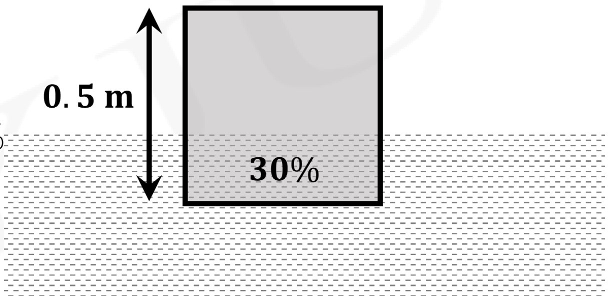
B. 87.5 kg

C. 65.4 kg

D. 30.1 kg

Given:
Side of the block $= 0.5 \text{ m}$

Submerged volume$= 30\%$



We know that according to Archimede's principle when a body floats then the weight of the body = upthrust

$$\therefore (50)^3 \times \frac{30}{100} \times (1) \times g = M_{cube} g \quad \ldots . (i)$$

Let m mass should be placed on the cube , then

$$(50)^3 \times (1) \times g = (M_{cube} + m) g \quad \ldots (ii)$$

Subtracting equation (i) $from$ (ii), we get

$$\Rightarrow mg = (50)^3 \times g(1 - 0.3) = 125 \times 0.7 \times 10^3 \ g$$

$$\Rightarrow m = 87.5 \text{ kg}$$

Hence option (B) is correct.

23. A submarine experiences a pressure of 5.05×10^6 Pa at depth of d_1 in a sea. When it goes further to a depth of d_2, it experiences a pressure of 8.08×10^6 Pa. Then $d_2 - d_1$ is approximately

(density of water $= 10^3$ kg/m^3 and acceleration due to gravity $= 10$ ms^{-2})

A. 300 m

B. 400 m

C. 600 m

D. 500 m

Given:
Pressure at depth of (d_1), $P_1 = 5.05 \times 10^6$ Pa
Pressure at depth of (d_2), $P_2 = 8.08 \times 10^6$ Pa.
Density of water $= 10^3$ kg/m^3

Now, absolute pressure at some depth in sea,
$P_1 = P_0 + \rho g d_1$
$P_2 = P_0 + \rho g d_2$
Pressure difference,

$$\Delta P = P_2 - P_1 = \rho g(d_2 - d_1) = \rho g \Delta d$$

$$\Rightarrow 3.03 \times 10^6 = 10^3 \times 10 \times \Delta d$$

$$\Rightarrow \Delta d \simeq 300 \text{ m}$$

Hence option (A) is correct.

Mechanical properties of solids and fluid mechanics

24. The elastic limit of brass is $379 \mathrm{~MPa}$. What should be the minimum diameter of a brass rod, if it is to support a $400 \mathrm{~N}$?

A. $1.0 \mathrm{~mm}$

B. $1.16 \mathrm{~mm}$

C. $0.90 \mathrm{~mm}$

D. $1.36 \mathrm{~mm}$

Given:
elastic limit of brass $=379 \mathrm{~MPa}$
Load to be supported $=400 \mathrm{~N}$
We know that,

$$\text{Stress}=\frac{F}{A}$$

$$\Rightarrow 379 \times 10^{6}=\frac{400 \times 4}{\pi d^{2}}$$

$$\Rightarrow d^{2}=\frac{400 \times 4}{379 \times 10^{6} \pi}$$

$d \approx 1.16 \mathrm{~mm}$.

Hence option (B) is correct.

25. Water from a tap emerges vertically downwards with an initial speed of 1.0 ms^{-1} . The cross-sectional area of the tap is 10^{-4} m^2. Assume that the pressure is constant throughout the stream of water and that the flow is streamlined. The cross-sectional area of the stream, 0.15 m below the tap would be :
[Take $g = 10 \text{ ms}^{-2}$]

A. $2 \times 10^{-5} \text{ m}^2$

B. $5 \times 10^{-5} \text{ m}^2$

C. $5 \times 10^{-4} \text{ m}^2$

D. $1 \times 10^{-5} \text{ m}^2$

Given:
Initial speed $v_1 = 1.0 \text{ ms}^{-1}$
Cross-sectional area of the tap $A_1 = 10^{-4} \text{ m}^2$
$h = 0.15 \text{ m}$

Using Bernoullie's equation

$$P + \frac{1}{2}\rho v_1^2 + \rho gh = P + \frac{1}{2}\rho v_2^2 + 0$$

$$\Rightarrow v_2^2 = v_1^2 + 2gh$$

$$\Rightarrow v_2 = \sqrt{v_1^2 + 2gh}$$

Equation of continuity

$$A_1 v_1 = A_2 v_2$$

$$(10^{-4}) \times (1 \text{ m/s}) = (A_2)\left(\sqrt{(1)^2 + 2 \times 10 \times \frac{15}{100}}\right)$$

$$10^{-4} \times 1 = A_2 \times 2$$

$$\therefore A_2 = \frac{10^{-4}}{2} = 5 \times 10^{-5} \text{ m}^2$$

Hence option (B) is correct.

26. The value of tension in a long, thin metal wire has been changed from T_1 to T_2. The lengths of the metal wire at two different values of tension T_1 and T_2 are l_1 and l_2 respectively. The actual length of the metal wire is :

A. $\dfrac{T_1 l_2 - T_2 l_1}{T_1 - T_2}$

B. $\dfrac{T_1 l_1 - T_2 l_2}{T_1 - T_2}$

C. $\dfrac{l_1 + l_2}{2}$

D. $\sqrt{T_1 T_2 l_1 l_2}$

We know that,

$$Y = \frac{FL}{A\Delta L}$$

$$\Rightarrow Y = \frac{T_1 l_0}{A(l_1 - l_0)} = \frac{T_2 l_0}{A(l_2 - l_0)}$$

$$\frac{T_1 l_0}{(l_1 - l_0)} = \frac{T_2 l_0}{(l_2 - l_0)}$$

$$1 = \frac{T_1(l_2 - l_0)}{T_2(l_1 - l_0)}$$

$$T_2 l_1 - T_2 l_0 = T_1 l_2 - T_1 l_0$$

$$(T_1 - T_2) l_0 = T_1 l_2 - T_2 l_1$$

$$\therefore l_0 = \frac{T_1 l_2 - T_2 l_1}{T_1 - T_2}$$

Hence, option (A) is the correct answer.

Mechanical properties of solids and fluid mechanics

27. Two wires of same length and radius, are joined end to end and loaded. The Young's modulii of the materials of the two wires are Y_1 and Y_2. The combination behaves as a single wire whose Young's modulus is :

A. $Y = \dfrac{2Y_1Y_2}{3(Y_1+Y_2)}$

B. $Y = \dfrac{2Y_1Y_2}{Y_1+Y_2}$

C. $Y = \dfrac{Y_1Y_2}{2(Y_1+Y_2)}$

D. $Y = \dfrac{Y_1Y_2}{Y_1+Y_2}$

For two wires joined in series $\Delta l = l_1 + l_2$

Now, $Y = \dfrac{\frac{F}{A}}{\frac{\Delta l}{l}} \Rightarrow \Delta l = \dfrac{Fl}{AY}$

Equivalent length of rod, after joining is $= 2l$
As, their lengths are same and force is also same in series.
$\Delta l = \Delta l_1 + \Delta l_2$

$$\frac{F\,l_{eq}}{A\,Y_{eq}} = \frac{Fl}{AY_1} + \frac{Fl}{AY_2}$$

$$\frac{l_{eq}}{Y_{eq}} = \frac{l}{Y_1} + \frac{l}{Y_2}$$

$$\Rightarrow \frac{2l}{Y} = \frac{l}{Y_1} + \frac{l}{Y_2}$$

Solving the above equation, we get,
$\therefore Y = \dfrac{2Y_1Y_2}{Y_1+Y_2}$

Hence, (B) is the correct answer.

Mechanical properties of solids and fluid mechanics

28. Two narrow bores of diameter $5.0 \mathrm{~mm}$ and $8.0 \mathrm{~mm}$ are joined together to form a U-shaped tube open at both ends. If this U-tube contains water, what is the difference in the level of two limbs of the tube.

Take surface tension of water, $T=7.3 \times 10^{-2} \mathrm{~Nm}^{-1}$
angle of contact $=0$
$g=10 \mathrm{~ms}^{-2}$
density of water $=1.0 \times 10^{3} \mathrm{~kgm}^{-3}$

A. $5.34 \mathrm{~mm}$

B. $3.62 \mathrm{~mm}$

C. $4.97 \mathrm{~mm}$

D. $2.19 \mathrm{~mm}$

$d_1=5 \mathrm{~mm}, d_2=8 \mathrm{~mm}$
$T=7.3 \times 10^{-2} \mathrm{~Nm}^{-1}$
$\theta=0^{\circ}$
$g=10 \mathrm{~ms}^{-2}$
$\rho=1.0 \times 10^{3} \mathrm{~kgm}^{-3}$

The level difference between the two limbs of the U-tube will be equal to the difference between the capillary rise in the two limbs.

Now, the capillary rise is given by, $h=\dfrac{2T\cos\theta}{r\rho g}$

$$\therefore h_1=\frac{2 \times 7.3 \times 10^{-2} \times 1}{2.5 \times 10^{-3} \times 10^{3} \times 10}=5.84 \mathrm{~mm}$$

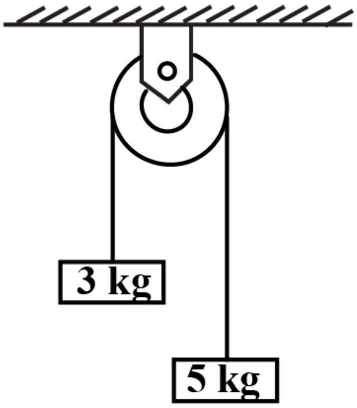
And, $h_2=\dfrac{2 \times 7.3 \times 10^{-2} \times 1}{4 \times 10^{-3} \times 10^{3} \times 10}=3.65 \mathrm{~mm}$

$\therefore$ The level difference is,
$\Delta h=h_1-h_2=5.84-3.65=2.19 \mathrm{~mm}$

Hence, (D) is the correct answer

Mechanical properties of solids and fluid mechanics

29. Two blocks of masses $3\ \mathrm{kg}$ and $5\ \mathrm{kg}$ are connected by a metal wire going over a smooth pulley. The breaking stress of the metal is $\frac{24}{\pi} \times 10^2\ \mathrm{Nm}^{-2}$. What is the minimum radius of the wire?
$(take\ g = 10\ \mathrm{ms}^{-2})$



A. $12.5\ \mathrm{cm}$

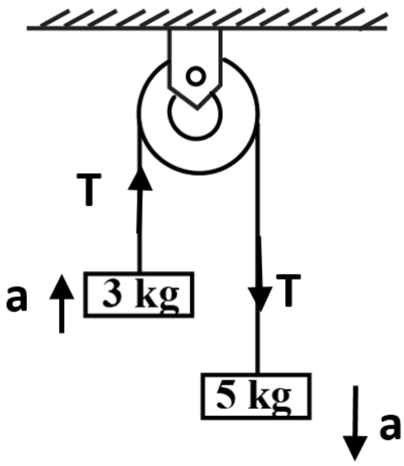
B. $125\ \mathrm{cm}$

C. $1250\ \mathrm{cm}$

D. $1.25\ \mathrm{cm}$

Mechanical properties of solids and fluid mechanics

Let the tension in the rope is wire is T and accelerations of the blocks is a as shown in the figure below.



For $5\ \mathrm{kg}$ block :
$5g - T = 5a \ldots (1)$

For $3\ \mathrm{kg}$ block :
$T - 3g = 3a \ldots (2)$

From (1) and (2) we have :
$a = 2.5\ \mathrm{m/s}^2, T = 37.5\ \mathrm{N}$

Now the minimum radius of the wire can be calculated using :

$$\text{Breaking stress} = \frac{T}{A_{min}}$$

$$\text{Breaking stress} = \frac{T}{\pi r^2}$$

$$r^2 = \frac{T}{\text{Breaking stress} \times \pi}$$

$$r = \sqrt{\frac{37.5}{\frac{24}{\pi} \times \pi \times 10^2}}$$

$$\Rightarrow r = 0.125\ \mathrm{m} = 125\ \mathrm{cm}$$

30. Four identical hollow cylindrical columns of mild steel support a big structure of mass 50×10^3 kg. The inner and outer radii of each column are 50 cm and 100 cm respectively. Assuming uniform local distribution, calculate the compression strain of each column. $\left[\text{use } Y = 2.0 \times 10^{11} \text{ Pa}, g = 9.8 \text{ m/s}^2\right]$

A. 2.60×10^{-7}

B. 3.60×10^{-8}

C. 1.87×10^{-3}

D. 7.07×10^{-4}

Given mass of the structure is 50000 kg

From symmetry weight acting on each column is $F = \dfrac{mg}{4} = 125 \times 10^3$ N

From Hooke's law compressive straign is given by $\epsilon = \dfrac{\Delta l}{l} = \dfrac{F}{AY}$

Here A is Area of cross section of column
i.e. $A = \pi(1)^2 - \pi(0.5)^2 = 0.75\pi$

On substitution we get

$$\epsilon = \frac{125000}{0.75\pi(2 \times 10^{11})} = \frac{250000}{3\pi(10^{11})} \approx 2.6 \times 10^{-7}$$