Cubes and Cube Roots Class 8 Notes: Chapter 7

Cubes and Cube Roots Class 8 Notes given here come in an easy to understand format and students can quickly go through all the topics given in chapter 7. The notes will help students to understand and remember the concepts better and prepare well for the exams.

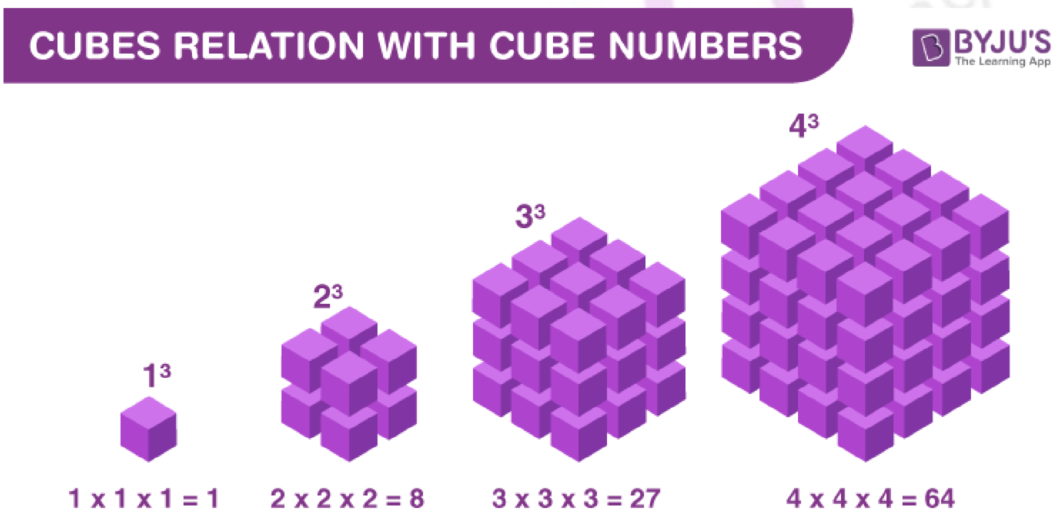
Introduction to Cube Numbers

Cube Numbers

If a natural number *m* can be expressed as n^3, where *n* is also a natural number, then *m* is called the cube number of *n*.
Numbers like 1, 8, 27 are cube number of the numbers 1, 2, and 3 respectively.
All perfect cube numbers are obtained by multiplying a number by itself three times.

Cubes Relation with Cube Numbers

In geometry, a cube is a solid figure where all edges are **equal** and are **perpendicular** to each other.



For example, take a cube of unit side. If we arrange these cubes to form a bigger cube of side 3 units, we find that there are a total of 27 such unit cubes that make up a cube of 3 units. Similarly, a cube of 4 units will have 64 such unit cubes.

Units Digits in Cube Numbers

Depending on whether a number is odd or even, its cube number is also odd or even respectively.
This is determined by the nature of the cube numbers' unit digit.

- If a number is odd, its cube numbers' unit digit is also odd.

- If a number is even, its cube numbers' unit digit is also even.

The table below shows the units digit of a number and the unit digit of the cube of that number:

Units digit of number	Units digit of its cube
1	1
2	8
3	7
4	4
5	5
6	6
7	3
8	2
9	9

Inside Cube Numbers

Adding Consecutive Odd Numbers

$1 = 1 = 1^3$

$3 + 5 = 8 = 2^3$

$7 + 9 + 11 = 27 = 3^3$

$13 + 15 + 17 + 19 = 64 = 4^3$

$21 + 23 + 25 + 27 + 29 = 125 = 5^3$

Thus, from the above pattern, if we need to find the n^3, n consecutive odd numbers will be needed, such that their sum is equal to n^3.
This pattern holds true for all natural numbers.
Also, if we need to find n^3 then we should add n consecutive natural numbers starting from ({[(n-1)n]/2}/1)th odd natural number.

Prime Factorisation Method to Find a Cube

In the **prime factorisation** of any number, if **each prime factor** appears **three times**, then the number is a **perfect cube**.

Consider, the number 216. By prime factorisation,
$216=2\times2\times2\times3\times3\times3=2^3\times3^3=6^3$
Hence, 216 is a perfect cube.

Consider, the number 500. By prime factorisation,
$500=2\times2\times5\times5\times5=2^2\times5^3$
In the above prime factorisation 2 appears twice.
Hence, 500 is not a perfect cube.

Cube roots

Finding the **cube root** is the **inverse operation** of finding the **cube**.
We know that $3^3=27$. We can also write the same equation as $\sqrt[3]{27}=3$. The symbol '$\sqrt[3]{}$' denotes '**cube root**'.

Smallest Multiple that is a Perfect Cube

Consider an example: 53240.

Now, we have to check whether the given number 53240 is a perfect cube or not.

So, to find whether the given number is a perfect cube or not, first we have to find the prime factorisation of 53240.

Hence, the prime factorisation of 53240 is $2 \times 2 \times 2 \times 5 \times 11 \times 11 \times 11$.

Here, the number 11 is repeated thrice and "2" is also repeated thrice. But we don't have three 5's.

Hence, the given number is not a perfect cube.

Thus, to make 53240 a perfect cube, we should multiply 25 on both sides.

53240 × 25 = 2 × 2 × 2 × 5 × 11 × 11 × 11 × 5 × 5 [Since 25 = 5 × 5]

So, we have three 2's, three 5's and three 11's.

$1331000 = 2^3 \times 5^3 \times 11^3$

$1331000 = (2 \times 5 \times 11)^3$

$1331000 = 110^3$

Hence, 1331000 is a perfect cube.

Therefore, the smallest natural number by which 53240 must be multiplied to make a perfect cube is 25.

Alternate Method:

As we know, the prime factorisation of 53240 = 2 × 2 × 2 × 5 × 11 × 11 × 11.

So, if we divide 53240 by 5 on both sides, we will get

(53240/ 5)= [(2 × 2 × 2 × 5 × 11 × 11 × 11) / 5]

On simplification, we get

10648 = 2 × 2 × 2 × 11 × 11 × 11

Hence, we have three 2's and three 11's, and we can say that 10648 is a perfect cube number.

I.e.,

$10648 = 2^3 \times 11^3$

$10648 = (2 \times 11)^3$

$10648 = 22^3$

Therefore, the smallest natural number by which 53240 must be divided to make a perfect cube is 5.

Cube root using prime factorisation

We can find the cube root of a number by prime factorisation method by the following steps:

- Resolve the number into its prime factors. Consider the number 5832. 5832 = (2×2×2)×(3×3×3)×(3×3×3).
- Make groups of three same prime factors.
- Take one **prime factor** from each group and multiply them. Their product is the required cube root.

Therefore, cube root of 5832= $\sqrt[3]{5832}$ =2×3×3=18

Be More Curious!

Cube Root of a Cube Number using estimation

If a **cube number** is given we can find out its cube root using the following steps:

- Take any cube number say 117649 and start making groups of three starting from the **rightmost** digit of the number. So 117649 has two groups, and **first group** (649) and the **second group** (117).
- The **unit's digit** of the **first group** (649) will decide the **unit digit of the cube root**. Since the number 649 ends with 9, the cube roots unit's digit is 9.
- Find the cube of numbers **between** which the **second group lies**. The other group is 117. We know that $4^3=64$ and $5^3=125$. 64 < 117 < 125. Take the smaller number between 4 and 5 as the ten's digit of the cube root. So, 49 is the cube root of 117649.

Differences of Squares of Triangular Numbers and Converse

Triangular numbers: It is a sequence of the numbers 1, 3, 6, 10, 15 etc. It is obtained by continued summation of the natural numbers. The **dot pattern** of a triangular number can be arranged as **triangles**.

Sum of two **consecutive triangular numbers** gives us a **square number**. For example, $1+3=4=2^2$ and $3+6=9=3^2$.

The **difference** between the **squares of two consecutive triangular** numbers is a **cube number**. For example, $3^2-1^2=9-1=8=2^3$ and $6^2-3^2=36-9=25=5^3$.

Also, if the **difference** between the **squares of two numbers** is a **cube number**, then these numbers are **consecutive triangular** numbers.