

Linear Equations In One Variable Class 8 Notes- Chapter 2

Introduction to Linear Equations in One Variable

The Linear equation in one variable is an equation, which consists of only one variable, and the highest power of the variables used in the equations is 1. The solution to this linear equation can be any rational number. This equation may consist expressions which are linear on both sides of the equal to sign.

Just like numbers, we can also transpose the variables from one side of the equation to the other side. The simplification of the equations which was formed by expressions and this can be done by bringing the equation into a linear form by equating the expression by multiplication using suitable techniques. Utilization of linear equation can be seen in diverse scenarios such as problems on numbers, perimeter, ages, currency, and even algebra has linear equations applications.

Variables and Constants

A **constant** is a value or number that never changes in an expression and it's constantly the same.

A **variable** is a letter representing some unknown value. Its value is not fixed, and it can take any value. On the other hand, the value of a constant is fixed.

For example, in the expression $4x+7$, 4 and 7 are the constants and x is a variable.

Algebraic Equation

The statement of **equality** of two **algebraic expressions** is an **algebraic equation**. It is of the form $P=Q$, where P and Q are algebraic expressions.

$6x + 5$ and $5x + 3$ are algebraic expressions. On equating the algebraic expressions we get an algebraic equation.

$6x + 5 = 5x + 3$ is an algebraic equation.

Linear Equations in One Variable

A **linear equation** is an algebraic equation in which each term is either a **constant** or the product of a **constant** and a **single variable**, where the highest power of the variable is **one**.

If the linear equation has only a **single variable** then it is called a **linear equation** in one variable.

For example, $7x + 4 = 5x + 8$ is a linear equation in one variable.

Solving Linear Equations

Performing Mathematical Operations on Equations

When we are doing mathematical operations on a linear equation, we should do it on both sides of the equality otherwise the equality won't hold true.

Suppose, $4x + 3 = 3x + 7$ is a linear equation. If we want to subtract 3 from the given equation, then we do it on both sides of the equality, so that the equality holds true.

$$4x+3-3=3x+7-3$$

$$\Rightarrow 4x=3x+4$$

Similarly, if we want to multiply or divide the equation, we multiply or divide all the terms on the left side of the equality and to the right side of the equality by the given number.

Note: we can not multiply or divide the equation by 0.

Solving Equations with Linear Expression on one side and numbers on the other Side

Suppose we have to find the solution of $2x-3=7$, where the linear expression is on the left-hand side, and numbers on the right-hand side.

Step 1: Transpose all the constant terms from the left-hand side to the right-hand side.

$$2x-7+3=10 \Rightarrow 2x=10$$

Step 2: Divide both sides of the equation by the coefficient of the variable.

In the above equation $2x$ is on the left-hand side. The coefficient of $2x$ is 2.

On dividing the equation by two, We get:

$$(\frac{1}{2}) \times (2x) = (\frac{1}{2}) \times (10)$$

It can be written as:

$$2x/2 = 10/2$$

$x = 5$, which is the required solution for the given linear equation in one variable $2x - 3 = 7$.

Let us consider an another example, $2x + 4 = 12$

Now, keep the term $2x$ on the left hand side and bring 4 on the right hand side of the equation.

So, we get

$$2x = 12 - 4$$

$$2x = 8$$

$$x = 8/2$$

$$x = 4$$

Therefore, 4 is the required solution for the linear equation $2x + 4 = 12$.

Solving Equations with variables on both sides

Suppose we have to solve $3x - 3 = x + 2$. In this equation, there are variables on both sides of the equation.

Step 1: Transpose all the terms with a variable from the right-hand side to the left-hand side of the equation and all the constants from the left-hand side to the right-hand side of the equation.

$$3x-x=2+3$$

$$\Rightarrow 2x=5$$

Step 2: Divide both sides of the equation by the coefficient of the variable.

$$(\frac{1}{2}) \times (2x) = (\frac{1}{2}) \times 5$$

We can write the above equation as follows:

$$2x/2 = 5/2$$

Now, cancel out 2 on the left hand side of the equation, we get

$x = 5/2$, which is the required solution.

Applications (Word Problems)

Question:

The sum of two numbers is 74. If one of the numbers is 10 more than the other number, find the two

numbers.

Solution:

Let one of the numbers be x .

Then the other number is $x + 10$.

Given that the sum of the two numbers is 74.

$$\text{So, } x + (x + 10) = 74$$

$$\Rightarrow 2x + 10 = 74$$

$$\Rightarrow 2x = 74 - 10 = 64$$

$$x = 64/2$$

$$x = 32$$

Therefore, one number is 32.

As given, the one of the numbers is 10 more than the other, we get $x + 10$

$$= 32 + 10$$

$$= 42.$$

Therefore, the two numbers are 32 and 42.

Equations Reducible to the Linear Form

Question:

$$(x+1)/(2x+3) = \frac{3}{8}$$

Solution:

$$\text{Given equation: } (x+1)/(2x+3) = \frac{3}{8}$$

Now, cross multiply the equation, we get

$$8(x+1) = 3(2x+3)$$

$$8x + 8 = 6x + 9$$

Now, bring the variables on one side and constants on the other side,

$$8x - 6x = 9 - 8$$

On simplifying the above equation,

$$2x = 1$$

$$x = \frac{1}{2}.$$

Now, this can be written as:

$$x - (\frac{1}{2}) = 0$$

Reducing Equations to Simpler Form

Question:

Simplify the equation: $[(6x+1)/3] + 1 = (x-3)/6$. Also, justify your answer.

Solution:

Given Equation: $[(6x + 1)/3] + 1 = (x - 3)/6$

To simplify the given equation, take the LCM of 3 and 6, which is 6.

Now, multiply both sides of the equation by 6, we get

$$[6(6x + 1)/3] + 1(6) = 6(x - 3)/6$$

$$2(6x+1) + 6 = x - 3$$

$$12x + 2 + 6 = x - 3$$

Now, bring all the terms with variables on one sides and constants on the other side.

So, we get

$$12x + 8 = x - 3$$

$$12x - x = -3 - 8$$

$$11x = -11$$

$$x = -11/11$$

$$x = -1$$

Hence, the simplification of $[(6x + 1)/3] + 1 = (x - 3)/6$ is $x = -1$, which is the required solution.

Justification:

Now, substitute $x = -1$ in the given equation.

$$\text{LHS: } [(6x + 1)/3] + 1$$

$$\text{LHS} = [(6(-1) + 1)/3] + 1$$

$$\text{LHS} = [(-6 + 1)/3] + 1$$

$$\text{LHS} = (-5/3) + 1$$

$$\text{LHS} = (-5 + 3) / 3$$

$$\text{LHS} = -2/3$$

Now, take the expression on the right side of the given equation.

$$\text{RHS} = (x - 3)/6$$

$$\text{RHS} = (-1 - 3)/6$$

$$\text{RHS} = -4/6$$

$$\text{RHS} = -2/3$$

Therefore, $\text{LHS} = \text{RHS}$.

Hence, verified.