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Mathematics JEE Solutions 2022

Mathematics

1. Find the remainder when 3^{2022} is divided by 5.

- (1) 1 (2) 2
(3) 4 (4) 0

Sol. Answer (3)

$$\begin{aligned} 3^{2022} &= (9)^{1011} \\ &= (10 - 1)^{1011} \\ &= -(1 - 10)^{1011} \\ &= \left[{}^{1011}C_0 - {}^{1011}C_1 \cdot 10 + {}^{1011}C_2 10^2 + \dots \right] \\ &= -[1 + 10k] \\ &= -[1 + 5k_2] \end{aligned}$$

$\therefore 3^{2022}$ is divided by 5, -1 is the negative.

Remainder so positive remainder is 4.

2. If sum of square of reciprocal of roots ' α ' and ' β ' of equation $3x^2 - \lambda x + 1 = 0$ is 15, then find $6(\alpha^3 + \beta^3)^2$.

- (1) $\frac{202}{3}$ (2) $\frac{200}{9}$
(3) $\frac{224}{9}$ (4) $\frac{221}{3}$

Sol. Answer (3)

$$\begin{aligned} 3x^2 - \lambda x + 1 &= 0 \quad \left\langle \begin{matrix} \alpha \\ \beta \end{matrix} \right. \\ \alpha + \beta &= \frac{\lambda}{3} \\ \alpha\beta &= \frac{1}{3} \\ (\alpha + \beta)^2 - 2\alpha\beta &= 15(\alpha\beta)^2 \\ \frac{1}{\alpha^2} + \frac{1}{\beta^2} &= 15 \\ \Rightarrow \frac{\alpha^2 + \beta^2}{(\alpha\beta)^2} &= 15 \end{aligned}$$

$$\Rightarrow \frac{\lambda^2}{9} - \frac{2}{3} = \frac{15}{9}$$

$$\Rightarrow \frac{\lambda^2 - 6}{9} = \frac{15}{9}$$

$$\lambda^2 = 21$$

$$\text{Now, } 6(\alpha^3 + \beta^3)^2 = 6((\alpha + \beta)(\alpha^2 + \beta^2 - \alpha\beta))^2$$

$$= 6\left(\left(\frac{\lambda}{3}\right)((\alpha + \beta)^2 - 3\alpha\beta)\right)^2$$

$$= 6\left(\left(\frac{\lambda}{3}\right)\left(\frac{\lambda^2}{9} - 1\right)\right)^2$$

$$= 6 \cdot \frac{\lambda^2}{9} \cdot \left(\frac{21}{9} - 1\right)^2 = 26 \cdot \frac{21}{9} \cdot \frac{12}{9^2}$$

$$= 14 \cdot \frac{12}{9} \cdot \frac{12}{9}$$

$$= \frac{14 \times 16}{9} = \frac{224}{9}$$

3. If $(\tan^{-1} x)^3 + (\cot^{-1} x)^3 = k\pi^3$, then find the range of k .

- (1) $\left(\frac{1}{32}, \frac{9}{8}\right)$ (2) $\left[\frac{1}{32}, \frac{7}{8}\right)$
(3) $\left[\frac{1}{32}, \frac{9}{8}\right]$ (4) $\left[\frac{1}{32}, 1\right]$

Sol. Answer (2)

$$\begin{aligned} (\tan^{-1} x)^3 + (\cot^{-1} x)^3 &= (\tan^{-1} x + \cot^{-1} x)^3 \\ &\quad - 3 \tan^{-1} x \cdot \cot^{-1} x (\tan^{-1} x + \cot^{-1} x) \end{aligned}$$

$$= \left(\frac{\pi}{2}\right)^3 - 3 \tan^{-1} x \cdot \cot^{-1} x \cdot \left(\frac{\pi}{2}\right)$$

$$= \left(\frac{\pi}{2}\right) \left[\frac{\pi^2}{4} - 3 \tan^{-1} x \left(\frac{\pi}{2} - \tan^{-1} x\right) \right]$$

$$= \frac{\pi}{2} \left[\frac{\pi^2}{4} + 3(\tan^{-1} x)^2 - \frac{3\pi}{2} \tan^{-1} x \right]$$

$$= \frac{\pi}{2} \left[3 \left(\tan^{-1} x - \frac{\pi}{4} \right)^2 + \frac{\pi^2}{16} \right]$$

$$\therefore \tan^{-1} x \in \left(-\frac{\pi}{2}, \frac{\pi}{2} \right), \text{ so } \left(\tan^{-1} x - \frac{\pi}{4} \right)^2 \in \left[0, \frac{9\pi^2}{16} \right]$$

$$\text{Hence } (\tan^{-1} x)^3 + (\cot^{-1} x)^3 \in \left[\frac{\pi^3}{32}, \frac{7\pi^3}{8} \right]$$

$$\Rightarrow k \in \left[\frac{1}{32}, \frac{7}{8} \right]$$

4. If $f(\theta) = \sin \theta + \int_{-\pi/2}^{\pi/2} (\sin \theta + t \cos \theta) f(t) d\theta$, then

$$\left| \int_0^{\pi/2} f(\theta) d\theta \right|$$

$$(1) 1 + \pi t f(t)$$

$$(2) 1 - \pi t f(t)$$

$$(3) 1 + \pi^2 t f(t)$$

$$(4) -1 + \pi t \cdot f(t)$$

Sol. Answer (1)

$$f(\theta) = \sin \theta + \int_{-\pi/2}^{\pi/2} (\sin \theta + t \cos \theta) f(t) d\theta$$

$$f(\theta) = \sin \theta + \int_{-\pi/2}^{\pi/2} \sin \theta d\theta + t f(t) \int_{-\pi/2}^{\pi/2} \cos \theta d\theta$$

$$f(\theta) = \sin \theta + 0 + t f(t) \cdot 2$$

$$f(\theta) = \sin \theta + 2t f(t)$$

$$\text{Now } \left| \int_0^{\pi/2} f(\theta) d\theta \right| = \left| \int_0^{\pi/2} \sin \theta d\theta + 2t f(t) \int_0^{\pi/2} 1 d\theta \right|$$

$$= |1 + \pi t f(t)|$$

5. $\langle a_i \rangle$ sequence is an A.P. with common difference is 1 and $\sum_{i=1}^n a_i = 192$, $\sum_{i=1}^{n/2} a_{2i} = 120$ then, find the value of n , where n is an even integer.

$$(1) 48$$

$$(2) 96$$

$$(3) 18$$

$$(4) 36$$

Sol. Answer (2)

Let $n = 2m$ $m \in N$ and a be the first term

$$\sum_{i=1}^{2m} = 192 \quad \dots(1)$$

$$d = 1$$

$$\Rightarrow \frac{2m}{2} [2a + (2m-1)1] = 192$$

$$\Rightarrow m(2a + 2m - 1) = 192 \quad \dots(3)$$

$$\sum_{i=1}^m a_{2i} = 120 \quad \dots(2)$$

$$\frac{2m}{2} [2a + (m-1)2] = 120$$

$$m(a_2 + m - 1) = 120$$

$$m(a + m) = 120 \quad \dots(4)$$

$$(3/4) \Rightarrow \frac{2a + 2m - 1}{a + m} = \frac{192}{120} = \frac{8}{5}$$

$$\Rightarrow 100 + 10m - 5 = 8a + 8m$$

$$\Rightarrow 2a + 2m = 5 \text{ put in (3)}$$

$$m(5 - 1) = 192$$

$$\Rightarrow m = \frac{192}{4} = 48$$

$$\text{So } n = 2m = 2 \times 48 = 96$$

6. If $A = \begin{bmatrix} 1 & 0 & a \\ 1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$, where 'a' is odd value from 1 to 50 and $\sum_{a=1}^{50} |\text{adj } A| = 100k$, then value of k is

$$(1) \frac{1723}{2}$$

$$(2) \frac{1717}{2}$$

$$(3) \frac{1719}{4}$$

$$(4) \frac{1821}{4}$$

Sol. Answer (4)

$$|A| = (a+1)$$

$$\sum_{a=1}^{50} |\text{Adj } A| = \sum_{a=1}^{50} (a+1)^2 = 100k$$

$$\Rightarrow 1 + 2^2 + \dots + 51^2 = 100k + 1$$

$$\Rightarrow \frac{51 \times 52 \times 103}{6} - 1 = 100k$$

$$\Rightarrow k = \frac{45525}{100} = \frac{1821}{4}$$

7. A tangent $ax - \mu y = 2$ to hyperbola $\frac{a^2 x^2}{\lambda^2} - \frac{b^2 y^2}{1} = 4$, then the value of $\left(\frac{\lambda}{a}\right)^2 - \left(\frac{\mu}{b}\right)^2$ is

- (1) 0 (2) 1
(3) 2 (4) 3

Sol. Answer (2)

Condition for tangent is $a^2 l^2 - b^2 m^2 = n^2$

$$\frac{4\lambda^2}{a^4} \times a^2 - \frac{4}{b^2} \times (-\mu)^2 = 2^2$$

$$\Rightarrow \frac{\lambda^2}{a^2} - \frac{\mu^2}{b^2} = 1$$

8. A tangent at (x, y) to the curve $y = x^3 + 2x^2 + 4$ and passes through origin then (x_1, y_1) is

- (1) (0, 4) (2) (-1, 5)
(3) (1, 7) (4) (2, 20)

Sol. Answer (3)

$$y = x^3 + 2x^2 + 4$$

when tangent is drawn from origin,

$$xf'(x) = f(x)$$

$$x(3x^2 + 4x) = x^3 + 2x^2 + 4$$

$$2x^3 + 2x^2 - 4 = 0$$

$$x^3 + x^2 - 2 = 0$$

$$(x-1)(x^2 + \underset{\substack{\downarrow \\ \text{no solutions}}}{2x} + 2) = 0$$

$$x = 1 \Rightarrow y = 7$$

$$P = (1, 7)$$

9. Find the domain of $\frac{\cos^{-1}\left(\frac{x^2-5x+6}{x^2-9}\right)}{\ln(x^2)}$

$$(1) x \in \left(-\infty, \frac{5}{2}\right]$$

$$(2) x \in \left(-\infty, \frac{5}{2}\right] - \{-3, 0\}$$

$$(3) x \in \left(-\infty, \frac{5}{2}\right] - \{0\}$$

$$(4) x \in \left(-\infty, \frac{5}{2}\right] - \{-3\}$$

Sol. Answer (*)

$$f(x) = \frac{\cos^{-1}\left(\frac{x^2-5x+6}{x^2-9}\right)}{\ln x^2}$$

$$x^2 \neq 1 \Rightarrow x \neq \pm 1 \text{ and } x \neq 0$$

$$\text{Now, } -1 \leq \frac{x^2-5x+6}{x^2-9} \leq 1$$

Case 1:

$$-1 \leq \frac{x^2-5x+6}{x^2-9} \leq 1$$

$$\frac{x^2-5x+6}{x^2-9} + 1 \geq 0$$

$$\Rightarrow \frac{x^2-5x+6+x^2-9}{x^2-9} \geq 0$$

$$\Rightarrow \frac{2x^2-5x-3}{(x-3)(x+3)} \geq 0$$

$$\Rightarrow \frac{2x^2-6x+x-3}{(x-3)(x+3)} \geq 0$$

$$\Rightarrow \frac{2x(x-3)+(x-3)}{(x-3)(x+3)} \geq 0$$

$$\Rightarrow \frac{(x-3)+(2x+1)}{(x-3)(x+3)} \geq 0$$

$$x \neq 3, \quad \frac{2x+1}{x+3} \geq 0$$

$$\begin{array}{c} + \quad , \quad - \quad , \quad + \\ -3 \quad -\frac{1}{2} \end{array}$$

$$x \in (-\infty, -3) \cup \left[-\frac{1}{2}, \infty\right) - \{3\}$$

Case 2:

$$\frac{x^2-5x+6}{x^2-9} \leq 1$$

$$\Rightarrow \frac{x^2-5x+6-x^2+9}{x^2-9} \leq 0$$

$$\Rightarrow \frac{15-5x}{(x-3)(x+3)} \leq 0$$

$$\Rightarrow \frac{5(3-x)}{(x-3)(x+3)} \leq 0$$

$$\Rightarrow \frac{(x-3)}{(x-3)(x+3)} \geq 0$$

$$x \neq 3 \quad x+3 \geq 0$$

$$x \geq -3$$

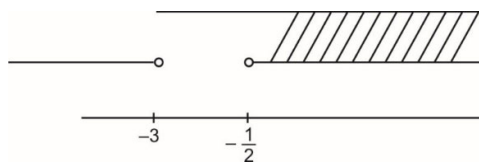
$$x \in [-3, \infty) - \{3\}$$

$$(1) \cap (2)$$

$$\text{Case-1 } (-\infty, -3) \cup \left[-\frac{1}{2}, \infty\right) - \{3\}$$

$$\text{Case-2 } x \in [-3, \infty) - \{3\}$$

$$t \quad x = \pm 1 \text{ and } x \pm 0$$



$$x \in \left[-\frac{1}{2}, \infty\right) - \{0, 1, 3\}$$

10. Solution of differential equation $x \frac{dy}{dx} - 2y$ is

$$(1) xy = c$$

$$(2) y = cx^2$$

$$(3) cx = y^2$$

$$(4) x^2 = cy^2$$

Sol. Answer (2)

$$\frac{x dy}{dx} = 2y$$

$$\Rightarrow \int \frac{dy}{y} = \int \frac{2dx}{x}$$

$$\ln y = 2 \ln(xc)$$

$$y = c^2 x^2$$

$$\Rightarrow \boxed{y = cx^2}$$

11. Consider a set $\Delta \in \{\vee, \wedge, \Rightarrow, \Leftrightarrow\}$ and $P \Delta q \Rightarrow (\sim P \Delta q) \Delta (\sim q \Delta P)$ is a tautology. Then number of arrangement is

$$(1) 1$$

$$(2) 2$$

$$(3) 3$$

$$(4) 4$$

Sol. Answer (3)

If $P \Delta q \Rightarrow (\sim P \Delta q) \Delta (\sim q \Delta P)$ is tautology

So, $P \Delta q = T$ and $(\sim P \Delta q) \Delta (\sim q \Delta P)$ is T

$P \Delta q = F$ and $(\sim P \Delta q) \Delta (\sim q \Delta P)$ is T

$P \Delta q = F$ and $(\sim P \Delta q) \Delta (\sim q \Delta P)$ is F

(1) for $D \equiv \vee$, if is a tautology

(2) for $D \equiv \wedge$, $P \wedge q = T \Rightarrow P = q = T$, not a tautology

$P \wedge q = f \Rightarrow P = f$ and $q = T$, not a tautology

$$P = T \text{ and } q = F$$

(3) for $D \equiv \Rightarrow$, $P \rightarrow q = T$ for $P = T$, $q = T$, is a tautology

$P = F_1$, $q = T$, is a tautology

$P = F_1$, $q = F$, is a tautology

(4) $D \equiv , P \rightarrow q T$

$$P = T_1 \theta = T$$

$$\text{RHS} = (F \Leftrightarrow T) \Leftrightarrow (F \Leftrightarrow T) \quad \left| \quad \text{RHS} = (T \Leftrightarrow F) \Leftrightarrow (T \Leftrightarrow F) \right.$$

$$= T$$

$$= F \Leftrightarrow F$$

= Tautology

= Tautology

12. The Boolean expression: $(p \Rightarrow q) \wedge (q \Rightarrow \sim p)$ is equivalent to

$$(1) \sim q$$

$$(2) q$$

$$(3) \sim p$$

$$(4) p$$

Sol. Answer (3)

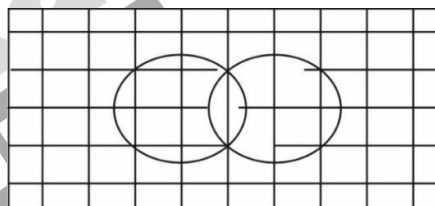
$$(P \Rightarrow q) \wedge q \Rightarrow \sim P$$

$$= (\sim P \vee q) \wedge (\sim q \vee \sim P)$$

$$= (\sim P \vee q) \wedge \sim(P \wedge q)$$

$$= \sim P$$

(from venn diagram)



13. Find number of solution in $\left[0, \frac{\pi}{2}\right]$ for

$$81^{\sin^2 x} + 81^{\cos^2 x} = 9$$

$$(1) \text{ Zero}$$

$$(2) \text{ Two}$$

$$(3) \text{ Three}$$

$$(4) \text{ One}$$

Sol. Answer (1)

$$81^{\sin^2 x} + 81^{1-\sin^2 x} = 9$$

$$(81^{\sin^2 x})^2 - 9(81^{\sin^2 x}) + 81 = 0$$

as $\Delta < 0$, no. solution

14. The coefficient of x^{20} in $(1+x)(1+2x)(1+4x)(1+8x) \dots (1+2^{20}x)$ is

$$(1) 2^{211} - 2^{190}$$

$$(2) 2^{191} - 2^{171}$$

$$(3) 2^{231} - 2^{209}$$

$$(4) 2^{161} - 2^{142}$$

Sol. Answer (1)

$$\text{Given expression} = (x+1)(2x+1)(2^2x+1) \dots (2^{20}x+1)$$

$$= 2^{210} (x+1) \left(x + \frac{1}{2}\right) \left(x + \frac{1}{2^2}\right) \cdots \left(x + \frac{1}{2^{20}}\right)$$

So coeff. of x^{20}

$$= 2^{210} \left[1 + \frac{1}{2} + \frac{1}{2^2} + \cdots + \frac{1}{2^{20}}\right]$$

$$= 2^{210} \cdot 1 \left[\frac{1 - \left(\frac{1}{2}\right)^{21}}{1 - \frac{1}{2}} \right]$$

$$= 2^{211} \left(1 - \frac{1}{2^{21}}\right) = 2^{211} - 2^{190}$$

15. Given, $f(x) = \frac{x^2 - 1}{x^2 + 1}$, find minimum value of $f(x)$

- (1) 0 (2) 1
(3) -1 (4) 2

Sol. Answer (3)

$$f(x) = \frac{x^2 - 1}{x^2 + 1} = y$$

$$\Rightarrow \frac{2x^2}{2} = \frac{1+y}{1-y}$$

$$\Rightarrow x^2 = \frac{y+1}{1-y} \geq 0$$

$$\Rightarrow \frac{y+1}{y-1} \leq 0$$

$$\Rightarrow y \in [-1, 1)$$

So min $f(x) = -1$

16. For a binomial probability distribution (33, p) $3P(x = 0) = P(x = 1)$ then find

$$\frac{P(x=15)}{P(x=18)} - \frac{P(x=16)}{P(x=17)} \text{ is}$$

- (1) 1000 (2) 1320
(3) 1221 (4) 1121

Sol. Answer (2)

$$x \sim B(33, p)$$

$\Rightarrow n = 33$ and probability of success p

$$\text{Now } 3P(x=0) = P(x=1)$$

$$3 \cdot {}^{33}C_0 (1-p)^{33} = {}^{33}C_1 \cdot p(1-p)^{32}$$

$$\Rightarrow 3(1-p) = 33p$$

$$\Rightarrow \boxed{p = \frac{1}{12}}$$

$$\frac{P(x=15)}{P(x=18)} - \frac{P(x=16)}{P(x=17)}$$

$$= \frac{{}^{33}C_{15} \left(\frac{1}{12}\right)^{15} \left(\frac{11}{12}\right)^{18} - {}^{33}C_{16} \left(\frac{1}{12}\right)^{16} \left(\frac{11}{12}\right)^{17}}{{}^{33}C_{18} \left(\frac{1}{12}\right)^{18} \left(\frac{11}{12}\right)^{15} - {}^{33}C_{17} \left(\frac{1}{12}\right)^{17} \left(\frac{11}{12}\right)^{16}}$$

$$= \left(\frac{11}{12}\right)^3 = 12^3 - \left(\frac{11}{12}\right) \cdot 12$$

$$= 11^3 - 11 = 1331 - 11 = 1320$$

17. $S = \{\theta : \theta \in [-\pi, \pi] - \left\{\pm \frac{\pi}{2}\right\} \text{ and } \sin \theta \tan \theta + \tan \theta = \sin 2\theta\}$

Let $T = \sum \cos 2\theta$ where $\theta \in S$, then $T + n(s) =$

- (1) 6 (2) 7
(3) 9 (4) 8

Sol. Answer (3)

$(\sin \theta + 0 \tan \theta = 2 \sin \theta \cos \theta = 0, \theta \in [-\pi, \pi] \text{ and}$

$$\theta \neq \pm \frac{\pi}{2}$$

$$\sin \theta = 0 \Rightarrow \theta = 0, \pm \pi$$

$$\text{and } \sin \theta + 1 - 2(1 - \sin^2 \theta) = 0$$

$$2 \sin^2 \theta + \sin \theta - 1 = 0$$

$$\sin \theta = -1, \sin \theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$\theta = -\frac{\pi}{2} \text{ not possible}$$

$\therefore$ no. of solution = 5

$$T = 2 \cos 2\theta$$

$$= \cos 0 + (\cos 2\pi + \cos(-2\pi) + \cos \frac{\pi}{3} + \cos \frac{5\pi}{3})$$

$$= 3 + \frac{1}{2} + \frac{1}{2} = 4$$

$$= T + n(s) = 4 + 5 = 9$$

18. A circle of equation $x^2 + y^2 + ax + by + c = 0$, passes through (0, 6) and touches $y = x^2$ at (2, 4). Find $a + c$.

- (1) 17 (2) 15
(3) 19 (4) 16

Sol. Answer (4)

Equation of tangent to $y = x^2$ at (2, θ) is

$$\frac{1}{2}(y+4) = 2x \Rightarrow 4x - y - 4 = 0$$

Given circle can be written as

$$(x-2)^2 + (y-4)^2 + \lambda(4x - y - 4) = 0$$

$$\dots(1)$$

$$\text{Sub } (0, 6) \Rightarrow 4 + 4 + \lambda(-4) = 0 \Rightarrow \lambda = \frac{4}{5}$$

$$(1) \Rightarrow$$

$$x^2 + y^2 - 4x - 8y + 20 + \frac{4}{5}(4x - y - 4) = 0$$

$$\Rightarrow x^2 + y^2 - \frac{4}{5}x - \frac{44}{5}y + \frac{84}{5} = 0$$

$$\text{Comparing with } x^2 + y^2 + ax + by + c = 0$$

$$a + c = 80 = 16$$

19. The equation of plane passing through the line of intersection of planes $x + 2y + 3z = 2$ and $x - y + z = 3$ and at a distance $\frac{2}{\sqrt{3}}$ from $(3, 1, -1)$ is

$$(1) 5x + 11y - z = 17 = 0$$

$$(2) 5x + 11y - z - 17 = 0$$

$$(3) 5x - 11y + z + 17 = 0$$

$$(4) 5x - 11y + z - 17 = 0$$

Sol. Answer (4)

$$\text{Equation res plane is } (x + 2y + 3z - 2) + \lambda(x - y + z - 3) = 0$$

$$\dots(1)$$

$$(1 + \lambda)x + (2 - \lambda)y + (3 + \lambda)z - (2 + 3\lambda) = 0$$

$$\text{dist from } (3, 1, -1) = \frac{2}{\sqrt{3}}$$

$$\Rightarrow \frac{|3 + 3\lambda + 2 - \lambda - 3 - \lambda - 2 - 3\lambda|}{\sqrt{(\lambda - 1)^2 + (2 - \lambda)^2 + (3 + \lambda)^2}} = \frac{2}{\sqrt{3}}$$

$$\Rightarrow 3\lambda^2 = 3\lambda^2 + 4\lambda + 14 \Rightarrow \lambda = -\frac{7}{2} \text{ sub in (1)}$$

$$-5x + 11y - z + 17 = 0$$

$$5x - 11y + z - 17 = 0$$

20. If the sides of a triangle and $x^2 + x + 1$, $x^2 - 1$, $2x + 1$, find the greatest angle of the triangle.

$$(1) 72^\circ \quad (2) 104^\circ$$

$$(3) 120^\circ \quad (4) 108^\circ$$

Sol. Answer (3)

$$\text{Sides } x^2 + x + 1 > x^2 - 1 > 2x + 1 \quad (\because x > 1, \text{ as}$$

$$x^2 - 1 > 0)$$

$$\cos A = \frac{(x^2 - 1)^2 + (2x + 1)^2 - (x^2 + x + 1)^2}{2(x^2 - 1)(2x + 1)}$$

$$= \frac{x^4 - 2x^2 + 1 + 4x^2 + 4x + 1 - (x^4 + x^2 + 1 + 2x^3 + 2x^2 + 2x)}{2(x^2 - 1)(2x + 1)}$$

$$= \frac{-2x^3 - x^2 + 2x + 1}{2(x^2 - 1)(2x + 1)} = \frac{-(x^2 - 1)(2x + 1)}{2(x^2 - 1)(2x + 1)} = \frac{-1}{2}$$

$$A = 120^\circ$$

21. A and B are two 3×3 matrices such that $AB = BA$ then

S-I: if A^3 is symmetric and B^2 is skew symmetric matrix, then $(AB)^6$ is a skew symmetric matrix.

S-II: If A^3 is a skew-symmetric and B^2 is symmetric, then $(AB)^6$ is symmetric.

(1) S-I is true and S-II is false

(2) S-I and S-II both are true

(3) S-I and S-II both are false

(4) S-I is false and S-II is true

Sol. Answer (2)

$$\text{Statement-I: } (A^3)^T = A^3, (B^2)^T = B^2$$

$$(AB)^6 = A^6 B^6 = (A^3)^2 (B^2)^3$$

$$[(AB)^6]^T = [(B^2)^T]^3 [(A^3)^T]^2 = (-B^2)^3 (A^3)^2$$

$$= -B^6 A^6 = -(BA)^6 = -(AB)^6$$

$\therefore$ It is skew symmetric

$$\text{Statement-II: } (A^3)^T = -A^3 \text{ and } (B^2)^T = B^2$$

$$[(AB)^6]^T = [(A^3)^2 (B^2)^3]^T = (B^2)^T (A^3)^T (A^3)^T (B^2)^T$$

$$(B^2)^3 (-A^3)^2 = (BA)^6 = (AB)^6,$$

22. Image of $A\left(\frac{3}{\sqrt{a}}, \sqrt{a}\right)$ in y -axis is B and image

of B in x -axis is C . Point $D(3 \cos \theta, a \sin \theta)$ lies in 4th quadrant and the maximum area of $\triangle ACD = 12$ sq. units. Then find a .

Sol. Answer ($a = 8$)

$$A = \left(\frac{3}{\sqrt{a}}, \sqrt{a}\right)$$

$$B = \left(-\frac{3}{\sqrt{a}}, \sqrt{a}\right), C = \left(-\frac{3}{\sqrt{a}}, -\sqrt{a}\right)$$

$$\Delta = (\cos \theta, a \sin \theta)$$

$$\text{Area of } \Delta ACD = \frac{1}{2} \begin{vmatrix} \frac{3}{\sqrt{a}} & \sqrt{a} & 1 \\ \frac{-3}{\sqrt{a}} & -\sqrt{a} & 1 \\ 3 \cos \theta & a \sin \theta & 1 \end{vmatrix}$$

$$R_1 \rightarrow R_1 + R_2 = \frac{1}{2} \begin{vmatrix} 0 & 0 & 2 \\ \frac{-3}{\sqrt{a}} & -\sqrt{a} & 1 \\ 3 \cos \theta & a \sin \theta & 1 \end{vmatrix}$$

$$= |-3\sqrt{a} \sin \theta + 3\sqrt{a} \cos \theta|$$

$$= 3\sqrt{a} |\cos \theta - \sin \theta| \leq 3\sqrt{a} \times \sqrt{2}$$

$$\text{Max Area} = 3\sqrt{2a} = 12 \Rightarrow 2a = 16$$

$$a = 8$$

23. If $\vec{a}$ and $\vec{b}$ are unit vectors and vectors $\vec{c}$ and $\vec{a}$ have an angle of $\frac{\pi}{12}$ between them such that

$$\vec{b} = \vec{c} + 2(\vec{c} \times \vec{a}) \text{ then the value of } |6\vec{c}|^2 \text{ is}$$

- (1) $3 + \sqrt{3}$ (2) $3 - \sqrt{3}$
(3) $2 + \sqrt{3}$ (4) $2 - \sqrt{3}$

Sol. Answer (1)

$$|\vec{a}| = |\vec{b}| = 1 \quad (\vec{c}, \vec{a}) = \frac{\pi}{12}$$

$$\text{Given, } \vec{b} - \vec{c} = 2(\vec{c} \times \vec{a}) \quad \dots (1)$$

$$(1) \cdot \vec{c} \Rightarrow \vec{b} \cdot \vec{c} = |\vec{c}|^2$$

$$|\vec{b} - \vec{c}|^2 = 4|\vec{c} \times \vec{a}|^2 \Rightarrow 1 + c^2 - 2c^2 = 4c^2 \left(\frac{\sqrt{3}-1}{2\sqrt{2}} \right)^2$$

$$\Rightarrow 2(1 - c^2) = c^2(4 - 2\sqrt{3})$$

$$\Rightarrow 2 = c^2(6 - 2\sqrt{3}) \Rightarrow c^2 = \frac{1}{3 - \sqrt{3}} = \frac{3 + \sqrt{3}}{6}$$

$$6c^2 = 3 + \sqrt{3}$$

24. A balloon, spherical in shape is inflated and its surface area is increasing with a constant rate, initially the radius is 3 units, after 5 sec radius is 7 units then the radius after 9 seconds is

- (1) 9 (2) 7
(3) 5 (4) 3

Sol. Answer (1)

$$\text{Given } \frac{ds}{dt} = k$$

$$\frac{d}{dt}(4\pi r^2) = k \Rightarrow 8\pi r \frac{dr}{dt} = k$$

$$\Rightarrow \int 8\pi r dr = \int k dt$$

$$\Rightarrow 4\pi r^2 = kt + c$$

$$\text{at } t = 0, r = 3 \Rightarrow c = 36\pi$$

$$\text{at } t = 5, r = 7 \Rightarrow 196\pi = 5k + 36\pi$$

$$\Rightarrow k = 32\pi$$

$$4\pi r^2 = 32\pi t + 36\pi \Rightarrow r^2 = 8t + 9$$

$$\text{at } t = 9, r^2 = 81 \Rightarrow r = 9$$

25. If $x(y) = x$ and $y \frac{dx}{dy} = 2x + y^3(y+1)e^y$ and $x(1) = 0$, then $x(e)$ is equal to

- (1) $(e^e - 1)$ (2) $e^3(e^e - 1)$
(3) $3^e - 1$ (4) $e^e - 3$

Sol. Answer (2)

$$y \frac{dx}{dy} = 2x + y^3(y+1)e^y$$

$$\Rightarrow \frac{dx}{dy} + \left(-\frac{2}{y}\right)x = y^2(y+1)e^y$$

$$P = -\frac{2}{y} \quad \theta = y^2(y+1)e^y$$

$$\text{I.F.} = e^{\int -\frac{2}{y} dy} = \frac{1}{y^2}$$

Solution is

$$\frac{x}{y^2} = \int \frac{1}{y^2} \cdot y^2(y+1)e^y dy + c$$

$$\frac{x}{y^2} = ye^y + c \quad x(1) = 0 \Rightarrow c = -e$$

$$\Rightarrow x = y^3 e^y - e y^2$$

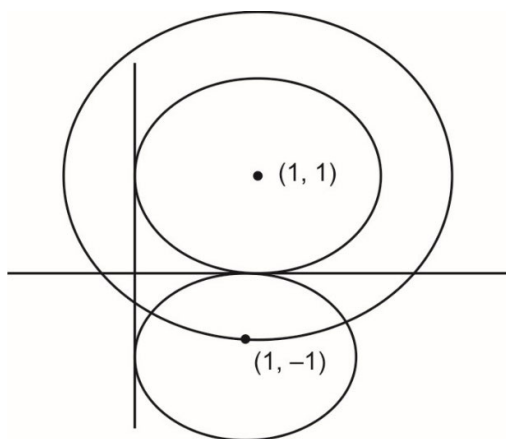
$$x(1) = e^3 \cdot e - e^3 = e^3(e^3 - 1)$$

26. If $S = \{z \in \mathbb{C} : 1 \leq |z - (1 + i)| \leq 2\}$ and $A = \{z \in \mathbb{C} : |z - (1 - i)| = 1\}$

- (1) null set (2) singleton set
(3) exactly two elements (4) infinite elements

Sol. Answer (4)

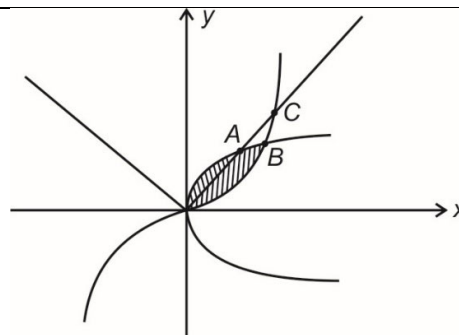
$$1 \leq |z - (1 + i)| \leq 2 \quad |z - (1 - i)| = 1$$



There are infinitely many point on the circle $|z - (1 - i)| = 1$, which satisfy $1 \leq |z - (1 + i)| \leq 2$

27. $y = 2|x|$ divides the area enclosed by $y = x^3$ and $y^2 = x$ into two parts of areas R_1 and R_2 where $R_2 > R_1$. Value of $\left(\frac{R_2}{R_1}\right)$ is

Sol. Answer (19)



$$y = 2|x|, y = x^3, y^2 = x$$

$$\text{Solving } y = x^3, y^2 = x$$

$$(0, 0) \text{ and } B = (1, 1)$$

$$\text{Solving } y = 2|x| \text{ and } y^2 = x$$

$$(0, 0) \text{ and } A = \left(\frac{1}{4}, \frac{1}{2}\right)$$

$$R_1 = \int_0^{1/4} \sqrt{x} - 2x dx = \frac{2}{3} \times \frac{1}{4} \times \frac{1}{2} = \frac{1}{16} = \frac{4-3}{48} = \frac{1}{48}$$

$$R_2 = \int_0^{1/4} 2x - x^3 dx + \int_{1/4}^1 \sqrt{x} - x^3 dx$$

$$= \frac{1}{16} - \frac{1}{1024} + \frac{2}{3} \left(1 - \frac{1}{8}\right) - \frac{1}{4} + \frac{1}{1024}$$

$$= \frac{1}{16} + \frac{7}{12} - \frac{1}{4} = \frac{3+28-12}{48} = \frac{19}{48} \Rightarrow \frac{R}{R_1} = 19$$

