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Class 8 Chapter 6 Square and Square Roots

Introduction to Square Numbers

If a natural number *m* can be expressed as n^2, where n is also a natural number, then *m* is a square number. Example: 1, 4, 9, 16 and 25.

Finding the Square of a Number

If *n* is a number, then its square is given as $n \times n = n^2$.
For example: Square of 5 is equal to $5 \times 5 = 25$

Properties of Square Numbers

Properties of square numbers are:

- If a number has 0, 1, 4, 5, 6 or 9 in the unit's place, then it may or may not be a square number. If a number has 2, 3, 7 or 8 in its units place then it is not a square number.
- If a number has 1 or 9 in unit's place, then it's square ends in 1.
- If a square number ends in 6, the number whose square it is, will have either 4 or 6 in unit's place.

- For example, consider the number 64. The unit digit of 64 is 4 and it is a square number. Because the square of 8 is 64 and 64 is considered to be a square number.
- Consider a number 11 (i.e., the unit's place of 11 is 1). Thus, the square of 11 is 121. Hence, square number 121 also has 1 in the unit's place.

Finding square of a number with unit's place 5

The square of a number **N5** is equal to $(N(N+1)) \times 100 + 25$, where **N** can have one or more than one digits.
For example: If N = 1, then $15^2 = (1 \times 2) \times 100 + 25 = 200 + 25 = 225$
If N = 20, then $205^2 = (20 \times 21) \times 100 + 25 = 42000 + 25 = 42025$

Perfect Squares

A number which is obtained from square of the other number is called perfect squares. For example, 81 is a perfect square number, which is obtained by taking the square of the number 9.

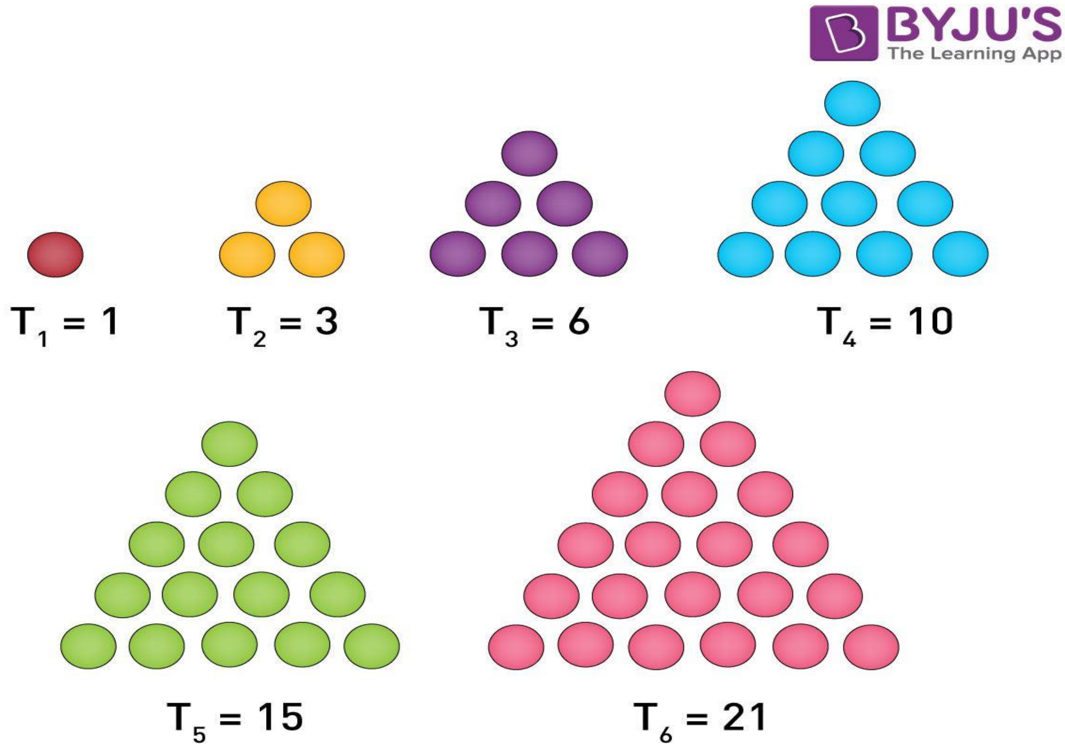
Interesting Patterns

There exists interesting patterns in:

- Adding triangular numbers
- Numbers between square numbers
- Adding odd numbers
- A sum of consecutive natural numbers
- Product of two consecutive even or odd natural numbers

Adding Triangular Numbers

Triangular numbers: It is a sequence of the numbers 1, 3, 6, 10, 15 etc. It is obtained by continued summation of the natural numbers. The **dot pattern** of a triangular number can be arranged as **triangles**.



If we add two consecutive triangular numbers, we get a square number.
Example: $1+3=4=2^2$ and $3+6=9=3^2$.

Numbers between Square Numbers

There are **2n** non-perfect square numbers between squares of the numbers *n* and (*n* + 1), where *n* is any natural number.

Example:

- There are two non-perfect square numbers (2, 3) between $1^2=1$ and $2^2=4$.
- There are four non-perfect square numbers (5, 6, 7, 8) between $2^2=4$ and $3^2=9$.

Addition of Odd Numbers

The sum of first n **odd natural numbers** is n^2.
Example:
$1+3=4=2^2$
$1+3+5=9=3^2$

Square of an odd number as a sum

Square of an odd number *n* can be expressed as sum of two consecutive positive integers

$(n^2-1)/2$ and $(n^2+1)/2$
.
For example: $3^2=9=4+5=$
$(3^2-1)/2 + (3^2+1)/2$

Similarly, $5^2=25=12+13=$
$(5^2-1)/2 + (5^2+1)/2$

Product of Two Consecutive Even or Odd Natural Numbers

The product of two even or odd natural number can be calculated as, $(a+1)\times(a-1)=(a^2-1)$, where *a* is a natural number, and $a-1$, $a+1$, are the consecutive odd or even numbers.
For example:
$11\times13=(12-1)\times(12+1)=12^2-1=144-1=143$

Random Interesting Patterns Followed by Square Numbers

Patterns in numbers like 1, 11, 111, … :
$1^2=$ 1
$11^2=$ 1 2 1
$111^2=$ 1 2 3 2 1
$1111^2=$ 1 2 3 4 3 2 1
$11111^2=$ 1 2 3 4 5 4 3 2 1
$11111111^2=$1 2 3 4 5 6 7 8 7 6 5 4 3 2 1
Patterns in numbers like 6, 67, 667, … :
$7^2=49$
$67^2=4489$
$667^2=444889$
$6667^2=44448889$
$66667^2=4444488889$
$666667^2=444444888889$

Pythagorean Triplets

For any natural number $m>1$, we have $(2m)^2+(m^2-1)^2=(m^2+1)^2$.
$2m$, (m^2-1) and (m^2+1) forms a Pythagorean triplet.
For $m=2$, $2m=4$, $m^2-1=3$ and $m^2+1=5$.
So, 3, 4, 5 is the required Pythagorean triplet.

Square Roots

Square Root of a Number

Finding the number **whose square is known** is known as finding the **square root**. Finding square root is **inverse operation** of finding the square of a number.
For example:
$1^2=1$, square root of 1 is 1.

$2^2=4$, square root of 4 is 2.
$3^2=9$, square root of 9 is 3.

Estimating the number of digits in the square root of a number

If a perfect square has *n* digits, then its square root will have n/2 digits if *n* is **even** and (n+1)/2 digits if *n* is **odd**.
For example: 100 has 3 digits, and its square root(10) has (3+1)/2 =2 digits.

Estimating Square Roots

Estimating the square root of 247:
Since: 100 < 247 < 400
i.e. $10<\sqrt{247}<20$
But it is not very close.
Also, $15^2=225<247$ and $16^2=256>247$
$15<\sqrt{247}<16$.
256 is much closer to 247 than 225.
Therefore, $\sqrt{247}$ is approximately equal to 16.

Finding square of a number using identity

Squares of numbers having two or more digits can easily be found by writing the number as the sum of two numbers.
For example:
$23^2=(20+3)^2=20(20+3)+3(20+3)$
$=20^2+20\times3+20\times3+3^2$
$=400+60+60+9$
$=529$

Square Roots of Perfect Squares

Finding square root through repeated subtraction

Every square number can be expressed as a sum of successive odd natural numbers starting from one.

The square root can be found through repeated subtraction. To find the square root of a number n:
Step 1: subtract successive odd numbers starting from one.
Step 2: stop when you get zero.

The number of successive odd numbers that are subtracted gives the square root of that number. Suppose we want to find the square root of 36.

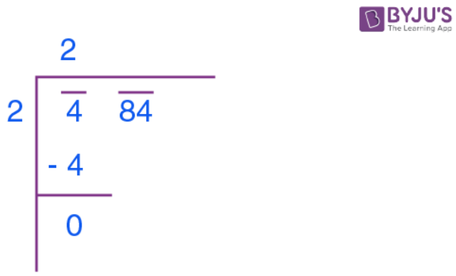
- $36-1=35$
- $35-3=32$
- $32-5=27$
- $27-7=20$
- $20-9=11$
- $11-11=0$

Here 6 odd numbers (1, 3, 5, 7, 9, 11) are subtracted to from 36 to get 0.
So, the square root of 36 is 6.

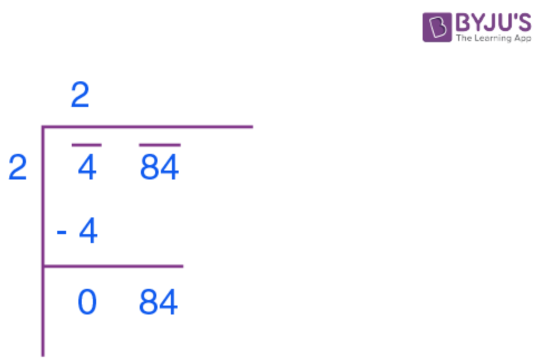
Square Roots – Generalised

Finding Square Root by Long Division Method

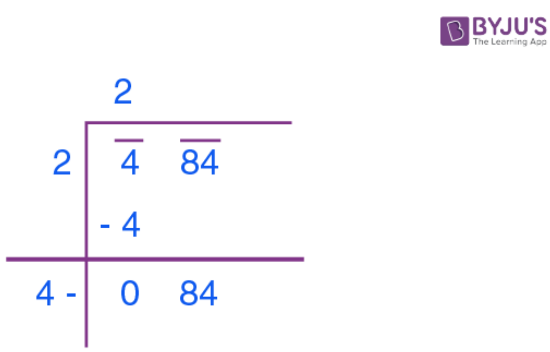
Steps involved in finding the square root of 484 by Long division method:
Step 1: Place a bar over every pair of numbers starting from the digit at units place. If the number of digits in it is odd, then the left-most single-digit too will have a bar.
Step 2: Take the largest number as divisor whose square is less than or equal to the number on the extreme left. Divide and write quotient.



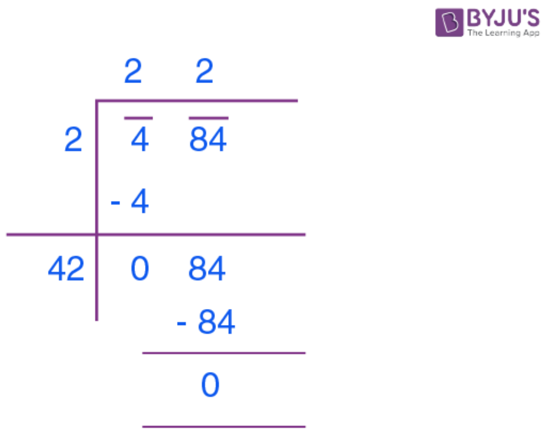
Step 3: Bring down the number which is under the next bar to the right side of the remainder.



Step 4: Double the value of the quotient and enter it with a blank on the right side.



Step 5: Guess the largest possible digit to fill the blank which will also become the new digit in the quotient, such that when the new divisor is multiplied to the new
quotient the product is less than or equal to the dividend.



The remainder is 0, therefore, $\sqrt{\boxed{484}}$=22.

Be More Curious!

Finding Pythagorean Triplets for Any Given Number

If we are given any member of a Pythagorean triplet, then we can find the Pythagorean triplet by using general form $2m$, $m^2–1$, m^2+1.
For example, If we want to find the Pythagorean triplet whose smallest number is 8.
Let, $m^2−1=8 \Rightarrow m=3$
$2m=6$ and $m^2+1=10$
The triplet is 6, 8 and 10.
But 8 is not the smallest number of this triplet.
So, we substitute $2m=8 \Rightarrow m=4$
$m^2−1=15$ and $m^2+1=17$.
Therefore, 8, 15, 17 is the required triplet.