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Mathematics JEE Solutions 2022

Mathematics

1. If a biased coin is tossed 5 times and if the probability of getting 4 heads is equal to the probability of getting 5 heads, then the probability of getting at most 2 heads is

(1) $\frac{46}{6^4}$ (2) $\frac{275}{6^5}$

(3) $\frac{41}{5^5}$ (4) $\frac{36}{5^4}$

Sol. Answer (1)

Let $P(H) = p \Rightarrow P(T) = 1 - p$

Now $P(4H) = P(5H)$

$\Rightarrow {}^5C_4 p^4(1-p) = {}^5C_5\ p^5$

$\Rightarrow 5(1-p) = p \Rightarrow p = \frac{5}{6}$

So P(at most $2H$)

$= P(No\ H) + P(1H) + P(2H)$

$= {}^5C_0\left(\frac{1}{6}\right)^5 + {}^5C_1\left(\frac{5}{6}\right)'\left(\frac{1}{6}\right)^4 + {}^5C_2\left(\frac{5}{6}\right)^2\left(\frac{1}{6}\right)^3$

$= \frac{1+25+250}{6^5}$

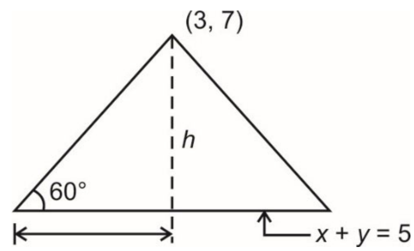
$= \frac{276}{6^6} = \frac{46}{6^4}$

2. The vertex of an equilateral triangle is (3, 7) and equation of opposite side is $x + y = 5$, then area of the triangle is

(1) $\frac{25}{\sqrt{3}}$ (2) $\frac{25}{2\sqrt{3}}$

(3) $25\sqrt{3}$ (4) 25

Sol. Answer (2)



$h = \frac{|3+7-5|}{\sqrt{2}} = \frac{5}{\sqrt{2}}$

Area of the equilateral triangle $= \frac{h^2}{\sqrt{3}} = \frac{25}{2\sqrt{3}}$

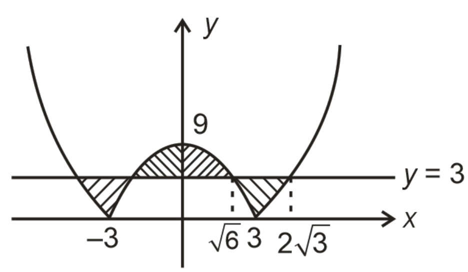
3. The area of region bounded by curve $y = |x^2 - 9|$ and line $y = 3$ is

(1) $4\left[2\sqrt{3}+\sqrt{6}-3\right]$ (2) $8\left[4\sqrt{3}-\sqrt{6}+9\right]$

(3) $4\left[2\sqrt{3}+\sqrt{6}+9\right]$ (4) $8\left[4\sqrt{3}+2\sqrt{6}-9\right]$

Sol. Answer (4)

$y = |x^2 - 9| = |(x-3)(x+3)|$



Required area =

$$2\left[\int_0^{\sqrt{6}}(9-x^2-3)dx + \int_{\sqrt{6}}^{3}(3-9+x^2)dx + \int_{3}^{2\sqrt{3}}(3-x^2+9)dx\right]$$

$$= 2\left[\int_0^{\sqrt{6}}(6-x^2)\,dx + \int_{\sqrt{6}}^{3}(x^2-6)\,dx + \int_{3}^{2\sqrt{3}}(12-x^2)\,dx\right]$$

$$= 2\left[\left(6x-\frac{x^3}{3}\right)_0^{\sqrt{6}} + \left(\frac{x^3}{3}-6x\right)_{\sqrt{6}}^{3} + \left(12x-\frac{x^3}{3}\right)_3^{2\sqrt{3}}\right]$$

$$= 2\left[\begin{matrix}6\sqrt{6}-2\sqrt{6}+(9-18)-\left(2\sqrt{6}-6\sqrt{6}\right)\\ +\left(24\sqrt{3}-8\sqrt{3}\right)-(36-9)\end{matrix}\right]$$

$$= 2\left[8\sqrt{6}-36+16\sqrt{3}\right]$$

$$= 8\left[2\sqrt{6}-6+4\sqrt{3}\right]$$

(No option is matching)

4. $\lim\limits_{x \to \frac{1}{\sqrt{2}}} \dfrac{\sin(\cos^{-1}x)-x}{1-\tan(\cos^{-1}x)}$ is

(1) $\dfrac{1}{2\sqrt{2}}$ (2) $\dfrac{-1}{2\sqrt{2}}$

(3) $-\dfrac{1}{\sqrt{2}}$ (4) $-\sqrt{2}$

Sol. Answer (3)

$$I = \lim_{x \to \frac{1}{\sqrt{2}}} \frac{\sin(\cos^{-1}x)-x}{1-\tan(\cos^{-1}x)}$$

$$I = \lim_{x \to \frac{1}{\sqrt{2}}} \frac{\cos(\cos^{-1}x)\cdot\dfrac{(-1)}{\sqrt{1-x^2}}-1}{-\sec(\cos^{-1}x)\dfrac{(-1)}{\sqrt{1-x^2}}}$$

$$I = \frac{\dfrac{1}{\sqrt{2}}\times\dfrac{(-1)}{\sqrt{1-\frac{1}{2}}}-1}{2\times\dfrac{1}{\sqrt{1-\frac{1}{2}}}} = -\frac{1}{\sqrt{2}}$$

5. The sum of absolute minimum and absolute maximum value of $f(x) = \left|3x-x^2+2\right|-x$ for $x \in [-1,\ 2]$ is

(1) 1 (2) 0

(3) $\dfrac{\sqrt{17}+3}{2}$ (4) $\dfrac{\sqrt{17}-3}{2}$

Sol. Answer (2)

$f(x) = \left|x^2-3x-2\right|-x \qquad x \in [-1, 2]$

for x^2-3x-2, $\dfrac{-b}{2a} = \dfrac{3}{2}$

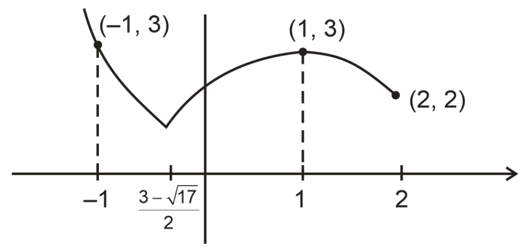
$$\frac{D}{4a} = \frac{-17}{4}$$

And roots of $x^2-3x-2=0$ are

$x = \dfrac{3 \pm \sqrt{17}}{2}$ out of which only

$x = \dfrac{3-\sqrt{17}}{2} \in [-1, 2] \qquad \dfrac{3-\sqrt{17}}{2} \simeq -0.56$

So $f(x) = \begin{cases} x^2-4x-2, & -1 \le x \le \dfrac{3-\sqrt{17}}{2} \\ -x^2+2x+2, & \dfrac{3-\sqrt{17}}{2} \le x \le 2 \end{cases}$



$$f\left(\frac{3-\sqrt{17}}{2}\right) = 0 + \frac{\sqrt{17}-3}{2} \simeq 0.56$$

So absolute maxima = 3

absolute minima = $\dfrac{\sqrt{17}-3}{2}$

So answer = $\dfrac{\sqrt{17}-3}{2}+3 = \dfrac{\sqrt{17}+3}{2}$

6. $\dfrac{x}{a}+\dfrac{y}{b}=2$ tangent $\left(\dfrac{x}{a}\right)^n+\left(\dfrac{y}{b}\right)^n = 2$ at $(a,\ b)$ $n \in s$, find s

(1) ϕ (2) {1}

(3) $\{2k, 1k \in N\}$ (4) N

Sol. Answer (4)

$$\left(\frac{x}{a}\right)^n+\left(\frac{y}{b}\right)^n = 2$$

$$n\left(\frac{x}{a}\right)^{n-1}\cdot\frac{1}{a}+n\left(\frac{y}{b}\right)^{n-1}\cdot\frac{1}{b}y' = 0$$

Point (a, b)

$$\frac{n}{a}+\frac{n}{b}y' = 0$$

$$\frac{1}{b}y' = -\frac{1}{a} \Rightarrow y' = -\frac{b}{a}$$

Equation of tangent at (a, b)

$$y-b = -\frac{b}{a}(x-a)$$

$ay - ab = -bx + ab$

$bx + ay = 2ab$

$$\frac{x}{a}+\frac{y}{b}=2$$

$\therefore$ $n \in$ Natural number

7. If $f(x)=\frac{x-1}{x+1}, f^{n+1}(x)=f(f^n(x)) \ \forall\ n \in N$, then the value of $f^6(6) + f^7(7)$ is

(1) $-\frac{3}{2}$ (2) $-\frac{2}{3}$

(3) $\frac{2}{3}$ (4) $\frac{3}{2}$

Sol. Answer (1)

$$f(x)=\frac{x-1}{x+1}$$

$$f^2(x)=f(f(x)=f\left(\frac{x-1}{x+1}\right)=\frac{\frac{x-1}{x+1}-1}{\frac{x-1}{x+1}+1}=\frac{x-1-x-1}{x-1+x+1}$$

$$=\frac{-2}{2x}=\frac{-1}{x}$$

$$f^3(x)=f\left(-\frac{1}{x}\right)=\frac{\frac{-1}{x}-1}{\frac{-1}{x}+1}=\frac{-1-x}{-1+x}=\frac{x+1}{1-x}$$

so $$f^6(x)=f^3(f^3(x))=\frac{\frac{x+1}{1-x}+1}{1-\frac{x+1}{1-x}}=\frac{2}{-2x}=\frac{-1}{x}$$

$$f^7(x)=f(f^6(x))=f\left(\frac{-1}{x}\right)=\frac{x+1}{1-x}$$

$$f^6(6)+f^7(7)=\frac{-1}{6}-\frac{8}{6}=\frac{-9}{6}=\frac{-3}{2}$$

8. $\frac{dy}{dx}=\frac{1}{1+\sin 2x}; y\left(\frac{\pi}{4}\right)=\frac{1}{2};$

$x \in (0, 2\pi)-\left\{\frac{\pi}{2},\frac{3\pi}{2},\frac{3\pi}{4},\frac{7\pi}{4}\right\}$

and $y(x)=\sqrt{2}\sin x$

Find the sum of x-co-ordinates of point of intersection of given curves.

(1) $\frac{7\pi}{2}$ (2) $\frac{7\pi}{3}$

(3) $\frac{9\pi}{4}$ (4) $\frac{9\pi}{2}$

Sol. Answer (1)

$$\frac{dy}{dx}=\frac{1}{1+\sin 2x}; \quad y\left(\frac{\pi}{4}\right)=\frac{1}{2}$$

$$x \in (0, 2\pi)-\left\{\frac{\pi}{2},\frac{3\pi}{2},\frac{3\pi}{4},\frac{7\pi}{4}\right\}$$

$$\frac{dy}{dx}=\frac{1}{(\sin x+\cos x)^2} \Rightarrow dy=\frac{\sec^2 \times dx}{(\tan x+1)^2}$$

$$\Rightarrow y=\frac{-1}{1+\tan x}+c$$

$$y\left(\frac{\pi}{4}\right)=\frac{1}{2} \Rightarrow c=1 \Rightarrow y=\frac{-1}{1+\tan x}+1$$

Now point of intersection of $\frac{-1}{1+\tan x}+1=\sqrt{2}\sin x$

$$\Rightarrow \frac{\tan x}{1+\tan x}=\sqrt{2}\sin x$$

$$\Rightarrow \frac{\sin x}{\sin x+\cos x}=\sqrt{2}\sin x$$

$$\Rightarrow \sin x = 0 \text{ or } \sin x+\cos x=\frac{1}{\sqrt{2}}$$

$$\Rightarrow x=\pi \text{ or } x=\frac{7\pi}{12},\frac{23\pi}{12}$$

So required sum = $\pi+\frac{7\pi}{12}+\frac{23\pi}{12} = \frac{7\pi}{2}$

9. If $\sin^2 10° \cdot \sin 20° \cdot \sin 40° \cdot \sin 50° \cdot \sin 70° = \alpha-\frac{1}{16}\sin 10°$ then find the value of $\alpha^{-1}+16$

(1) 80 (2) 60

(3) 64 (4) 16

Sol. Answer (1)

$$\sin^2 10 \ \sin 20 \ \sin 40 \ \sin 50 \ \sin 70=\alpha-\frac{1}{16}\sin 10°$$

$$\sin A \ \sin(60-A)\sin(60+A)=\frac{1}{4}\sin 3A$$

$$\therefore \ \sin 10 \ \sin 50 \ \sin 70 = \frac{1}{4} \ \sin 30=\frac{1}{8}$$

$$\therefore \ \sin 10 \ \underbrace{\sin 10 \ \sin 20 \ \sin 40 \ \sin 50}\ \underbrace{\sin 70}$$

$$=\frac{1}{8}\sin 10° \ \sin 20° \ \sin 40°$$

$$=\frac{1}{8}\cdot\frac{1}{2}\sin 10\left(\cos 20-\frac{1}{2}\right)$$

$$=\frac{1}{8}\cdot\frac{1}{2}\left(\frac{1}{2}\cdot 2\sin 10\cos 20-\frac{1}{2}\sin 10°\right)$$

$$= \frac{1}{16}\left(\frac{1}{2}\left(\frac{1}{2} - \sin 10\right) - \frac{1}{2}\sin 10^\circ\right)$$

$$= \frac{1}{16}\left[\frac{1}{4} - \sin 10^\circ\right] = \frac{1}{64} - \frac{1}{16}\sin 10^\circ$$

$$\therefore \ \alpha = \frac{1}{64} \ \Rightarrow \ \frac{1}{\alpha} + 16 = 64 + 16 = 80$$

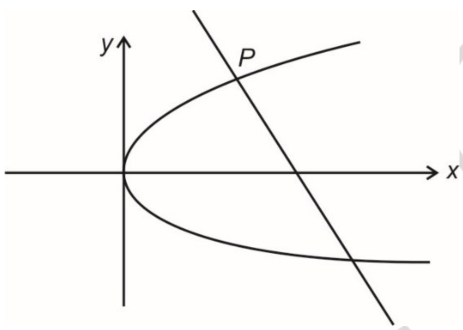
10. Normal to $y^2 = 6x$ at P, passes through (5, –8) then ordinate of point intersection of Directrix and tangent at P is

(1) $\frac{9}{4}$ (2) $\frac{-9}{4}$

(3) $\frac{-9}{2}$ (4) $\frac{9}{2}$

Sol. Answer (2)

$y^2 = 6x$ …(1)



Equation of normal: $y = mx - 3m - \frac{3}{2}m^3$

(5, –8) satisfy it

$$\Rightarrow \ -8 = 5m - 3m - \frac{3}{2}m^3$$

$$\Rightarrow \ 2m - \frac{3}{2}m^3 = -8 \ \Rightarrow \boxed{m = 2}$$

$\Rightarrow$ Slope of tangent at P = $\frac{-1}{2} = m_T$

So equation of tangent $\rightarrow$ $y = m_T x + \frac{3}{2m_T}$

$$y = \frac{-x}{2} - 3$$

and equation of directrix is $x = \frac{-3}{2}$

$$\Rightarrow \ y = \frac{-3}{2}\left(\frac{-1}{2}\right) - 3 = \frac{-9}{4}$$

So ordinate = $\frac{-9}{4}$

11. From a group of 10 boys $B_1, B_2 \ldots B_{10}$, 5 girls $G_1, G_2 \ldots G_5$, the number of ways of selection of group of 3 boys & 3 girls, such that B_1 & B_2 are not together in group

Sol. Answer (1120)

$B_1 \ldots B_2 \rightarrow$ 10 Boys

$G_1 \ldots G_2 \rightarrow$ 5 girls

Number of ways to select 3 boys as per condition = ${}^{10}C_3$ — when (B_1B_2) are selected

$= {}^{10}C_3 - {}^{8}C_1 \cdot 1$

= 112

Number of ways for selected girls ${}^{5}C_3$

Total no. of ways = $112 \times {}^{5}C_3$

= 112 × 10

= 1120

12. Find the remainder when $(2021)^{2023}$ is divided by 7

Sol. Answer (5)

$$(2021)^{2023} = (7 \times 289 - 2)^{2023}$$

$$= (7t - 2)^{2023}$$

$$= {}^{2023}C_0(7t)^{2023} - {}^{2023}C_1(7t)^{2023}(2) + \ldots - {}^{2023}C_{2023}(2)^{2023}$$

$$= 7P - 2^{2023}$$

Now, $2^{2023} = 2 \cdot 2^{2022}$

$= 2(2^3)^{674}$

$= 2(1 + 7)^{674}$

$= 7m + 2$

$$\therefore (2021)^{2023} = 7P - (7m + 2)$$

$$= 7P - 7m - 2$$

Negative Remainder = –2

So, +ve remainder is 5

Remainder r = 5

13. Let $a = \sum_{i=i}^{10}\sum_{j=1}^{10}\min(i, j)$ and $b = \sum_{i=i}^{10}\sum_{j=1}^{10}\max(i, j)$. Then value of $(a + b)$ is

Sol. Answer (100)

$$a = \sum_{i=1}^{10}\sum_{j=1}^{10}\min(i, j)$$

$= \sum_{i=1}^{10}\{\min(i,1)+\min(i,2)+...+\min(i,10)\}$

$= (\underbrace{1+1+...1}_{19\text{ times}})+(\underbrace{2+2+...+2}_{17\text{ times}})+(\underbrace{3+3+...3}_{15\text{ times}})+...\ \underbrace{(1)}_{1\text{ time}}$

$b = (\underbrace{10+10+...10}_{19\text{ times}})+(\underbrace{9+9+...9}_{17\text{ times}})+...+\underbrace{1}_{1\text{ times}}$

$\therefore a+b = 20(1+2+...+10) = 20\times\frac{10\times 11}{2} = 100$

14. $S_1 = \{n \mid \text{HCF}(n,45)=1\}$

$S_2 = \{2k \mid k \in \{1,2,....10\}\}$

Then $n(S_1 \cap S_2)$ is equal to

Sol. Answer (5)

$45 = 3^2 \times 5$

HCF $(n,45) = 1 \Rightarrow n$ is neither multiple of 3 nor multiple of 5.

Given $S_2 = \{2,4,....20\}$

$S_1 \cap S_2$ gives the even numbers that are neither multiple of 3 nor multiple of 5

$S_1 \cap S_2 = \{2,4,8,14,16\}$

$n(S_1 \cap S_2) = 5$

15. Let $f(x) = \max\{|x+1|,|x+2|,|x+3|,|x+4|,|x+5|\}$

Then the value of $\int_{-6}^{0} f(x)\,dx$ is

Sol. Answer (21)

$$f(x) = \begin{cases} -(x+1) & x \le -3 \\ (x+5) & x \ge -3 \end{cases}$$

Required Area

$$\int_{-6}^{-3} -(x+1)\,dx + \int_{-3}^{0}(x+5)\,dx$$

$$= -\frac{(x+1)^2}{2}\Bigg|_{-6}^{-3} + \frac{(x+5)^2}{2}\Bigg|_{-3}^{0}$$

$$= \frac{-1}{2}(4-25)+\frac{1}{2}(25-4) = 21$$

16. If mean of 5 elements a, b, 8, 5, 10 is 6 and variance is 6.8, then mean deviation about mean is M then 25 M is

Sol. Answer (60)

$$\text{Mean} = \frac{a+b+8+5+10}{5} = 6$$

$\Rightarrow a+b = 7 \qquad ...(1)$

$$\text{Variance} = \frac{a^2+b^2+64+25+100}{5} - 36 = 68$$

$\Rightarrow a^2+b^2+189 = 214$

$\Rightarrow a^2+b^2 = 25 \qquad ...(2)$

From (1) & (2), a = 4, b = 3

Mean deviation about Mean $= \frac{|4-6|+|3-6|+|8-6|+|5-6|+|10-6|}{5}$

$$M = \frac{2+3+2+1+4}{5} = \frac{12}{5}$$

$\Rightarrow 5M = 12$

$\Rightarrow 25M = 60$

17. The value of

$27\left[\vec{a}\times\left(\vec{b}\times\vec{c}\right)\quad \vec{b}\times\left(\vec{c}\times\vec{a}\right)\quad \vec{c}\times\left(\vec{a}\times\vec{b}\right)\right]$ for

$\vec{a}\cdot\vec{b} = 1$, $\vec{b}\cdot\vec{c} = 2$, $\vec{a}\cdot\vec{c} = 3$ is equal to

Sol. Answer (0)

$\vec{a}\cdot\vec{b} = 1,\ \vec{b}\cdot\vec{c} = 2,\ \vec{a}\cdot\vec{c} = 3$

Now, $\vec{a}\times\left(\vec{b}\times\vec{c}\right) = \left(\vec{b}\,.\,\vec{c}\right)\vec{b} - \left(\vec{a}\cdot\vec{b}\right)\vec{c}$

$= 3\vec{b} - \vec{c}$

$\vec{b}\times\left(\vec{c}\times\vec{a}\right) = \vec{c} - 2\vec{a}$

$\vec{c}\times\left(\vec{a}\times\vec{b}\right) = 2\vec{a} - 3\vec{b}$

$\Rightarrow \vec{a}\times\left(\vec{b}\times\vec{c}\right) + \vec{b}\times\left(\vec{c}\times\vec{a}\right) + \vec{c}\times\left(\vec{a}\times\vec{b}\right) = 0$

So, given expression,

$= \left[3\vec{b}-\vec{c}\quad \vec{c}-2\vec{a}\quad 2\vec{a}-3\vec{b}\right] = 0$

18. A 3 × 3 invertible matrices given [A].

If $|\text{adj}.24A| = |\text{adj}.3(\text{adj}.2A)|$, then $|A|^2$ is

Sol. Answer (64)

$|adj(24A)| = |adj3(adj2A)|$

$|24A|^2 = |3adj(2A)|^2$

$\Rightarrow \left(24^3|A|\right)^2 = \left(3^3|adj(2A)|\right)^2$

$= 3^6\left(|2A|^2\right)^2$

$$\left(24^3|A|\right)^2 = 3^6 \cdot 2^{12} \cdot |A|^4$$

$$\Rightarrow 24^4 \cdot |A|^2 = 3^6 \cdot 2^{12}\,|A|^4$$

$$|A|^2 = \frac{24^6}{3^6 \cdot 2^{12}} = 64$$

19. Find the value of $\frac{48}{\pi^4}\int_0^{\pi}\left(\frac{3\pi}{2}x^2 - x^3\right)\frac{\sin x}{1+\cos^2 x}dx$.

Sol. Answer (6)

$$I = \frac{4B}{\pi^4}\int_0^{\pi}\left(\frac{3\pi}{2}x^2 - x^3\right)\frac{\sin x}{1+\cos^2 x}dx \quad \ldots(1)$$

$$I = \frac{48}{\pi^4}\int_0^{\pi}\left(\frac{3\pi}{2}(\pi - x)^2 - (\pi - x)^3\right)\frac{\sin x}{1+\cos^2 x}dx$$

$$I = \frac{48}{\pi^4}\int_0^{\pi}\left[\frac{3\pi^3}{2} - 3\pi^2 x + \frac{3\pi}{2}x^2 - \left(\pi^3 - x^3 - 3\pi^2 x + 3\pi x^2\right)\right]\frac{\sin x}{1+\cos^2 x}$$

$$I = \frac{48}{\pi^4}\int_0^{\pi}\left(\frac{\pi^3}{2} - \frac{3\pi}{2}x^2 + x^3\right)\frac{\sin x}{1+\cos^2 x}dx \quad \ldots(2)$$

Adding (1) & (2)

$$\Rightarrow 2I = \frac{24\pi^3}{\pi^4}\int_0^{\pi}\frac{\sin x}{1+\cos^2 x}dx$$

$$I = -\frac{12}{\pi}\left[\tan^{-1}(\cos x\right]_0^{\pi}$$

$$I = -\frac{12}{\pi}\left[-\frac{\pi}{4} - \left(+\frac{\pi}{4}\right)\right]$$

$$I = \frac{12}{2} = 6$$

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