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Mathematics JEE Solutions 2022

Mathematics

1. 2 sin (12°) – sin(72°) =

(1) $\frac{1-\sqrt{5}}{8}$ (2) $\frac{\sqrt{5}(1-\sqrt{3})}{4}$

(3) $\frac{\sqrt{3}(1-\sqrt{5})}{2}$ (4) $\frac{\sqrt{3}(1-\sqrt{5})}{4}$

Sol. Answer (4)

2 sin 12° – sin 72° = sin 12° + sin 12° – sin 72°

= sin 12° – 2 cos 42° · sin 30°

= –(cos 42° – sin 12°) = –(sin 48° – sin 12°)

$= -2\cos 30^\circ \sin 18^\circ = -2\cdot\frac{\sqrt{3}}{2}\left(\frac{\sqrt{5}-1}{4}\right)$

$= \frac{\sqrt{3}\left(1-\sqrt{5}\right)}{4}$

2. $y = y(x)$ is the solution of the differential equation $2x^2\frac{dy}{dx} - 2xy + 8y^2 = 0$, $y(e) = \frac{e}{3}$, then $y(1)$ is equal to

(1) $\frac{2}{3}$ (2) 3

(3) $\frac{3}{2}$ (4) –1

Sol. Answer (4)

$2x^2\frac{dy}{dx} - 2xy + 8y^2 = 0$

$\frac{dy}{dx} = \frac{xy-4y^2}{x^2}$, $y = vx$

$v + \frac{xdv}{dx} = v - 4v^2 \Rightarrow \int\frac{dv}{v^2} = -4\int\frac{dx}{x}$

$\Rightarrow -\frac{1}{v} = -4\ln x - \ln c \Rightarrow \frac{x}{y} = \ln cx^4$

$x = e,\ y = \frac{e}{3} \Rightarrow 3 = \ln c + 4 \Rightarrow \ln c = -1 \Rightarrow c = e^{-1}$

$\frac{x}{y} = \ln\frac{x^4}{e}$

$x = 1 \Rightarrow \frac{1}{y} = \ln\frac{1}{e} = -\ln e = -1 \Rightarrow y = -1$

3. $A = \{x \in R : |x+1| < 2\}$, $B = \{x \in R : |x-1| \ge 2\}$ then

(1) $A \cup B = R - [1, 3]$

(2) $A \cap B = [-1, 1]$

(3) $A \cap B = (-3, -1]$

(4) $B - A = R - (-3, 1]$

Sol. Answer (3)

$|x+1| < 2 \Rightarrow x+1 \in (-2, 2) \Rightarrow x \in (-3, 1) = A$

$|x-1| \ge 2 \Rightarrow x-1 \le -2$ or $x-1 \ge 2$

$\Rightarrow x \le -1$ or $x \ge 3$

$\Rightarrow x \in (-\infty, -1] \cup [3, \infty) = B$

$A \cup B = (-\infty, 1) \cup [3, \infty) = R - [1, 3)$

$A \cap B = (-3, -1]$

$B - A = (-\infty, -3] \cup [3, \infty) = R - (-3, 3)$

4. Find the sum $S = 1 + 2.3 + 3.3^2 + \cdots + 10.3^9$

(1) $\frac{1}{4}(19.3^{10}+1)$ (2) $\frac{1}{4}(19.3^{10}-1)$

(3) $\frac{1}{2}(19.3^{10}+1)$ (4) $\frac{1}{2}(19.3^{10}-1)$

Sol. Answer (1)

$s = 1 + 2.3 + 3.3^2 + \cdots + 10 \times 10 \times 3^9$

$3s = 0 + 3 + 2.3^2 + \cdots + 9 \times 3^9 + 10 \times 3^{10}$

Subtracting $-2s = 1 + 3 + 3^2 + \cdots + 3^9 - 10 \times 3^{10}$

$$-2s = \frac{3^{10}-1}{3-1} - 10 \times 3^{10}$$

$$\Rightarrow\ s = 5 \times 3^{10} - \frac{(3^{10}-1)}{4} = \frac{19.3^{10}+1}{4}$$

5. Water increasing in a right circular cone with rate 1 cm^3/sec. then rate change of lateral surface of cone is (where height and diameter of cone is 35 and 14 respectively) at h = 10 cm.

(1) $\frac{\sqrt{26}}{10}$ (2) $\frac{\sqrt{26}}{5}$

(3) $\frac{\sqrt{21}}{5}$ (4) 5

Sol. Answer (2)

$$\frac{h}{r} = \frac{35}{7} \Rightarrow h = 5r$$

$$v = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^2 \cdot 5r = \frac{5}{3}\pi r^3$$

$$\frac{dv}{dt} = \frac{5}{3}\pi \cdot 3r^2 \frac{dr}{dt} = 1 \ \Rightarrow\ \frac{dr}{dt} = \frac{1}{5\pi r^2}$$

Now $s = \pi r l = \pi r\sqrt{r^2+h^2}$

$$= \pi r\sqrt{r^2+25r^2} = \pi r^2\sqrt{26}$$

$$\frac{ds}{dt} = 2\pi r \cdot \frac{dr}{dt} \cdot \sqrt{26} = \frac{2\pi r}{5\pi r^2} \cdot \sqrt{26} = \frac{2\cdot\sqrt{26}}{5r}$$

when $h = 10 \Rightarrow r = 2$

$$\frac{ds}{dt} = \frac{\sqrt{26}}{5}$$

6. 2, 4, 8, 16, 32, 32 written on a faces of a biased dice probability of showing up face with mark n is $\frac{1}{n}$ of dice is rolled 3 times. Find probability of sum coming as 48.

(1) $\frac{7}{2^{12}}$ (2) $\frac{3}{2^{10}}$

(3) $\frac{7}{2^{11}}$ (4) $\frac{13}{2^{12}}$

Sol. Answer (4)

P(sum = 48) = $P(16) \times P(16) \times P(16)$

$+ P(8) \times P(32) \times P(8)$

$$= \frac{1}{16} \times \frac{1}{16} \times \frac{1}{16} + {}^3C_1 \times \left(\frac{1}{8}\right)^2 \times \left(\frac{2}{32}\right)$$

$$= \frac{1+12}{16^3} = \frac{13}{16^3} = \frac{13}{2^{12}}$$

7. In the series

$$(5+x)^{500} + x(5+x)^{499} + \cdots + x^{500}$$

find coefficient of x^{101}

(1) 5^{300} (2) 5^{399}

(3) $5^{399}\ {}^{501}C_{101}$ (4) $-5^{399}\ {}^{501}C_{101}$

Sol. Answer (3)

$$\frac{(5+x)^{500}\left(\left(\frac{x}{5+x}\right)^{501} - 1\right)}{\left(\frac{x}{5+x}\right) - 1} = \frac{x^{501} - (5+x)^{501}}{(-5)}$$

$$= \frac{1}{5}(5+x)^{501} - \frac{x^{501}}{5}$$

for coefficient of x^{101}, consider only $\frac{1}{5}(5+x)^{501}$

$$\therefore \frac{1}{5}\left({}^{501}C_{101} \cdot 5^{400} \cdot x^{101}\right)$$

$\Rightarrow$ coefficient of x^{101} is $5^{399} \cdot {}^{501}C_{101}$

8. Negation of the statement $(\sim p \vee q) \Rightarrow (\sim q \wedge p)$

(1) $\sim p \Rightarrow q$ (2) $p \Rightarrow q$

(3) $\sim q \Rightarrow p$ (4) $p \Leftrightarrow q$

Sol. Answer (2)

$$(\sim p \vee q) \Rightarrow (\sim q \wedge p) \equiv \sim(\sim p \vee q) \vee (p \wedge \sim q)$$

$$\equiv (\sim q \wedge p) \vee (\sim q \wedge p) \equiv (p \wedge \sim q)$$

Now $\sim\left[(\sim p \vee q) \Rightarrow (\sim q \wedge p)\right] \equiv \sim(p \wedge \sim q)$

$$\sim\left[(\sim p \vee q) \Rightarrow (\sim q \wedge p)\right] \equiv \sim(p \wedge \sim q)$$

$$\equiv \sim p \vee q$$

$$\equiv p \Rightarrow q$$

9. If $b_n = \int_0^{\pi/2} \frac{\cos^2 nx}{\sin x} dx$ then

(1) $\frac{1}{b_3 - b_2}, \frac{1}{b_4 - b_3}, \frac{1}{b_5 - b_4}$ are in A.P. with common difference = –2

(2) $\frac{1}{b_3-b_2}, \frac{1}{b_4-b_3}, \frac{1}{b_5-b_4}$ are in A.P with common difference = 2

(3) $b_3 - b_2$, $b_4 - b_3$, $b_5 - b_4$ are in G.P.

(4) $b_3 - b_2$, $b_4 - b_3$, $b_5 - b_4$ are in A.P. with common difference = – 2

Sol. Answer (1)

$$b_n - b_{n-1} = \int_0^{\pi/2} \frac{\cos^2 nx - \cos^2(n-1)x}{\sin x}dx$$

$$= \int_0^{\pi/2} \frac{\sin(2n-1)x(-\sin x)}{\sin x}dx$$

$$= \frac{-1}{2n-1}$$

$$n = 3, b_3 - b_2 = \frac{-1}{5}$$

$$n = 4, b_4 - b_3 = \frac{-1}{7}$$

$$n = 5, b_5 - b_\phi = \frac{-1}{9}$$

$$\therefore \frac{1}{b_3-b_2}, \frac{1}{b_4-b_3}, \frac{1}{b_5-b_4}$$

are in A.P with common difference is –2

10. Equation of tangent at p on parabola $y = x - x^2$ is $y = 4 + kx$, $x > 0$ and let v be the vertex of parabola, then slope of line joining p and v is

(1) $-\frac{3}{2}$ (2) $\frac{26}{9}$

(3) $\frac{5}{2}$ (4) $\frac{23}{6}$

Sol. Answer (2)

$$(x+1)y^1 = y = e^{3x}(x+1)^2$$

$$y^2 - \frac{1}{x+1}y = e^{3x}(x+1) \quad y(0) = \frac{1}{3}$$

$$\text{I.F.} = e^{\int \frac{-1}{x+1}dx} = e^{-\log(x+1)}$$

so solution is $y \cdot \frac{1}{x+1} = \int e^{3x}dx + c$

$$\frac{y}{x+1} = \frac{e^{3x}}{3} + c$$

$$x = 0 \Rightarrow \frac{1}{3} = \frac{1}{3} + c \Rightarrow c = 0$$

$$y = \frac{(x+1)}{3}e^{3x}$$

$$x = \frac{-4}{3} \Rightarrow y = \frac{-1}{9e^4}$$

11. If system of equation

$-kx + 3y - 14z = 25$

$-15x + 4y - kz = 3$

$-4x + y + 32 = 4$

consistent then k belongs

(1) $R - \{-11, 13\}$ (2) $R - \{-11, 11\}$

(3) $R - \{13\}$ (4) for all $k \in R$

Sol. Answer (2)

$$\Delta = \begin{vmatrix} -k & 3 & -14 \\ -15 & 4 & -k \\ -4 & 1 & 3 \end{vmatrix} \neq 0$$

$$\Rightarrow -k(12+k) - 3(-45-4k) - 14(-15+16) \neq 0$$

$$\Rightarrow -12k - k^2 + 135 + 12k - 14 \neq 0$$

$$k^2 \neq 121 \Rightarrow k \neq \pm 11$$

$$k \in R - \{-11, 11\}$$

12. Find the value of $\tan^{-1}\left[\frac{\cos\left(\frac{15\pi}{4}\right) - 1}{\sin\frac{\pi}{4}}\right]$

(1) $-\frac{\pi}{8}$ (2) $-\frac{4\pi}{9}$

(3) $-\frac{5\pi}{12}$ (4) $-\frac{\pi}{4}$

Sol. Answer (1)

$$\tan^{-1}\left(\frac{\cos\frac{15\pi}{4}}{\sin\frac{\pi}{4}}\right) = \tan^{-1}\left(\frac{\cos\left(4\pi - \frac{\pi}{4}\right) - 1}{\sin\frac{\pi}{4}}\right)$$

$$= \tan^{-1}\left(\frac{\cos\frac{\pi}{4} - 1}{\sin\frac{\pi}{4}}\right) = \tan^{-1}\left(\frac{-2\sin^2\frac{\pi}{8}}{2\sin\frac{\pi}{8}\cos\frac{\pi}{8}}\right)$$

$$= \tan^{-1}\left(\tan\frac{-\pi}{8}\right) = -\frac{\pi}{8}$$

13. $ax^2 - 2bx + 15 = 0$, $(a, b \in R)$ α is a repeated root and $x^2 - 2bx + 21 = 0$ have roots α and β then $\alpha^2 + \beta^2 =$

(1) 58 (2) 56

(3) 57 (4) 60

Sol. Answer (1)

α and β are roots of $x^2 - 2bx + 21 = 0$

$\Rightarrow \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= 4b^2 - 2 \times 21 = 4b^2 - 42$

Further, α is repeated root of $ax^2 - 2bx + 15 = 0$

$\Rightarrow \alpha.\alpha = \dfrac{15}{a}$ and $2\alpha = \dfrac{2b}{a}$

$\Rightarrow \alpha^2 = \dfrac{15}{a}$ and $\boxed{\alpha = \dfrac{b}{a}}$

Now, substituting α in $x^2 - 2bx + 21 = 0$

$\Rightarrow \dfrac{b^2}{a^2} - \dfrac{2b^2}{a} + 21 = 0$

$\Rightarrow b^2 - 2b^2a + 21a^2 = 0$

$\Rightarrow 15a - 30a^2 + 12a^2 = 0$

$\Rightarrow 15a = 9a^2$

$\Rightarrow a = 0$ (Not possible) or $a = \dfrac{5}{3}$

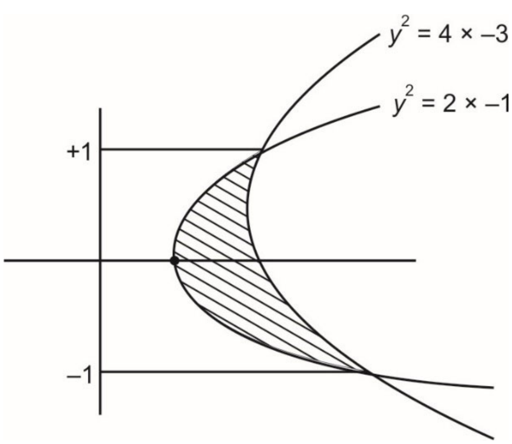
$\therefore \alpha^2 + \beta^2 = 4b^2 - 42 = 4 \times 15a - 42$

$= 60 \times \dfrac{5}{3} - 42 = 100 - 42 = 58$

14. Area of the region bounded by the curves $y^2 = 2x - 1$ and $y^2 = 4x - 3$ is

(1) $\dfrac{1}{6}$ (2) $\dfrac{1}{3}$

(3) $\dfrac{2}{3}$ (4) $\dfrac{1}{2}$



$y^2 = 2x - 1$ $\qquad$ $y^2 = 4x - 3$

Point of intersection is (1, –1) and (1, 1)

$\text{Area} = \int_{-1}^{+1} \left[\left(\dfrac{y^2 + 3}{4} \right) - \left(\dfrac{y^2 + 1}{2} \right) \right] dy$

$\int_{-1}^{1} \left(\dfrac{1}{4} - \dfrac{y^2}{4} \right) dy = 2\int_{0}^{1} \left(\dfrac{1}{4} - \dfrac{y^2}{4} \right) dy$

$= 2\left(\dfrac{1}{4} - \dfrac{1}{12} \right) = \dfrac{1}{3}$

15. If sum of first n terms of tow AP's are in ratio of $3n + 8 : 7n + 15$ then the ratio of their 12th term is

(1) 8 : 7 (2) 7 : 16

(3) 74 : 169 (4) 13 : 47

Sol. Answer (2)

$\dfrac{S_n}{S'_n} = \dfrac{3n + 8}{7n + 15}$

$\Rightarrow \dfrac{2a + (n-1)d}{2A + (n-1)D} = \dfrac{3n + 8}{7n + 15}$

Put $n = 23 \Rightarrow \dfrac{a + 11d}{A + 11D} = \dfrac{77}{176} = \dfrac{7}{16}$

$\dfrac{t_{12}}{t'_{12}} = \dfrac{7}{16}$

16. The value of $\lim\limits_{x \to \frac{\pi}{2}} \tan^2 x \left(\sqrt{2\sin^2 x + 3\sin x + 4} - \sqrt{\sin^2 x + 6\sin x + 2} \right)$ is

(1) $-\dfrac{1}{12}$ (2) $\dfrac{1}{18}$

(3) $\dfrac{1}{12}$ (4) $-\dfrac{1}{18}$

Sol. Answer (3)

$\lim\limits_{x \to \frac{\pi}{2}} \tan^2 x \left(\sqrt{2\sin^2 x + 3\sin x + 4} - \sqrt{\sin^2 x + 6\sin x + 2} \right)$

$(0 \times \infty \text{ form})$

$\lim\limits_{x \to \frac{\pi}{2}} \tan^2 x \left(\dfrac{2\sin^2 x + 3\sin x + 4 - (\sin^2 x + 6\sin x + 2)}{\sqrt{2\sin^2 x + 3\sin x + 4} + \sqrt{\sin^2 x + 6\sin x + 2}} \right)$

$\lim\limits_{x \to \frac{\pi}{2}} \dfrac{\tan^2 x (\sin^2 x - 3\sin x + 2)}{(3 + 3)}$

$$\lim_{x\to\frac{\pi}{2}} \frac{\sin^2 x}{6\cos^2 x}(\sin x - 1)(\sin x - 2)$$

$$\lim_{x\to\frac{\pi}{2}} \frac{1\times(1-2)}{6}\frac{(\sin x - 1)}{1-\sin^2 x}$$

$$= \lim_{x\to\frac{\pi}{2}} \frac{-1}{6}\times\frac{(-1)}{1+\sin x}$$

$$= \frac{+1}{6}\times\frac{1}{2} = \frac{1}{12}$$

17. Find the number of 3 digit numbers which has exactly 2 digits identical

Sol. Answer (243)

Case-I: $\underline{a}$ $\underline{a}$ $\underline{b}$

$\underline{a}$ $\underline{b}$ $\underline{a}$

Number of such number = 9 × 9 × 2 (a can not be zero) = 162

Case-II: $\underline{b}$ $\underline{a}$ $\underline{a}$

Number of such number = 9 × 9 (*b* can not be zero) = 81

Number of such number 162 + 81 = 243

18. Mean deviation about mean = $\frac{5(n+1)}{n}$ of data 1, 2, 3, … *n*. Where *n* is odd, then values of *n* is equal to

Sol. Answer (21)

Mean of $1, 2, \ldots n = \frac{n+1}{2} = \frac{2k+1+1}{2} = k+1$,

where *n* = 2*k* + 1

Mean deviation =

$$\frac{|1-(k+1)|+|2-(k+1)|+\ldots+|(2k+1)-(k+1)|}{2k+1}$$

$$= \frac{2(1+2+\cdots+k)}{2k+1} = \frac{k(k+1)}{2k+1} = \frac{(n-1)(n+1)}{4n}$$

Given $\frac{n^2-1}{4n} = \frac{5(n+1)}{n}$

$\Rightarrow (n-1)(n+1) = 20(n+1)$

$\Rightarrow n = 21$

19. $12\int_3^b \frac{dx}{(x^2-4)(x^2-1)} = \ln\frac{49}{50}$, then number of possible values of *b* are

Sol. Answer (4)

Given $12\int_3^b \frac{dx}{\left(x^2-4\right)\left(x^2-1\right)} = \ln\frac{49}{50}$

$$\frac{12}{3}\left(\int_3^b\left(\frac{1}{x^2-4}-\frac{1}{x^2-1}\right)dx\right) = \ln\frac{49}{50}$$

$$\Rightarrow 4\left(\frac{1}{4}\ln\left|\frac{x-2}{x+2}\right| - \frac{1}{2}\ln\left|\frac{x-1}{x+1}\right|\right)\Big|_3^b = \ln\frac{49}{50}$$

$$\Rightarrow \ln\left|\frac{b-2}{b+2}\right| - \ln\left|\frac{1}{5}\right| - 2\left(\ln\left|\frac{b-1}{b+1}\right| - \ln\left|\frac{2}{4}\right|\right) = \ln\frac{49}{50}$$

$$\Rightarrow \ln\left(\frac{b-2}{b+2}\times 5\times\left(\frac{b+1}{b-1}\right)^2\right) = \ln\frac{49}{50}$$

$$\Rightarrow \frac{(b-2)(b+1)^2}{(b+2)(b-1)^2} = \frac{49\times 4}{50\times 5} = \frac{98}{125}$$

$$\Rightarrow \frac{b^3+2b^2+b-2b^2-4b-2}{b^3-2b^2+b+2b^2-4b+2} = \frac{98}{125}$$

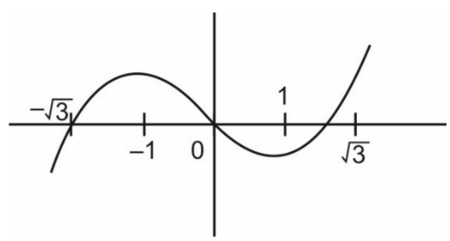
$$\Rightarrow \frac{b^3-3b-2}{b^3-3b+2} = \frac{98}{125}$$

Using componendo and Dividendo,

$$\Rightarrow \frac{b^3-3b}{2} = \frac{223}{27} \text{ or } \frac{27}{223}$$

$$\therefore b^3-3b = \frac{446}{27} \text{ or } b^3-3b = \frac{54}{223}$$

Now, let $f(b) = b^3-3b$



f(*b*) has local maximum at *x* = 1

f(–1) = –1 + 3 =2

$\therefore b^2-3b = \frac{444}{27}$ has only one solution

$b^2-3b = \frac{54}{223}$ has 3 solution

$\therefore$ 4 distinct values of *b* are possible.

20. If sum of coefficients of positive even powers of x in the expansion of $\left(2x^3+\frac{3}{x}\right)^{10}$ is $5^{10} - \beta \cdot 3^9$, then value of β is

Sol. Answer (83)

Given Binomial expression $\left(2x^3+\frac{3}{x}\right)^{10}$

General term = ${}^{10}C_r = \left(2x^3\right)^4\left(\frac{3}{x}\right)^{10-r} = {}^{10}C_r 2^4 3^{10-r}\ x^{3r+r-10}$

$= {}^{10}C_4 2^4\ 3^{10-4}\ x^{4r-10}$

$\therefore$ r cannot be 0,1,2

$\therefore$ Required sum = $5^{10} - \left({}^{10}C_0 2^0 3^{10} + {}^{10}C_1 2^1 3^9 + {}^{10}C_2 2^2 3^8\right)$

$= 5^{10} - \left(3^{10} + 10\times 2\times 3^9 + 45\times 4\times 3^8\right)$

$= 5^{10} - (3^{10} + 20\times 3^9 + 60\times 3^9)$

$= 5^{10} - 83\times 3^9$

$\therefore \beta = 83$

21. Hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ has eccentricity $\frac{5}{4}$. If normal to the hyperbola at $\left(\frac{8}{\sqrt{5}}, \frac{12}{5}\right)$ is $8\sqrt{5}x+\beta y=\lambda$. Then the value of $(\lambda-\beta)$ is

Sol. Answer (85)

$b^2 = a^2\left(e^2-1\right)$

$\Rightarrow \frac{b^2}{a^2} = \frac{25}{16} - 1 = \frac{9}{16}$

$\Rightarrow \frac{b}{a} = \frac{3}{4}$

Now equation of normal at (x_1, y_1) is

$\Rightarrow \frac{a^2x}{x_1} + \frac{b^2y}{y_1} = a^2+b^2 = a^2e^2$

$\Rightarrow \frac{a^2x}{8/\sqrt{5}} + \frac{b^2y}{12/5} = a^2.\frac{25}{16}$ dividing by a^2

$\Rightarrow \frac{\sqrt{5}x}{8} + \frac{b^2}{a^2}.\frac{5y}{12} = \frac{25}{16}$

$\Rightarrow \frac{\sqrt{5}x}{8} + \frac{9}{16}.\frac{5y}{12} = \frac{25}{16}$

$\Rightarrow 8\sqrt{5}x + 15y = 100$

So, $\lambda - \beta = 100 - 15 = 85$

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