Subject: Mathematics

1. A man is walking on a straight line. The arithmetic mean of the reciprocals of the intercepts of this line on the coordinate axes is $\frac{1}{4}$. Three stones A, B and C are placed at the points $(1,1), (2,2)$ and $(4,4)$ respectively. Then which of these stones is/are on the path of the man?

   A. B only

   B. A only

   C. the three

   D. C only

2. In a triangle PQR, the co-ordinates of the points P and Q are $(-2,4)$ and $(4,-2)$ respectively. If the equation of the perpendicular bisector of PR is $2x - y + 2 = 0$, then the centre of the circumcircle of the ΔPQR is:

   A. $(-2,-2)$

   B. $(0,2)$

   C. $(-1,0)$

   D. $(1,4)$

3. Let $A(-1,1), B(3,4)$ and $C(2,0)$ be given three points. A line $y = mx, m > 0$, intersects lines AC and BC at point P and Q respectively. Let A_1 and A_2 be the areas of ΔABC and ΔPQC respectively, such that $A_1 = 3A_2$, then the value of m is equal to :

   A. $\frac{4}{15}$

   B. 1

   C. 2

   D. 3

4. The number of integral values of m so that the abscissa of point of intersection of lines $3x + 4y = 9$ and $y = mx + 1$ is also an integer, is:

   A. 3

   B. 2

   C. 1

   D. 0

5. If C be the centroid of the triangle having vertices $(3, -1), (1, 3)$ and $(2, 4)$. Let P be the point of intersection of the lines $x + 3y - 1 = 0$ and $3x - y + 1 = 0$, then the line passing through the points C and P also passes through the point :

   A. $(-9, -7)$

   B. $(-9, -6)$

   C. $(7, 6)$

   D. $(9, 7)$

6. The locus of the mid-points of the perpendiculars drawn from points on the line, $x = 2y$ to the line $x = y$ is :

   A. $2x - 3y = 0$

   B. $3x - 2y = 0$

   C. $5x - 7y = 0$

   D. $7x - 5y = 0$

7. Two sides of a parallelogram are along the lines $4x + 5y = 0$ and $7x + 2y = 0$. If the equation of one of the diagonals of the parallelogram is $11x + 7y = 9$, then other diagonal passes through the point

A. $(2, 2)$

B. $(2, 1)$

C. $(1, 3)$

D. $(1, 2)$

8. Let a, b, c be in arithmetic progression. Let the centroid of the triangle with vertices (a, c), $(2, b)$ and (a, b) be $\left(\frac{10}{3}, \frac{7}{3}\right)$. If α, β are the roots of the equation $ax^2 + bx + 1 = 0$, then the value of $\alpha^2 + \beta^2 - \alpha\beta$ is

A. $\frac{71}{256}$

B. $-\frac{69}{256}$

C. $\frac{69}{256}$

D. $-\frac{71}{256}$

9. Let $A(a, 0), B(b, 2b + 1)$ and $C(0, b),\ b \neq 0, |b| \neq 1$, be points such that the area of triangle ABC is 1 sq. unit, then the sum of all possible values of a is

A. $\frac{2b}{b+1}$

B. $\frac{-2b^2}{b+1}$

C. $\frac{2b^2}{b+1}$

D. $\frac{-2b}{b+1}$

10. Let the centroid of an equilateral triangle ABC be at the origin. Let one of the sides of the equilateral triangle be along the straight line $x + y = 3$. If R and r be the radius of circumcircle and incircle respectively of $\triangle ABC$, then $(R + r)$ is equal to

A. $2\sqrt{2}$

B. $3\sqrt{2}$

C. $7\sqrt{2}$

D. $\frac{9}{\sqrt{2}}$

11. The image of the point $(3, 5)$ in the line $x - y + 1 = 0$, lies on :

A. $(x-2)^2 + (y-4)^2 = 4$

B. $(x-4)^2 + (y+2)^2 = 16$

C. $(x-4)^2 + (y-4)^2 = 8$

D. $(x-2)^2 + (y-2)^2 = 12$

12. Let the equations of two sides of a triangle be $3x - 2y + 6 = 0$ and $4x + 5y - 20 = 0$. If the orthocentre of this triangle is at $(1, 1)$, then the equation of its third side is :

A. $122y - 26x - 1675 = 0$

B. $26x - 122y - 1675 = 0$

C. $26x + 61y + 1675 = 0$

D. $122y + 26x + 1675 = 0$

13. A point P moves on the line $2x - 3y + 4 = 0$. If $Q(1, 4)$ and $R(3, -2)$ are fixed points, then the locus of the centroid of ΔPQR is a line :

A. parallel to x-axis

B. parallel to y-axis

C. with slope $\frac{3}{2}$

D. with slope $\frac{2}{3}$

14. A triangle ABC lying in the first quadrant has two vertices as $A(1, 2)$ and $B(3, 1)$. If $\angle BAC = 90^\circ$, and $ar(\Delta ABC) = 5\sqrt{5}$ sq. units, then the abscissa of the vertex C is

A. $1 + \sqrt{5}$

B. $1 + 2\sqrt{5}$

C. $2\sqrt{5} - 1$

D. $2 + \sqrt{5}$

15. A straight line L at a distance of 4 units from the origin makes positive intercepts on the coordinate axes and the perpendicular from the origin to this line makes an angle of 60° with the line $x + y = 0$. Then an equation of the line L is :

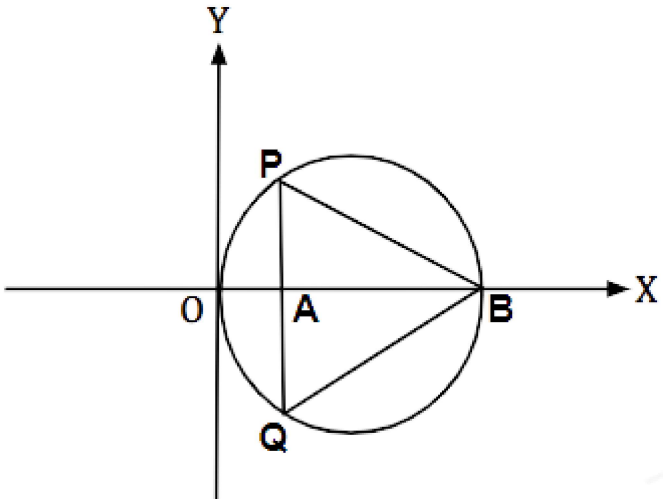
A. $(\sqrt{3} + 1)x + (\sqrt{3} - 1)y = 8\sqrt{2}$

B. $(\sqrt{3} - 1)x + (\sqrt{3} + 1)y = 8\sqrt{2}$

C. $\sqrt{3}x + y = 8$

D. $x + \sqrt{3}y = 8$

16. In the circle given below, let $OA = 1$ unit, $OB = 13$ unit and $PQ \perp OB$. Then, the area of the triangle PQB (in square units) is :



A. $26\sqrt{3}$

B. $24\sqrt{2}$

C. $24\sqrt{3}$

D. $26\sqrt{2}$

17. Let $A(1, 4)$ and $B(1, -5)$ be two points. Let P be a point on the circle $(x-1)^2 + (y-1)^2 = 1$ such that $(PA)^2 + (PB)^2$ have maximum value, then the points P, A and B lie on:

A. a parabola

B. a straight line

C. a hyperbola

D. an ellipse

18. Let r_1 and r_2 be the radii of the largest and smallest circles, respectively, which pass through the point $(-4, 1)$ and having their centres on the circumference of the circle $x^2 + y^2 + 2x + 4y - 4 = 0$. If $\frac{r_1}{r_2} = a + b\sqrt{2}$, then $a + b$ is equal to:

A. 3

B. 7

C. 11

D. 5

19. Let the lengths of intercepts on x-axis and y-axis made by the circle $x^2 + y^2 + ax + 2ay + c = 0$, $(a < 0)$ be $2\sqrt{2}$ and $2\sqrt{5}$, respectively. Then the shortest distance from origin to a tangent to this circle which is perpendicular to the line $x + 2y = 0$, is equal to :

A. $\sqrt{10}$

B. $\sqrt{6}$

C. $\sqrt{11}$

D. $\sqrt{7}$

20. Let the tangent to the circle $x^2 + y^2 = 25$ at the point $R(3,\ 4)$ meet $x-$axis and $y-$axis at points P and Q, respectively. If r is the radius of the circle passing through the origin O and having centre at the incentre of the triangle OPQ, then r^2 is equal to :

A. $\frac{625}{72}$

B. $\frac{585}{66}$

C. $\frac{125}{72}$

D. $\frac{529}{64}$

21. Choose the correct statement about two circles whose equations are given below:
$x^2 + y^2 - 10x - 10y + 41 = 0$
$x^2 + y^2 - 22x - 10y + 137 = 0$

A. circles have no meeting point

B. circles have two meeting point

C. circles have only one meeting point

D. circles have same centre

22. Let $S_1 : x^2 + y^2 = 9$ and $S_2 : (x-2)^2 + y^2 = 1$. Then the locus of center of a variable circle S which touches S_1 internally and S_2 externally always passes through the points:

A. $\left(\frac{1}{2}, \pm\frac{\sqrt{5}}{2}\right)$

B. $\left(2, \pm\frac{3}{2}\right)$

C. $(1, \pm 2)$

D. $\left(0, \pm\sqrt{3}\right)$

23. Choose the incorrect statement about the two circles whose equations are given below:
$x^2 + y^2 - 10x - 10y + 41 = 0$ and $x^2 + y^2 - 16x - 10y + 80 = 0$

A. Distance between two centres is the average of radii of both the circles.

B. Circles have two intersection points.

C. Both circles centres lie inside region of one another.

D. Both circles pass through the centre of each other.

24. Let $A = \{(x, y) \in \mathbb{R} \times \mathbb{R} | 2x^2 + 2y^2 - 2x - 2y = 1\}$,
$B = \{(x, y) \in \mathbb{R} \times \mathbb{R} | 4x^2 + 4y^2 - 16y + 7 = 0\}$ and
$C = \{(x, y) \in \mathbb{R} \times \mathbb{R} | x^2 + y^2 - 4x - 2y + 5 \leq r^2\}$.
Then the minimum value of $|r|$ such that $A \cup B \subseteq C$ is equal to:

A. $\frac{2 + \sqrt{10}}{2}$

B. $\frac{3 + 2\sqrt{5}}{2}$

C. $1 + \sqrt{5}$

D. $\frac{3 + \sqrt{10}}{2}$

25. A circle C touches the line $x = 2y$ at the point $(2, 1)$ and intersects the circle $C_1 : x^2 + y^2 + 2y - 5 = 0$ at two points P and Q such that PQ is a diameter of C_1. Then the diameter of C is

A. $\sqrt{285}$

B. 15

C. $4\sqrt{15}$

D. $7\sqrt{5}$

26. If $y + 3x = 0$ is the equation of a chord of the circle, $x^2 + y^2 - 30x = 0$, then the equation of the circle with this chord as diameter is

A. $x^2 + y^2 - 3x - 9y = 0$

B. $x^2 + y^2 + 3x + 9y = 0$

C. $x^2 + y^2 - 3x + 9y = 0$

D. $x^2 + y^2 + 3x - 9y = 0$

Subject: Mathematics

1. Let $\tan\alpha, \tan\beta, \tan\gamma; \alpha, \beta, \gamma \neq \frac{(2n-1)\pi}{2}, n \in \mathbb{N}$ be the slopes of three line segment OA, OB and OC, respectively, where O is origin. If the circumcentre of $\triangle ABC$ coincides with origin and its orthocentre lies on $y-$ axis, then the value of $\left(\frac{\cos 3\alpha + \cos 3\beta + \cos 3\gamma}{\cos\alpha\cos\beta\cos\gamma}\right)^2$ is equal to

2. Let a point P be such that its distance from the point $(5, 0)$ is thrice the distance of P from the point $(-5, 0)$. If the locus of the point P is a circle of radius r, then $4r^2$ is equal to

3. Let $ABCD$ be a square of side of unit length. Let a circle C_1 centered at A with unit radius is drawn. Another circle C_2 which touches C_1 and the lines AD and AB are tangent to it, is also drawn. Let a tangent line from the point C to the circle C_2 meet the side AB at E. If the length of EB is $\alpha + \sqrt{3}\beta$ where α, β are integers, then $\alpha + \beta$ is equal to

4. Let B be the centre of the circle $x^2 + y^2 - 2x + 4y + 1 = 0$. Let the tangents at two points P and Q on the circle intersect at the point $A(3, 1)$. Then $8 \cdot \left(\frac{\text{area}\triangle APQ}{\text{area}\triangle BPQ}\right)$ is equal to