Subject: Mathematics

1. If the vectors $\overrightarrow{AB} = 3\hat{i} + 4\hat{k}$ and $\overrightarrow{AC} = 5\hat{i} - 2\hat{j} + 4\hat{k}$ are the sides of a triangle ABC, then the length of the median through A is :

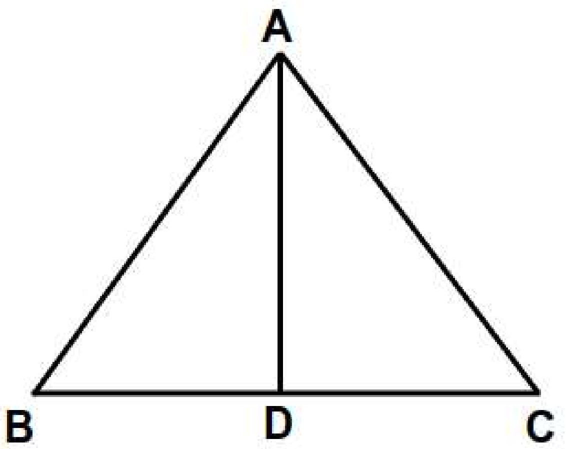
A. $\sqrt{18}$

B. $\sqrt{72}$

C. $\sqrt{33}$

D. $\sqrt{45}$

Median through any vertex divide the opposite side into two equal parts



$$\overrightarrow{AD} = \frac{\overrightarrow{AB} + \overrightarrow{AC}}{2}$$
$$= \frac{(3\hat{i} + 4\hat{k}) + (5\hat{i} - 2\hat{j} + 4\hat{k})}{2}$$
$$= 4\hat{i} - \hat{j} + 4\hat{k}$$

Hence, $|\overrightarrow{AD}| = \sqrt{4^2 + (-1)^2 + 4^2}$
$= \sqrt{33}$

2. A perpendicular is drawn from a point on the line $\frac{x-1}{2}=\frac{y+1}{-1}=\frac{z}{1}$ to the plane $x+y+z=3$ such that the foot of the perpendicular Q also lies on the plane $x-y+z=3$. Then the co-ordinates of Q are :

A. $(2,0,1)$

B. $(1,0,2)$

C. $(-1,0,4)$

D. $(4,0,-1)$

$$\frac{x-1}{2}=\frac{y+1}{-1}=\frac{z}{1}=\lambda$$

Then the any point P on the line is
$(2\lambda+1,-\lambda-1,\lambda)$

Let point $Q=(x_2,y_2,z_2)$ and $P=(x_1,y_1,z_1)$

Then, foot of perpendicular Q drawn from point P to the plane $ax+by+cz+d=0$ is given by

$$\frac{x_2-x_1}{a}=\frac{y_2-y_1}{b}=\frac{z_2-z_1}{c}$$
$$=-\frac{ax_1+by_1+cz_1+d}{a^2+b^2+c^2}$$

Foot of perpendicular Q is given by

$$\frac{x-2\lambda-1}{1}=\frac{y+\lambda+1}{1}=\frac{z-\lambda}{1}=-\frac{(2\lambda-3)}{3}$$

Q lies on $x+y+z=3$ and $x-y+z=3$

$\Rightarrow x+z=3$ and $y=0$

$$\therefore y=0\Rightarrow\lambda+1=\frac{-2\lambda+3}{3}$$
$$\Rightarrow\lambda=0$$

$Q=(2,0,1)$

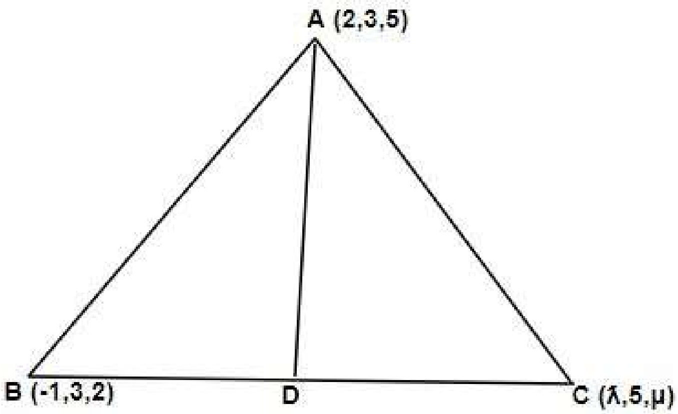
3. ABC is a triangle in a plane with vertices $A(2,3,5), B(-1,3,2)$ and $C(\lambda, 5, \mu)$. If the median through A is equally inclined to the coordinate axes, then the value of $(\lambda^3 + \mu^3 + 5)$ is:

A. 1348

B. 1130

C. 1077

D. 676



Coordinates of $D = \left(\frac{\lambda-1}{2}, 4, \frac{\mu+2}{2}\right)$

$\therefore \overrightarrow{AD} = \left(\frac{\lambda-1}{2} - 2\right)\hat{i} + (4-3)\hat{j} + \left(\frac{\mu+2}{2} - 5\right)\hat{k}$

$= \left(\frac{\lambda-5}{2}\right)\hat{i} + \hat{j} + \left(\frac{\mu-8}{2}\right)\hat{k}$

Since, $\overrightarrow{AD}$ makes equal angle with the coordinate axes, the direction ratios are equal.

$\Rightarrow$ Direction ratios are equal.

i.e., $\frac{\lambda-5}{2} = 1 = \frac{\mu-8}{2}$

$\Rightarrow \lambda = 7, \mu = 10$

$\therefore \lambda^3 + \mu^3 + 5 = 1348$

4. The equation of the line passing through $(-4,3,1)$, parallel to the plane $x+2y-z-5=0$ and intersecting the line $\frac{x+1}{-3}=\frac{y-3}{2}=\frac{z-2}{-1}$ is:

A. $\frac{x+4}{-1}=\frac{y-3}{1}=\frac{z-1}{1}$

B. $\frac{x+4}{1}=\frac{y-3}{1}=\frac{z-1}{3}$

C. $\frac{x+4}{3}=\frac{y-3}{-1}=\frac{z-1}{1}$

D. $\frac{x+4}{2}=\frac{y-3}{1}=\frac{z-1}{4}$

Vector passing through $(-4,3,1)$ and $(-1,3,2)=3\hat{i}+\hat{k}$
Normal vector of plane containing two intersecting lines is parallel to the vector.

$$\vec{r_1}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 0 & 1\\ -3 & 2 & -1\end{vmatrix}=-2\hat{i}+6\hat{k}$$

$\therefore$ Required line is parallel to the vector.

$$\vec{r_2}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & -1\\ -2 & 0 & 6\end{vmatrix}=3\hat{i}-\hat{j}+\hat{k}$$

$\therefore$ Required equation of line passing through $(-4,3,1)$ is $\frac{x+4}{3}=\frac{y-3}{-1}=\frac{z-1}{1}$

5. Volume of parallelopiped whose coterminous edges are given by $\vec{u}=\hat{i}+\hat{j}+\lambda\hat{k}$, $\vec{v}=\hat{i}+\hat{j}+3\hat{k}$ and $\vec{w}=2\hat{i}+\hat{j}+\hat{k}$ is 1 cu. unit. If θ be the angle between the edges $\vec{u}$ and $\vec{w}$, then $\cos\theta$ can be:

A. $\frac{7}{6\sqrt{6}}$

B. $\frac{5}{7}$

C. $\frac{7}{6\sqrt{3}}$

D. $\frac{5}{3\sqrt{3}}$

Volume of parallelepiped $=[\vec{u}\ \ \vec{v}\ \ \vec{w}]$

$$\Rightarrow\begin{vmatrix}1 & 1 & \lambda\\ 1 & 1 & 3\\ 2 & 1 & 1\end{vmatrix}=\pm1$$

$\Rightarrow \lambda=2$ or 4
For $\lambda=4$,

$$\cos\theta=\frac{2+1+4}{\sqrt{18}\sqrt{6}}=\frac{7}{6\sqrt{3}}$$

6. Let $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ be two vectors. If $\vec{c}$ is a vector such that $\vec{b} \times \vec{c} = \vec{b} \times \vec{a}$ and $\vec{c} \cdot \vec{a} = 0$, then $\vec{c} \cdot \vec{b}$ is equal to :

A. $\frac{1}{2}$

B. $-\frac{3}{2}$

C. $-\frac{1}{2}$

D. -1

$\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$
$\vec{b} = \hat{i} - \hat{j} + \hat{k}$
$\vec{b} \times \vec{c} = \vec{b} \times \vec{a}$
$\Rightarrow \vec{a} \times (\vec{b} \times \vec{c}) = \vec{a} \times (\vec{b} \times \vec{a})$
$\Rightarrow (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c} = (\vec{a} \cdot \vec{a})\vec{b} - (\vec{a} \cdot \vec{b})\vec{a}$
$\Rightarrow -(\vec{a} \cdot \vec{b})\vec{c} = (\vec{a} \cdot \vec{a})\vec{b} - (\vec{a} \cdot \vec{b})\vec{a}$
$\Rightarrow -4\vec{c} = 6(\hat{i} - \hat{j} + \hat{k}) - 4(\hat{i} - 2\hat{j} + \hat{k})$
$\Rightarrow \vec{c} = -\frac{1}{2}(\hat{i} + \hat{j} + \hat{k})$
$\therefore \vec{b} \cdot \vec{c} = -\frac{1}{2}$

7. A plane which bisects the angle between the two given planes $2x - y + 2z - 4 = 0$ and $x + 2y + 2z - 2 = 0$, passes through the point :

A. $(1, -4, 1)$

B. $(2, 4, 1)$

C. $(2, -4, 1)$

D. $(1, 4, -1)$

Equation of angle bisectors are

$$\frac{2x - y + 2z - 4}{\sqrt{2^2 + (-1)^2 + 2^2}} = \pm \frac{x + 2y + 2z - 2}{\sqrt{1^2 + (2)^2 + 2^2}}$$

Taking positive sign, we get
$x - 3y - 2 = 0 \quad \ldots(1)$
Taking negative sign, we get
$3x + y + 4z - 6 = 0 \quad \ldots(2)$

Now by putting all the given options in the equation (1) and (2), we find that $(2, -4, 1)$ satisfies the equation (2)

8. Let P be the plane, which contains the line of intersection of the planes, $x+y+z-6=0$ and $2x+3y+z+5=0$ and it is perpendicular to the xy-plane. Then the distance of the point $(0,0,256)$ from P is equal to :

A. $63\sqrt{5}$

B. $\frac{11}{\sqrt{5}}$

C. $\frac{205}{\sqrt{5}}$

D. $\frac{17}{\sqrt{5}}$

Equation of plane P is :
$(x+y+z-6)+\lambda(2x+3y+z+5)=0$
$\Rightarrow (1+2\lambda)x+(1+3\lambda)y+(1+\lambda)z+(-6+5\lambda)=0$
Since plane P is perpendicular to xy-plane,
$\therefore 1+\lambda=0 \Rightarrow \lambda=-1$
Hence, equation of plane $P: x+2y+11=0$
Perpendicular distance of plane from $(0,0,256)=\frac{11}{\sqrt{5}}$

9. The perpendicular distance from the origin to the plane containing the two lines, $\frac{x+2}{3}=\frac{y-2}{5}=\frac{z+5}{7}$ and $\frac{x-1}{1}=\frac{y-4}{4}=\frac{z+4}{7}$, is:

A. 11

B. $11\sqrt{6}$

C. $\frac{11}{\sqrt{6}}$

D. $6\sqrt{11}$

The plane containing is the lines

$L_1 : \frac{x+2}{3}=\frac{y-2}{5}=\frac{z+5}{7}$ and

$L_2 : \frac{x-1}{1}=\frac{y-4}{4}=\frac{z+4}{7}$

$\therefore$ normal vector of plane is -

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 5 & 7 \\ 1 & 4 & 7 \end{vmatrix}$$
$$= \hat{i}(35-28) - \hat{j}(21-7) + \hat{k}(12-5)$$
$$= 7\hat{i} - 14\hat{j} + 7\hat{k}$$

$\therefore$ Equation of plane is

$7(x+2) - 14(y-2) + 7(z+5) = 0$

$\Rightarrow 7x - 14y + 7z + 77 = 0$

$\Rightarrow x - 2y + z + 11 = 0$

Now, perpendicular distance from $(0,0,0)$ to plane is given by -

$$d = \left|\frac{0-0+0+11}{\sqrt{1+4+1}}\right|$$

$$\Rightarrow d = \frac{11}{\sqrt{6}}$$

10. The equation of a plane containing the line of intersection of the planes $2x - y - 4 = 0$ and $y + 2z - 4 = 0$ and passing through the point $(1, 1, 0)$ is:

A. $x - y - z = 0$

B. $x + 3y + z = 4$

C. $x - 3y - 2z = -2$

D. $2x - z = 2$

Let equation of planes be P_1 and P_2
Now according to question,
$\Rightarrow P_1 + \lambda P_2 = 0$
$\Rightarrow (2x - y - 4) + \lambda(y + 2z - 4) = 0 \quad \ldots(1)$

The plane passes through the point $(1, 1, 0)$
Put $(1, 1, 0)$ in equation (1)
$\Rightarrow 1 + \lambda = 0 \Rightarrow \lambda = -1$
$\therefore x - y - z = 0$

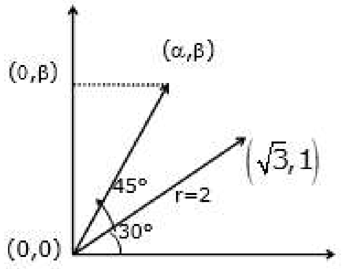
11. Let a vector $\alpha\hat{i} + \beta\hat{j}$ be obtained by rotating the vector $\sqrt{3}\hat{i} + \hat{j}$ by an angle 45° about the origin in counterclockwise direction in the first quadrant. Then the area of triangle having vertices $(\alpha, \beta), (0, \beta)$and $(0, 0)$ is equal to:

A. 1

B. $\frac{1}{2}$

C. $\frac{1}{\sqrt{2}}$

D. $2\sqrt{2}$



$(\alpha, \beta) \equiv (2\cos 75^\circ, 2\sin 75^\circ)$

$\text{Area} = \frac{1}{2}(2\cos 75^\circ)(2\sin 75^\circ)$

$= \sin(150^\circ) = \frac{1}{2}$sq. unit

12. If vectors $\overrightarrow{a}_1 = x\hat{i} - \hat{j} + \hat{k}$ and $\overrightarrow{a}_2 = \hat{i} + y\hat{j} + 2\hat{k}$ are collinear, then a possible unit vector parallel to the vector $x\hat{i} + y\hat{j} + z\hat{k}$ is :

A. $\frac{1}{\sqrt{2}}(-\hat{j} + \hat{k})$

B. $\frac{1}{\sqrt{2}}(\hat{i} - \hat{j})$

C. $\frac{1}{\sqrt{3}}(\hat{i} - \hat{j} + \hat{k})$

D. $\frac{1}{\sqrt{3}}(\hat{i} + \hat{j} - \hat{k})$

Collinear condition

$\frac{x}{1} = -\frac{1}{y} = \frac{1}{z} = \lambda$ (let)

Unit vector parallel to $x\hat{i} + y\hat{j} + z\hat{k} = \pm\frac{(\lambda\hat{i} - \frac{1}{\lambda}\hat{j} + \frac{1}{\lambda}\hat{k})}{\sqrt{\lambda^2 + \frac{2}{\lambda^2}}}$

For $\lambda = 1$, it is $\pm\frac{(\hat{i} - \hat{j} + \hat{k})}{\sqrt{3}}$

13. If (x, y, z) be an arbitrary point lying on a plane P which passes through the points $(42, 0, 0)$, $(0, 42, 0)$ and $(0, 0, 42)$, then the value of the expression $3 + \frac{x-11}{(y-19)^2(z-12)^2} + \frac{y-19}{(x-11)^2(z-12)^2} + \frac{z-12}{(x-11)^2(y-19)^2} - \frac{x+y+z}{14(x-11)(y-19)(z-12)}$ is equal to :

A. 3

B. 0

C. 39

D. -45

Equation of plane is $x + y + z = 42$

or, $(x-11) + (y-19) + (z-12) = 0$

Now, $\frac{x-11}{(y-19)^2(z-12)^2} + \frac{y-19}{(x-11)^2(z-12)^2} + \frac{z-12}{(z-11)^2(y-19)^2}$

$= \frac{(x-11)^3 + (y-19)^3 + (z-12)^3}{(x-11)^2(y-19)^2(z-12)^2}$

$= \frac{3(x-11)(y-19)(z-12)}{(x-11)^2(y-19)^2(z-12)^2}$

[If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$]

$= \frac{3}{(x-11)(y-19)(z-12)}$

$\therefore$ The given expression is equal to

$3 + \frac{3}{(x-11)(y-19)(z-12)} - \frac{3}{(x-11)(y-19)(z-12)} = 3$

14. Let $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ and $\vec{b} = 7\hat{i} + \hat{j} - 6\hat{k}$ If $\vec{r} \times \vec{a} = \vec{r} \times \vec{b}$, $\vec{r} \cdot (\hat{i} + 2\hat{j} + \hat{k}) = -3$, then $\vec{r} \cdot (2\hat{i} - 3\hat{j} + \hat{k})$ is equal to:

A. 10

B. 13

C. 12

D. 8

Given : $\vec{a} = (2, -3, 4)$, $\vec{b} = (7, 1, -6)$
Now,
$\vec{r} \times \vec{a} = \vec{r} \times \vec{b}$
$\Rightarrow \vec{r} \times \vec{a} - \vec{r} \times \vec{b} = 0$
$\Rightarrow \vec{r} \times (\vec{a} - \vec{b}) = 0$
$\Rightarrow \vec{r} = \lambda(\vec{a} - \vec{b})$
$\Rightarrow \vec{r} = \lambda(-5\hat{i} - 4\hat{j} + 10\hat{k})$

We know that
$\vec{r} \cdot (1, 2, 1) = -3$
$\Rightarrow \lambda(-5 - 8 + 10) = -3$
$\Rightarrow \lambda = 1$
$\therefore (-5, -4, 10) \cdot (2, -3, 1) = 12$

15. The equation of the plane which contains the $y-$ axis and passes through the point $(1, 2, 3)$ is:

A. $3x + z = 6$

B. $3x - z = 0$

C. $x + 3z = 10$

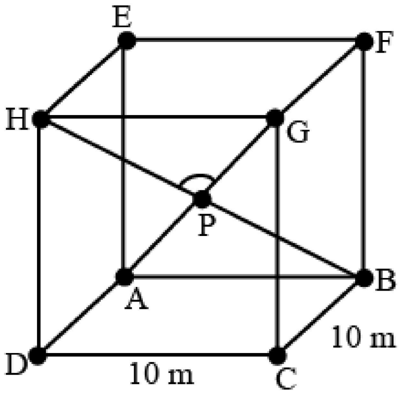
D. $x + 3z = 0$

Let $A = (1, 2, 3)$ and O be the origin
D.R.'s of $y-$ axis are $0, 1, 0$
$\overrightarrow{OA} = (1, 2, 3)$
Therefore, the required equation of the plane

$$\begin{vmatrix} x & y & z \\ 0 & 1 & 0 \\ 1 & 2 & 3 \end{vmatrix} = 0$$

$\Rightarrow 3x - z = 0$

16. A hall has a square floor of dimension $10\ m \times 10\ m$ (see the figure) and vertical walls. If the angle GPH between the diagonals AG and BH is $\cos^{-1}\frac{1}{5}$, then the height of the hall (in meters) is

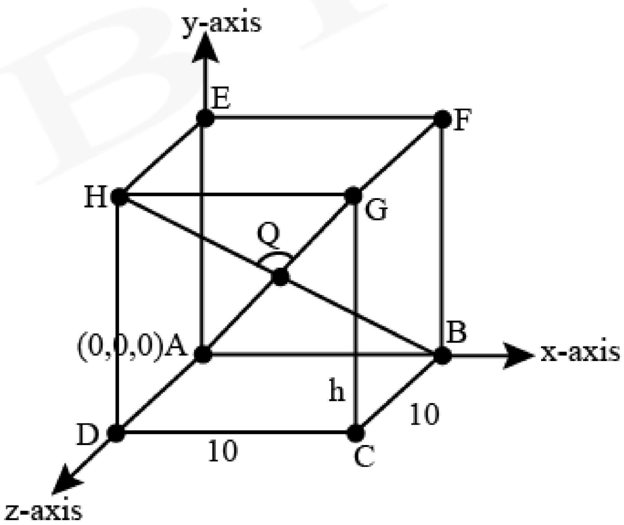


A. $2\sqrt{10}$

B. $5\sqrt{2}$

C. $5\sqrt{3}$

D. 5



Let height be h.
$A \equiv (0,0,0)$
$G \equiv (10,h,10)$
$B \equiv (10,0,0)$
$H \equiv (0,h,10)$

DRs of $AG \equiv (10,h,10)$
DRs of $BH \equiv (10,-h,-10)$

$$\cos\theta = \left|\frac{10\times 10 + h(-h) + 10(-10)}{\sqrt{10^2+h^2+10^2}\times\sqrt{10^2+h^2+10^2}}\right|$$

$$\Rightarrow \frac{1}{5} = \frac{h^2}{200+h^2}$$

$$\Rightarrow h = 5\sqrt{2}$$

17. If for some $\alpha \in \mathbb{R}$, the lines $L_1 : \frac{x+1}{2} = \frac{y-2}{-1} = \frac{z-1}{1}$ and $L_2 : \frac{x+2}{\alpha} = \frac{y+1}{5-\alpha} = \frac{z+1}{1}$ coplanar, then the line L_2 passes through the point:

A. $(2, -10, -2)$

B. $(10, -2, -2)$

C. $(10, 2, 2)$

D. $(-2, 10, 2)$

$A(-1, 2, 1), B(-2, -1, -1)$

$$\left[\overrightarrow{AB} \ \vec{b_1} \ \vec{b_2}\right] = 0$$

$$\Rightarrow \begin{vmatrix} -1 & -3 & -2 \\ 2 & -1 & 1 \\ \alpha & 5-\alpha & 1 \end{vmatrix} = 0$$

$\Rightarrow -1(-1+\alpha-5) + 3(2-\alpha) - 2(10-2\alpha+\alpha) = 0$
$\Rightarrow 6 - \alpha + 6 - 3\alpha + 2\alpha - 20 = 0$
$\Rightarrow -8 - 2\alpha = 0$
$\Rightarrow \alpha = -4$

Checking Point satisfies L_2

$$L_2 : \frac{x+2}{-4} = \frac{y+1}{9} = \frac{z+1}{1}$$

Any point on L_2 is $(-4k-2, 9k-1, k-1)$

From given options, for point $(2, -10, -2)$:

$$\frac{4}{-4} = \frac{-9}{9} = \frac{-1}{1}$$

Therefore, option (a) is correct.

18. The mirror image of the point $(1, 2, 3)$ in a plane is $\left(-\frac{7}{3}, -\frac{4}{3}, -\frac{1}{3}\right)$. Which of the following points lies on this plane?

A. $(1, -1, 1)$

B. $(-1, -1, 1)$

C. $(1, 1, 1)$

D. $(-1, -1, -1)$

Image of point $P(1, 2, 3)$ w.r.t. a plane $ax + by + cz + d = 0$ is
$Q\left(-\frac{7}{3}, -\frac{4}{3}, -\frac{1}{3}\right)$

Direction ratios of PQ : $\left(-\frac{7}{3} - 1, -\frac{4}{3} - 2, -\frac{1}{3} - 3\right)$

$$= \left(-\frac{10}{3}, -\frac{10}{3}, -\frac{10}{3}\right) = (1, 1, 1)$$

Direction ratios of normal to plane is $(1, 1, 1)$
Mid-point of line PQ lies on the plane.

$\therefore$ The mid-point of $PQ = \left(-\frac{2}{3}, \frac{1}{3}, \frac{4}{3}\right)$
$\therefore$Equation of plane is $x + \frac{2}{3} + y - \frac{1}{3} + z - \frac{4}{3} = 0$
$\Rightarrow x + y + z = 1$
Hence point $(1, -1, 1)$ satisfies the equation of the plane.

19. The angle between the straight lines, whose direction cosines are given by the equations $2l+2m-n=0$ and $mn+nl+lm=0$, is

A. $\frac{\pi}{3}$

B. $\frac{\pi}{2}$

C. $\pi-\cos^{-1}\left(\frac{4}{9}\right)$

D. $\cos^{-1}\left(\frac{8}{9}\right)$

$\because 2l+2m-n=0\cdots(i)$
and $mn+nl+lm=0\cdots(ii)$
From equation (i) and (ii)
$(m+l)(2l+2m)+lm=0$
$\Rightarrow 2l^2+5lm+2m^2=0$
$\Rightarrow 2l^2+4lm+lm+2m^2=0$
$\Rightarrow 2l(l+2m)+m(l+2m)=0$
$\therefore (2l+m)(l+2m)=0$
Direction ratios of lines are $(1,-2,-2)$ and $(2,-1,2)$.
Here $l_1l_2+m_1m_2+n_1n_2=0$
$\therefore$ Lines are perpendicular to each other.

20. Let $\overrightarrow{a}=2\hat{i}+\hat{j}-2\hat{k}$ and $\overrightarrow{b}=\hat{i}+\hat{j}$. If $\overrightarrow{c}$ is a vector such that $\overrightarrow{a}\cdot\overrightarrow{c}=\left|\overrightarrow{c}\right|$, $\left|\overrightarrow{c}-\overrightarrow{a}\right|=2\sqrt{2}$ and the angle between $\left(\overrightarrow{a}\times\overrightarrow{b}\right)$ and $\overrightarrow{c}$ is $\frac{\pi}{6}$, then the value of $\left|\left(\overrightarrow{a}\times\overrightarrow{b}\right)\times\overrightarrow{c}\right|$ is

A. 4

B. $\frac{2}{3}$

C. $\frac{3}{2}$

D. 3

$\overrightarrow{a}=2\hat{i}+\hat{j}-2\hat{k}$

$\Rightarrow\left|\overrightarrow{a}\right|=\sqrt{4+1+4}=3$

Now, $\left|\overrightarrow{c}-\overrightarrow{a}\right|=2\sqrt{2}$

$\Rightarrow\left|\overrightarrow{c}-\overrightarrow{a}\right|^2=8$

$\Rightarrow\left|\overrightarrow{c}\right|^2+\left|\overrightarrow{a}\right|^2-2\overrightarrow{c}\cdot\overrightarrow{a}=8$

$\Rightarrow\left|\overrightarrow{c}\right|^2+9-2\left|\overrightarrow{c}\right|=8\ \left(\because\overrightarrow{a}\cdot\overrightarrow{c}=\left|\overrightarrow{c}\right|\right)$

$\Rightarrow|\overrightarrow{c}|^2-2|\overrightarrow{c}|+1=0$

$\Rightarrow\left(\left|\overrightarrow{c}\right|-1\right)^2=0$

$\therefore\left|\overrightarrow{c}\right|=1$

And angle between $\overrightarrow{a}\times\overrightarrow{b}$ and $\overrightarrow{c}$ is $\theta=\frac{\pi}{6}$ then $\left|\left(\overrightarrow{a}\times\overrightarrow{b}\right)\times\overrightarrow{c}\right|=\left|\overrightarrow{a}\times\overrightarrow{b}\right|\cdot\left|\overrightarrow{c}\right|\cdot\sin\theta$

$=\left|2\hat{i}-2\hat{j}+\hat{k}\right|\cdot1\cdot\sin\frac{\pi}{6}$

$=\frac{3}{2}$

21. The shortest distance between the lines $\frac{x-1}{0}=\frac{y+1}{-1}=\frac{z}{1}$ and $x+y+z+1=0, 2x-y+z+3=0$ is:

A. 1

B. $\frac{1}{\sqrt{2}}$

C. $\frac{1}{\sqrt{3}}$

D. $\frac{1}{2}$

Plane through line of intersection is
$x+y+z+1+\lambda(2x-y+z+3)=0$

It should be parallel to given line
$0(1+2\lambda)-1(1-\lambda)+1(1+\lambda)=0$
$\Rightarrow \lambda=0$
Plane : $x+y+z+1=0$
Shortest distance of $(1,-1,0)$ from this plane
$=\frac{|1-1+0+1|}{\sqrt{1^2+1^2+1^2}}=\frac{1}{\sqrt{3}}$

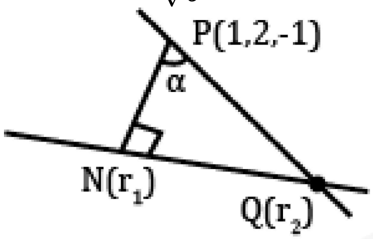
22. Let the foot of perpendicular from a point $P(1,2,-1)$ to the straight line $L: \frac{x}{1}=\frac{y}{0}=\frac{z}{-1}$ be N. Let a line be drawn from P parallel to the plane $x+y+2z=0$ which meets L at point Q. If α is the acute angle between the lines PN and PQ, then $\cos\alpha$ is equal to

A. $\frac{\sqrt{3}}{2}$

B. $\frac{1}{2\sqrt{3}}$

C. $\frac{1}{\sqrt{3}}$

D. $\frac{1}{\sqrt{5}}$



$L: \frac{x}{1}=\frac{y}{0}=\frac{z}{-1}=r$

$N \equiv (r_1, 0, -r_1)$

$P(1,2,-1)$

D.R's of $NP = (r_1-1, -2, -r_1+1) \perp$ with D.R's $(1,0,-1)$

$\Rightarrow r_1 - 1 + 0 + r_1 - 1 = 0$

$\Rightarrow r_1 = 1$

$\Rightarrow N(1,0,-1)$

$Q(r_2, 0, -r_2)$

D.R's of normal of plane $x+y+2z=0$ are $(1,1,2)$

D.R's of $PQ = (r_2-1, -2, -r_2+1)$ are perpendicular to $(1,1,2)$

$\Rightarrow r_2 - 1 - 2 - 2r_2 + 2 = 0$

$\Rightarrow r_2 = -1$

$\Rightarrow Q(-1,0,1)$

$NP = 2,\ PQ = 2\sqrt{3}$

$\cos\alpha = \frac{NP}{PQ} = \frac{1}{\sqrt{3}}$

23. Let P be a plane passing through the points $(2,1,0),(4,1,1)$ and $(5,0,1)$ and R be any point $(2,1,6)$. Then the image of R in the plane P is:

A. $(6,5,2)$

B. $(6,5,-2)$

C. $(4,3,2)$

D. $(3,4,-2)$

Points $A(2,1,0), B(4,1,1) C(5,0,1)$
$\overrightarrow{AB}=(2,0,1)$
$\overrightarrow{AC}=(3,-1,1)$
$\vec{n}=\overrightarrow{AB}\times\overrightarrow{AC}=(1,1,-2)$
Equation of the plane is $x+y-2z=3$ ....(1)
Let the image of point $(2,1,6)$ is (l,m,n)
$\frac{l-2}{1}=\frac{m-1}{1}=\frac{n-6}{-2}=\frac{-2(-12)}{6}=4$
$\Rightarrow l=6, m=5, n=-2$
Hence the image of R in the plane P is $(6,5,-2)$

24. If the foot of the perpendicular drawn from the point $(1,0,3)$ on a line passing through $(\alpha,7,1)$ is $\left(\frac{5}{3},\frac{7}{3},\frac{17}{3}\right)$, then α is equal to

Accepted Answers

4 4.0 4.00

Solution:

Given points $P(1,0,3)$ and $Q\left(\frac{5}{3},\frac{7}{3},\frac{17}{3}\right)$

Direction ratios of PQ : $\left(\frac{2}{3},\frac{7}{3},\frac{8}{3}\right)$

Direction ratios of line L : $\left(\alpha-\frac{5}{3}, 7-\frac{7}{3}, 1-\frac{17}{3}\right)$

$=\left(\frac{3\alpha-5}{3},\frac{14}{3},-\frac{14}{3}\right)$

As line L is perpendicular to PQ,
so, $\left(\frac{2}{3}\right)\left(\frac{3\alpha-5}{3}\right)+\left(\frac{7}{3}\right)\left(\frac{14}{3}\right)+\left(\frac{8}{3}\right)\left(-\frac{14}{3}\right)=0$

$\Rightarrow 6\alpha-10+98-112=0$
$\Rightarrow 6\alpha=24 \Rightarrow \alpha=4$

25. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors such that $|\vec{a}| = \sqrt{3}, |\vec{b}| = 5, \vec{b} . \vec{c} = 10$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{3}$.

If $\vec{a}$ is perpendicular to vector $\vec{b} \times \vec{c}$, then $|\vec{a} \times (\vec{b} \times \vec{c})|$ is equal to

Accepted Answers

30 30.0 30.00

Solution:

$|\vec{a} \times (\vec{b} \times \vec{c})| = |\vec{a}||\vec{b} \times \vec{c}| \sin\theta$ where θ is angle between $\vec{a}$ and $\vec{b} \times \vec{c}$

$\theta = \frac{\pi}{2}$ (given)

$\Rightarrow |\vec{a} \times (\vec{b} \times \vec{c})| = \sqrt{3}|\vec{b} \times \vec{c}| \sin\frac{\pi}{2}$

$\Rightarrow |\vec{a} \times (\vec{b} \times \vec{c})| = \sqrt{3}|\vec{b}||\vec{c}| \sin\frac{\pi}{3}$

$\Rightarrow |\vec{a} \times (\vec{b} \times \vec{c})| = \sqrt{3} \times 5 \times |\vec{c}| \times \frac{\sqrt{3}}{2}$

$\Rightarrow |\vec{a} \times (\vec{b} \times \vec{c})| = \frac{15}{2}|\vec{c}|$

Now, $|\vec{b}||\vec{c}| \cos\theta = 10$

$\Rightarrow 5|\vec{c}|\frac{1}{2} = 10$

$\Rightarrow |\vec{c}| = 4$

$\Rightarrow |\vec{a} \times (\vec{b} \times \vec{c})| = 30$

26. If the vector $\overrightarrow{p}=(a+1)\hat{i}+a\hat{j}+a\hat{k}, \overrightarrow{q}=a\hat{i}+(a+1)\hat{j}+a\hat{k}$ and $\overrightarrow{r}=a\hat{i}+a\hat{j}+(a+1)\hat{k},\ (a\in R)$ are coplanar and $3(\overrightarrow{p}\,.\,\overrightarrow{q})^2-\lambda|\overrightarrow{r}\times\overrightarrow{q}|^2=0$, then value of` λ is

Accepted Answers

1 1.0 1.00

Solution:

As $\vec{p}, \vec{q}, \vec{r}$ are coplanar,

$$\begin{vmatrix} a+1 & a & a \\ a & a+1 & a \\ a & a & a+1 \end{vmatrix} = 0$$

$R_1 \to R_1 + R_2 + R_3$

$$\Rightarrow \begin{vmatrix} 3a+1 & 3a+1 & 3a+1 \\ a & a+1 & a \\ a & a & a+1 \end{vmatrix} = 0$$

$$\Rightarrow (3a+1)\begin{vmatrix} 1 & 1 & 1 \\ a & a+1 & a \\ a & a & a+1 \end{vmatrix} = 0$$

$C_2 \to C_2 - C_1\ ,\ C_3 \to C_3 - C_1$

$$\Rightarrow (3a+1)\begin{vmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ a & 0 & 1 \end{vmatrix} = 0$$

$\Rightarrow 3a + 1 = 0$

$\Rightarrow a = -\dfrac{1}{3}$

$\vec{p} = \dfrac{1}{3}(2\hat{i} - \hat{j} - \hat{k}), \vec{q} = \dfrac{1}{3}(-\hat{i} + 2\hat{j} - \hat{k}),$

$\vec{r} = \dfrac{1}{3}(-\hat{i} - \hat{j} + 2\hat{k})$

$$\vec{r} \times \vec{q} = \frac{1}{9}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -1 & 2 \\ -1 & 2 & -1 \end{vmatrix}$$

$$\begin{aligned} \vec{r} \times \vec{q} &= \frac{1}{9}(-3\hat{i} - 3\hat{j} - 3\hat{k}) \\ &= -\frac{1}{3}(\hat{i} + \hat{j} + \hat{k}) \end{aligned}$$

$|\vec{r} \times \vec{q}|^2 = \dfrac{1}{3}$

$\vec{p}\,.\,\vec{q} = \dfrac{1}{9}(-2 - 2 + 1) = -\dfrac{1}{3}$

$3(\vec{p}\,.\,\vec{q})^2 - \lambda|\vec{r} \times \vec{q}|^2 = 0$

$\Rightarrow \dfrac{1}{3} - \lambda \times \dfrac{1}{3} = 0$

$\Rightarrow \lambda = 1$

27. Let P be a plane containing the line $\frac{x-1}{3}=\frac{y+6}{4}=\frac{z+5}{2}$ and parallel to the line $\frac{x-3}{4}=\frac{y-2}{-3}=\frac{z+5}{7}$. If the point $(1,-1,\alpha)$ lies on the plane P, then the value of $|5\alpha|$ is equal to

Accepted Answers

38.0 38.00 38

Solution:

Let $L_1 : \frac{x-1}{3}=\frac{y+6}{4}=\frac{z+5}{2}$ and $L_2 : \frac{x-3}{4}=\frac{y-2}{-3}=\frac{z+5}{7}$

The plane contains the points
$(1,-6,-5)$ and $(1,-1,\alpha)$
Vector joining both points is
$\vec{V}=5\hat{j}+(\alpha+5)\hat{k}$
So, $\vec{V}\cdot\vec{n}=0$, where $\vec{n}$ is normal vector to the plane
Now,

$$\begin{vmatrix} 0 & 5 & \alpha+5 \\ 3 & 4 & 2 \\ 4 & -3 & 7 \end{vmatrix}=0$$

$\Rightarrow -5(13)+(\alpha+5)(-25)=0$

$\Rightarrow -13-25-5a=0$

$\Rightarrow 5a=-38$

$\therefore |5a|=38$

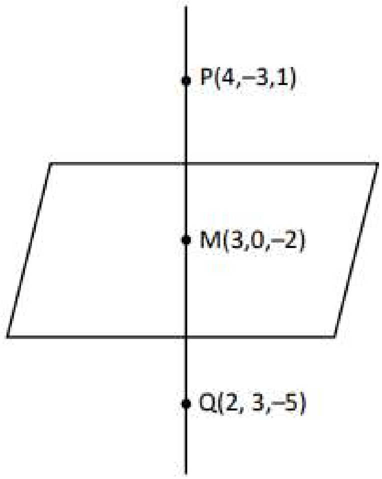
28. Let the plane $ax + by + cz + d = 0$ bisect the line joining the points $(4, -3, 1)$ and $(2, 3, -5)$ at the right angles. If a, b, c, d are integers, then the minimum value of $(a^2 + b^2 + c^2 + d^2)$ is

Accepted Answers

28 28.0 28.00

Solution:

Normal of plane $= \overrightarrow{PQ} = -2\hat{i} + 6\hat{j} - 6\hat{k}$
$a = -2, b = 6, c = -6$
Equation of plane is
$-2x + 6y - 6z + d = 0$



Mid point of $P(4, -3, 1)$ and $Q(2, 3, -5)$ is $M(3, 0, -2)$.
Since, plane passes through M,
$\therefore d = -6$
So, equation of plane is
$-2x + 6y - 6z - 6 = 0$
$\Rightarrow x - 3y + 3z + 3 = 0$
$(a^2 + b^2 + c^2 + d^2)_{\min} = 1^2 + 9 + 9 + 9 = 28$

29. Let the line L be the projection of the line $\frac{x-1}{2}=\frac{y-3}{1}=\frac{z-4}{2}$ in the plane $x-2y-z=3$. If d is the distance of the point $(0,0,6)$ from L, then d^2 is equal to

Accepted Answers

26 26.0 26.00

Solution:

$L: \frac{x-1}{2}=\frac{y-3}{1}=\frac{z-4}{2}$
$P_1: x-2y-z=3$
Equation of a plane P_2 which contains L and perpendicular to P_1,
$\begin{vmatrix} x-1 & y-3 & z-4 \\ 1 & -2 & -1 \\ 2 & 1 & 2 \end{vmatrix}=0$
$\Rightarrow 3x+4y-5z+5=0$
Distances of point $(0,0,6)$ from P_1 and P_2 are
$d_1=\frac{25}{\sqrt{50}}=\sqrt{\frac{25}{2}}$ and $d_2=\frac{9}{\sqrt{6}}=\sqrt{\frac{27}{2}}$
$\therefore d^2=d_1^2+d_2^2=26$

30. If the projection of the vector $\hat{i}+2\hat{j}+\hat{k}$ on the sum of the two vectors $2\hat{i}+4\hat{j}-5\hat{k}$ and $-\lambda\hat{i}+2\hat{j}+3\hat{k}$ is 1, then λ is equal to

Accepted Answers

5 5.0 5.00 05

Solution:

$\overrightarrow{v_1}=\hat{i}+2\hat{j}+\hat{k}$,

$\overrightarrow{v_2}=2\hat{i}+4\hat{j}-5\hat{k}$,

$\overrightarrow{v_3}=-\lambda\hat{i}+2\hat{j}+3\hat{k}$
$\overrightarrow{v_2}+\overrightarrow{v_3}=(2-\lambda)\hat{i}+6\hat{j}-2\hat{k}=\overrightarrow{v_4}$

Projection of $\overrightarrow{v_1}$ on $\overrightarrow{v_4}=\overrightarrow{v_1}\cdot\frac{\overrightarrow{v_4}}{|\overrightarrow{v_4}|}$
$\Rightarrow \frac{1\times(2-\lambda)+2\times 6+1\times(-2)}{\sqrt{(2-\lambda)^2+6^2+(-2)^2}}=1$
$\Rightarrow (12-\lambda)^2=(2-\lambda)^2+40$
On solving, we get
$\lambda=5$