Subject: Mathematics

1. If the vectors $\overrightarrow{AB} = 3\hat{i} + 4\hat{k}$ and $\overrightarrow{AC} = 5\hat{i} - 2\hat{j} + 4\hat{k}$ are the sides of a triangle ABC, then the length of the median through A is :

A. $\sqrt{18}$

B. $\sqrt{72}$

C. $\sqrt{33}$

D. $\sqrt{45}$

2. A perpendicular is drawn from a point on the line $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z}{1}$ to the plane $x + y + z = 3$ such that the foot of the perpendicular Q also lies on the plane $x - y + z = 3$. Then the co-ordinates of Q are :

A. $(2, 0, 1)$

B. $(1, 0, 2)$

C. $(-1, 0, 4)$

D. $(4, 0, -1)$

3. ABC is a triangle in a plane with vertices $A(2, 3, 5), B(-1, 3, 2)$ and $C(\lambda, 5, \mu)$. If the median through A is equally inclined to the coordinate axes, then the value of $(\lambda^3 + \mu^3 + 5)$ is:

A. 1348

B. 1130

C. 1077

D. 676

4. The equation of the line passing through $(-4, 3, 1)$, parallel to the plane $x + 2y - z - 5 = 0$ and intersecting the line $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z-2}{-1}$ is:

A. $\frac{x+4}{-1} = \frac{y-3}{1} = \frac{z-1}{1}$

B. $\frac{x+4}{1} = \frac{y-3}{1} = \frac{z-1}{3}$

C. $\frac{x+4}{3} = \frac{y-3}{-1} = \frac{z-1}{1}$

D. $\frac{x+4}{2} = \frac{y-3}{1} = \frac{z-1}{4}$

5. Volume of parallelopiped whose coterminous edges are given by $\vec{u} = \hat{i} + \hat{j} + \lambda\hat{k}, \vec{v} = \hat{i} + \hat{j} + 3\hat{k}$ and $\vec{w} = 2\hat{i} + \hat{j} + \hat{k}$ is 1 cu. unit. If θ be the angle between the edges $\vec{u}$ and $\vec{w}$, then $\cos\theta$ can be:

A. $\frac{7}{6\sqrt{6}}$

B. $\frac{5}{7}$

C. $\frac{7}{6\sqrt{3}}$

D. $\frac{5}{3\sqrt{3}}$

6. Let $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ be two vectors. If $\vec{c}$ is a vector such that $\vec{b} \times \vec{c} = \vec{b} \times \vec{a}$ and $\vec{c} \cdot \vec{a} = 0$, then $\vec{c} \cdot \vec{b}$ is equal to :

A. $\frac{1}{2}$

B. $-\frac{3}{2}$

C. $-\frac{1}{2}$

D. -1

7. A plane which bisects the angle between the two given planes $2x - y + 2z - 4 = 0$ and $x + 2y + 2z - 2 = 0$, passes through the point :

A. $(1, -4, 1)$

B. $(2, 4, 1)$

C. $(2, -4, 1)$

D. $(1, 4, -1)$

8. Let P be the plane, which contains the line of intersection of the planes, $x+y+z-6=0$ and $2x+3y+z+5=0$ and it is perpendicular to the xy-plane. Then the distance of the point $(0,0,256)$ from P is equal to :

A. $63\sqrt{5}$

B. $\frac{11}{\sqrt{5}}$

C. $\frac{205}{\sqrt{5}}$

D. $\frac{17}{\sqrt{5}}$

9. The perpendicular distance from the origin to the plane containing the two lines, $\frac{x+2}{3}=\frac{y-2}{5}=\frac{z+5}{7}$ and $\frac{x-1}{1}=\frac{y-4}{4}=\frac{z+4}{7}$, is:

A. 11

B. $11\sqrt{6}$

C. $\frac{11}{\sqrt{6}}$

D. $6\sqrt{11}$

10. The equation of a plane containing the line of intersection of the planes $2x-y-4=0$ and $y+2z-4=0$ and passing through the point $(1,1,0)$ is:

A. $x-y-z=0$

B. $x+3y+z=4$

C. $x-3y-2z=-2$

D. $2x-z=2$

11. Let a vector $\alpha\hat{i}+\beta\hat{j}$ be obtained by rotating the vector $\sqrt{3}\hat{i}+\hat{j}$ by an angle 45° about the origin in counterclockwise direction in the first quadrant. Then the area of triangle having vertices $(\alpha,\beta),(0,\beta)$and $(0,0)$ is equal to:

A. 1

B. $\frac{1}{2}$

C. $\frac{1}{\sqrt{2}}$

D. $2\sqrt{2}$

12. If vectors $\vec{a}_1=x\hat{i}-\hat{j}+\hat{k}$ and $\vec{a}_2=\hat{i}+y\hat{j}+2\hat{k}$ are collinear, then a possible unit vector parallel to the vector $x\hat{i}+y\hat{j}+z\hat{k}$ is :

A. $\frac{1}{\sqrt{2}}(-\hat{j}+\hat{k})$

B. $\frac{1}{\sqrt{2}}(\hat{i}-\hat{j})$

C. $\frac{1}{\sqrt{3}}(\hat{i}-\hat{j}+\hat{k})$

D. $\frac{1}{\sqrt{3}}(\hat{i}+\hat{j}-\hat{k})$

13. If (x,y,z) be an arbitrary point lying on a plane P which passes through the points $(42,0,0)$, $(0,42,0)$ and $(0,0,42)$, then the value of the expression

$$3+\frac{x-11}{(y-19)^2(z-12)^2}+\frac{y-19}{(x-11)^2(z-12)^2}+\frac{z-12}{(x-11)^2(y-19)^2}-\frac{x+y+z}{14(x-11)(y-19)(z-12)}$$ is equal to :

A. 3

B. 0

C. 39

D. -45

14. Let $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ and $\vec{b} = 7\hat{i} + \hat{j} - 6\hat{k}$ If $\vec{r} \times \vec{a} = \vec{r} \times \vec{b}$, $\vec{r} \cdot (\hat{i} + 2\hat{j} + \hat{k}) = -3$, then $\vec{r} \cdot (2\hat{i} - 3\hat{j} + \hat{k})$ is equal to:

A. 10

B. 13

C. 12

D. 8

15. The equation of the plane which contains the $y-$ axis and passes through the point $(1, 2, 3)$ is:

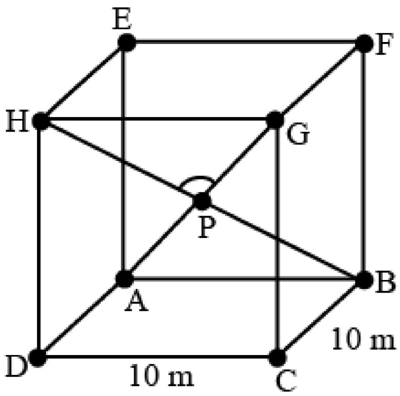
A. $3x + z = 6$

B. $3x - z = 0$

C. $x + 3z = 10$

D. $x + 3z = 0$

16. A hall has a square floor of dimension $10\ m \times 10\ m$ (see the figure) and vertical walls. If the angle GPH between the diagonals AG and BH is $\cos^{-1}\frac{1}{5}$, then the height of the hall (in meters) is



A. $2\sqrt{10}$

B. $5\sqrt{2}$

C. $5\sqrt{3}$

D. 5

17. If for some $\alpha \in \mathbb{R}$, the lines $L_1 : \frac{x+1}{2} = \frac{y-2}{-1} = \frac{z-1}{1}$ and $L_2 : \frac{x+2}{\alpha} = \frac{y+1}{5-\alpha} = \frac{z+1}{1}$ coplanar, then the line L_2 passes through the point:

A. $(2, -10, -2)$

B. $(10, -2, -2)$

C. $(10, 2, 2)$

D. $(-2, 10, 2)$

18. The mirror image of the point $(1, 2, 3)$ in a plane is $\left(-\frac{7}{3}, -\frac{4}{3}, -\frac{1}{3}\right)$. Which of the following points lies on this plane?

A. $(1, -1, 1)$

B. $(-1, -1, 1)$

C. $(1, 1, 1)$

D. $(-1, -1, -1)$

19. The angle between the straight lines, whose direction cosines are given by the equations $2l + 2m - n = 0$ and $mn + nl + lm = 0$, is

A. $\frac{\pi}{3}$

B. $\frac{\pi}{2}$

C. $\pi - \cos^{-1}\left(\frac{4}{9}\right)$

D. $\cos^{-1}\left(\frac{8}{9}\right)$

20. Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$. If $\vec{c}$ is a vector such that $\vec{a} \cdot \vec{c} = |\vec{c}|$, $|\vec{c} - \vec{a}| = 2\sqrt{2}$ and the angle between $(\vec{a} \times \vec{b})$ and $\vec{c}$ is $\frac{\pi}{6}$, then the value of $\left|(\vec{a} \times \vec{b}) \times \vec{c}\right|$ is

A. 4

B. $\frac{2}{3}$

C. $\frac{3}{2}$

D. 3

21. The shortest distance between the lines $\frac{x-1}{0}=\frac{y+1}{-1}=\frac{z}{1}$ and $x+y+z+1=0, 2x-y+z+3=0$ is:

A. 1

B. $\frac{1}{\sqrt{2}}$

C. $\frac{1}{\sqrt{3}}$

D. $\frac{1}{2}$

22. Let the foot of perpendicular from a point $P(1,2,-1)$ to the straight line $L: \frac{x}{1}=\frac{y}{0}=\frac{z}{-1}$ be N. Let a line be drawn from P parallel to the plane $x+y+2z=0$ which meets L at point Q. If α is the acute angle between the lines PN and PQ, then $\cos\alpha$ is equal to

A. $\frac{\sqrt{3}}{2}$

B. $\frac{1}{2\sqrt{3}}$

C. $\frac{1}{\sqrt{3}}$

D. $\frac{1}{\sqrt{5}}$

23. Let P be a plane passing through the points $(2,1,0),(4,1,1)$ and $(5,0,1)$ and R be any point $(2,1,6)$. Then the image of R in the plane P is:

A. $(6,5,2)$

B. $(6,5,-2)$

C. $(4,3,2)$

D. $(3,4,-2)$

24. If the foot of the perpendicular drawn from the point $(1,0,3)$ on a line passing through $(\alpha,7,1)$ is $\left(\frac{5}{3},\frac{7}{3},\frac{17}{3}\right)$, then α is equal to

25. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors such that $|\vec{a}|=\sqrt{3}, |\vec{b}|=5, \vec{b}\cdot\vec{c}=10$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{3}$.

If $\vec{a}$ is perpendicular to vector $\vec{b}\times\vec{c}$, then $|\vec{a}\times(\vec{b}\times\vec{c})|$ is equal to

26. If the vector $\overrightarrow{p} = (a+1)\hat{i} + a\hat{j} + a\hat{k}, \overrightarrow{q} = a\hat{i} + (a+1)\hat{j} + a\hat{k}$ and $\overrightarrow{r} = a\hat{i} + a\hat{j} + (a+1)\hat{k}, \ (a \in R)$ are coplanar and $3(\overrightarrow{p}\,.\,\overrightarrow{q})^2 - \lambda|\overrightarrow{r} \times \overrightarrow{q}|^2 = 0$, then value of λ is

27. Let P be a plane containing the line $\frac{x-1}{3} = \frac{y+6}{4} = \frac{z+5}{2}$ and parallel to the line $\frac{x-3}{4} = \frac{y-2}{-3} = \frac{z+5}{7}$. If the point $(1, -1, \alpha)$ lies on the plane P, then the value of $|5\alpha|$ is equal to

28. Let the plane $ax + by + cz + d = 0$ bisect the line joining the points $(4, -3, 1)$ and $(2, 3, -5)$ at the right angles. If a, b, c, d are integers, then the minimum value of $(a^2 + b^2 + c^2 + d^2)$ is

29. Let the line L be the projection of the line $\frac{x-1}{2} = \frac{y-3}{1} = \frac{z-4}{2}$ in the plane $x - 2y - z = 3$. If d is the distance of the point $(0, 0, 6)$ from L, then d^2 is equal to

30. If the projection of the vector $\hat{i} + 2\hat{j} + \hat{k}$ on the sum of the two vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $-\lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is 1, then λ is equal to