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Mathematics JEE Solutions 2022

Mathematics

1. The value of

$$\cos^{-1}\left(\frac{3}{10}\cos\left(\tan^{-1}\left(\frac{4}{3}\right)\right)+\frac{2}{5}\sin\left(\tan^{-1}\left(\frac{4}{3}\right)\right)\right)$$

(1) 0 (2) $\frac{\pi}{3}$

(3) $\frac{\pi}{6}$ (4) $\frac{\pi}{2}$ or $\frac{\pi}{4}$

Sol. Answer (2)

Let $\tan^{-1}\frac{4}{3}=\theta \Rightarrow \tan\theta=\frac{4}{3}$

$\Rightarrow \cos\theta=\frac{3}{5} \Rightarrow \sin\theta=\frac{4}{5}$

$$\Rightarrow \cos^{-1}\left(\frac{3}{10}\cos\theta+\frac{2}{5}\sin\theta\right)=\cos^{-1}\left(\frac{3}{10}\times\frac{3}{5}+\frac{2}{5}\times\frac{4}{5}\right)$$

$$=\cos^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{3}$$

2. $\lim\limits_{x\to 0}\frac{\cos(\sin x)-\cos x}{x^4}$ is equal to

(1) $\frac{1}{4}$ (2) $\frac{1}{12}$

(3) $\frac{1}{6}$ (4) $\frac{1}{8}$

Sol. Answer (3)

$$\lim_{x\to 0}\frac{\cos(\sin x)-\cos x}{x^4}$$

$$=\lim_{x\to 0}\frac{\cos\left(x-\frac{x^3}{3!}+\frac{x^5}{5!}-\cdots\right)-\cos x}{x^4}$$

$$=\lim_{x\to 0}\frac{\left(1-\frac{\left(x-\frac{x^3}{3!}\right)^2}{2!}+\frac{\left(x-\frac{x^3}{3!}\right)^4}{4!}-\cdots\right)-\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}\cdots\right)}{x^4}$$

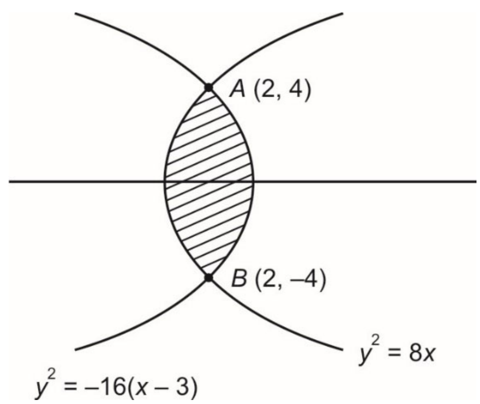
$$=\frac{\text{Coefficient of } x^4 \text{ in Numerator}}{\text{Coefficient of } x^4 \text{ in Denominator}}=\frac{1}{6}+\frac{1}{24}-\frac{1}{24}=\frac{1}{6}$$

3. Find the area bounded between curve $y^2 = 8x$ and $y^2 = 16(3 - x)$

(1) 16 (2) 8

(3) 32 (4) 64

Sol. Answer (1)



By solving 2 parabola

we get $A \equiv (2,4)$ and $B(2,-4)$

$$\text{Area required }=\int_{-4}^{4}\left(3-\frac{y^2}{16}-\frac{y^2}{8}\right)dy$$

$$=\int_{-4}^{4}\left(3-\frac{3y^2}{16}\right)dy=\left(3y-\frac{y^3}{16}\right)_{-4}^{4}=16$$

4. The value of 16 sin 20° · sin 40° · sin 80° is

(1) $2\sqrt{3}$ (2) 3

(3) $\sqrt{3}$ (4) $4\sqrt{3}$

Sol. Answer (1)

$$\because\ \sin\left(\frac{\pi}{3}-\theta\right)\sin\theta\,\sin\left(\frac{\pi}{3}+\theta\right)=\frac{1}{4}\sin 3\theta$$

$\therefore\ 16\sin(60^\circ-20^\circ)\sin 20^\circ\sin\left(60^\circ+20^\circ\right)$

$$=16\times\frac{1}{4}\sin 60^\circ=4\times\frac{\sqrt{3}}{2}=2\sqrt{3}$$

5. $\frac{dy}{dx}+\frac{2y}{x}=e^x$ be a differential equation and $y(x)$ be a solution satisfying $y(1) = 0$, then $z(x) = x^2\cdot y(x) - e^x$ is

(1) $z(x)=e^x(x^2+2x+1)-e$

(2) $z(x)=e^x(x-1)^2-e$

(3) $z(x)=e^x(x^2+1)-e$

(4) $z(x)=e^x(x^2+1)+e$

Sol. Answer (2)

$$\frac{dy}{dx}+\frac{2y}{x}=e^x\ \Rightarrow I.F.\ e^{\int\frac{2}{x}dx}=x^2$$

$$\Rightarrow\ yx^2=\int e^x x^2 dx$$

$$=x^2e^x-\int 2xe^x dx$$

$$=x^2e^x-\left\{2xe^x-\int 2e^x\right\}+c$$

$$\Rightarrow\ yx^2=x^2e^x-2xe^x+2e^x+c$$

$$y(1)=0\ \Rightarrow\ 0=e-2e+2e+c\ \Rightarrow \boxed{c=-e}$$

Here $\boxed{z(x)=x^2e^x-2xe^x+2e^x-e-e^x}$

$$=e^x(x^2-2x+1)-e$$

$$=e^x(x-1)^2-e$$

6. $\int\frac{1}{x}\sqrt{\frac{1-x}{1+x}}\,dx=g(x)+c$, then find the value of $g\left(\frac{1}{2}\right)$

(1) $\ln\left(2-\sqrt{3}\right)-\frac{\pi}{6}$ (2) $\ln\left(2+\sqrt{3}\right)+\frac{\pi}{3}$

(3) $\ln\left(2+\sqrt{3}\right)-\frac{\pi}{6}$ (4) $\ln\left(2-\sqrt{3}\right)+\frac{\pi}{3}$

Sol. Answer (4)

Put $x=\cos\theta\Rightarrow dx=-\sin\theta\,d\theta$

$$g(x)=\int\frac{1}{\cos\theta}\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}\times-\sin\theta\,d\theta$$

$$=-\int\frac{1}{\cos\theta}\times\tan\left(\frac{\theta}{2}\right)\times 2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)d\theta$$

$$=-\int\frac{2\sin^2\frac{\theta}{2}}{\cos\theta}d\theta$$

$$=+\int\frac{\cos\theta-1}{\cos\theta}d\theta$$

$$=\theta-\ln\left|\sec\theta+\tan\theta\right|+c$$

$$=\cos^{-1}x-\ln\left|\frac{1}{x}+\frac{\sqrt{1-x^2}}{x}\right|+c$$

$$g\left(\frac{1}{2}\right)=\cos^{-1}\left(\frac{1}{2}\right)-\ln\left(2+\sqrt{3}\right)$$

$$=\frac{\pi}{3}+\ln\left(2-\sqrt{3}\right)$$

7. If $A=\sum_{n=1}^{\infty}\frac{1}{(3+(-1)^n)^n}$ and $B=\sum_{n=1}^{\infty}\frac{(-1)^n}{(3+(-1)^n)^n}$. The value of $\frac{A}{B}$ is

(1) $\frac{-11}{9}$ (2) $\frac{-11}{3}$

(3) $\frac{-11}{6}$ (4) -11

Sol. Answer (1)

$$A=\sum_{n=1}^{\infty}\frac{1}{\left[3+(-1)^n\right]^n}$$

$$=\frac{1}{2}+\frac{1}{4^2}+\frac{1}{2^3}+\frac{1}{4^4}+\cdots$$

$$=\left(\frac{1}{2}+\frac{1}{2^3}+\cdots\right)+\left(\frac{1}{4^2}+\frac{1}{4^4}+\cdots\right)$$

$$=\frac{\frac{1}{2}}{1-\frac{1}{4}}+\frac{\frac{1}{16}}{1-\frac{1}{16}}=\frac{2}{3}+\frac{1}{15}=\frac{11}{15}$$

$$B=\sum_{n=1}^{\infty}\frac{(-1)^n}{[3+(-1)^n]^n}$$

$$=\left(-\frac{1}{2}-\frac{1}{2^3}-\cdots\right)+\left(\frac{1}{4^2}+\frac{1}{4^4}+\cdots\right)$$

$$=-\frac{2}{3}+\frac{1}{15}=\frac{-9}{15}=\frac{-3}{5}\Rightarrow\frac{A}{B}=\frac{-11}{9}$$

8. If function $f(x)=x-1$ and $g(x)=\frac{x^2}{x^2+1}$, then fog is

(1) one-one and onto

(2) one-one but not onto

(3) onto but not one-one

(4) Neither one-one nor onto

Sol. Answer (4)

fog = $f(g(x)) = g(x) - 1$

$= \frac{x^2}{x^2+1} - 1 = \frac{-1}{x^2+1}$

fog is even $\Rightarrow$ $\boxed{\text{many-one}}$

and $x^2 \geq 0 \Rightarrow x^2 + 1 \geq 1$

$\Rightarrow 0 < \frac{1}{x^2+1} \leq 1 \Rightarrow 0 > \frac{-1}{x^2+1} \geq -1$

Thus range is [–1 0)

fog is into

9. The sides of a cuboid are $2x$, $4x$, $5x$. There is a closed hemisphere of radius r. The sum of their surface area is a constant k. What is the ratio $x : r$ such that the sum of their volume is maximum

(1) $\frac{19}{45}$ (2) $\frac{45}{19}$

(3) $\frac{19}{24}$ (4) $\frac{24}{7}$

Sol. Answer (1)

Surface area of cuboid = $2(8x^2 + 20x^2 + 10x^2) = 76x^2$

Surface area of hemisphere = $2\pi r^2 + \pi r^2 = 3\pi r^2$

Sum of S.As(s) = $3\pi r^2 + 76x^2 = k$

Total volume (v) = $\frac{2}{3}\pi r^3 + 40x^3$

Now $\frac{dv}{dx} = 2\pi r^2 \frac{dr}{dx} + 120x^2 = 0$ ...(1)

But s is constant

$\Rightarrow \frac{ds}{dx} = 0 \Rightarrow \frac{d}{dx}(3\pi r^2 + 76x^2) = 0$

$\Rightarrow 6\pi r \frac{dr}{dx} + 152x = 0 \Rightarrow \boxed{\frac{dr}{dx} = \frac{-76x}{3\pi r}}$

Put this (1)

$\Rightarrow 2\pi r^2 \left(\frac{-76x}{3\pi r}\right) + 120x^2 = 0 \Rightarrow \boxed{\frac{x}{r} = \frac{19}{45}}$

10. If the given system $\alpha x + y + z = 5$, $x + 2y + 4z = 4$ and $x + 3y + 5z = \beta$ has infinitely many solution, then (α, β) is

(1) (0, 9) (2) (–1, –3)

(3) (–1, 3) (4) 1, –3)

Sol. Answer (1)

$(x + 2y + 4z - 4) + \lambda(x + 3y + 5z - \beta) = 0$ ...(1)

$\alpha x + y + z - 5 = 0$ ...(2)

Planes (1) and (2) are coincident

$\Rightarrow \frac{1+\lambda}{\alpha} = \frac{2+3\lambda}{1} = \frac{4+5\lambda}{1} = \frac{-4-\lambda\beta}{-5}$

$\Rightarrow 2 + 3\lambda = 4 + 5\lambda$

$\Rightarrow \lambda = -1$

$\Rightarrow \frac{0}{\alpha} = -1 = -1 = \frac{-4+\beta}{-5}$

$5 + 4 = \beta$

$\beta = 9$

$\alpha = 0$

11. Consider the function

$f(x) = \min\{1, 1 + x\sin x\}$, $x \in [0, \pi]$

(1) Continuous and differentiable in $[0, \pi]$

(2) Discontinuous at $x = \frac{\pi}{2}$

(3) Continuous but not differentiable at $\frac{\pi}{2}$

(4) Discontinuous at two points in $[0, \pi]$

Sol. Answer (1)

$f(x) = \min\{1, 1 + x\sin x\}$, $x \in [0, \pi]$

as $x \in [0, \pi]$, $x\sin x \geq 0$

as $1 + x\sin x \geq 1$

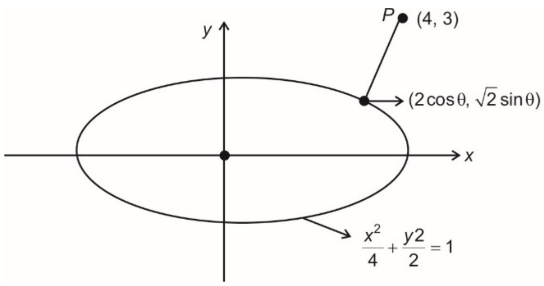
$\therefore$ $f(x) = 1$, continuous and differentiable on $[0, \pi]$

12. Find the eccentricity of locus of midpoint of line segments joining (4, 3) to each point on ellipse $\frac{x^2}{4} + \frac{y^2}{2} = 1$.

(1) $\frac{\sqrt{3}}{2}$ (2) $\frac{1}{2}$

(3) $\frac{1}{\sqrt{3}}$ (4) $\frac{1}{\sqrt{2}}$

Sol. Answer (4)



Let P = (4, 3)

$Q = \left(2\cos\theta, \sqrt{2}\sin\theta\right)$

Let the mid point of PQ be R = (h, k)

$\Rightarrow\ 2h = 4 + 2\cos\theta$ and $2k = 3 + \sqrt{2}\sin\theta$

$$\Rightarrow (h-2)^2 + \left(\frac{2k-3}{\sqrt{2}}\right)^2 = 1$$

$$\Rightarrow \frac{(x-2)^2}{1} + \frac{4\left(y-\frac{3}{2}\right)^2}{2} = 1$$

$$\therefore \text{ locus is } \frac{(x-2)^2}{1} + \frac{\left(y-\frac{3}{2}\right)^2}{\frac{1}{2}} = 1$$

$$\Rightarrow e^2 = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\Rightarrow e = \frac{1}{\sqrt{2}}$$

13. $\dfrac{24}{\pi}\displaystyle\int_0^{\sqrt{2}} \frac{2-X^2}{(2+x^2)\sqrt{4+x^4}}\,dx$ equals

(1) 0 (2) $\frac{1}{12}$

(3) 3 (4) $\frac{1}{8}$

Sol. Answer (3)

$$\frac{24}{\pi}\int_0^{\sqrt{2}} \frac{2-x^2}{(2+x^2)\sqrt{x^4+4}}\,dx$$

$$\frac{24}{\pi}\int_0^{\sqrt{2}} \frac{\frac{2}{x^2}-1}{\left(x+\frac{2}{x}\right)}\frac{dx}{\sqrt{x^2+\frac{4}{x^2}}}$$

$$= -\frac{24}{\pi}\int_0^{\sqrt{2}} \frac{\left(1-\frac{2}{x^2}\right)}{\left(x+\frac{2}{x}\right)\sqrt{\left(x+\frac{2}{x}\right)^2-4}}\,dx$$

Let $x + \frac{2}{x} = z$

$\left(1-\frac{2}{x^2}\right)dx = dz$

$$= \frac{-24}{\pi}\int \frac{dz}{z\sqrt{z^2-4}}$$

$$= \frac{-12}{\pi}\sec^{-1}\left(\frac{z}{2}\right)$$

$$= \frac{-12}{\pi}\sec^{-1}\left(\frac{1}{2}\left(x+\frac{2}{x}\right)\right)\Bigg]_0^{\sqrt{2}}$$

$$= \frac{-12}{\pi}\left[\sec^{-1}\left(\sqrt{2}\right) - \frac{\pi}{2}\right]$$

$$= \left(\frac{\pi}{4} - \frac{\pi}{2}\right) \times \frac{-12}{\pi} = 3$$

14. Let a_1, a_2, a_3, … are in G.P. such that $a_2 + a_4 = 2a_3 + 1$, $a_2 + a_3 = 2a_4$. Then the value of $a_2 + a_3 + 2a_4$ is

(1) $\frac{4}{27}$ (2) $\frac{4}{9}$

(3) $\frac{3}{14}$ (4) $\frac{3}{19}$

Sol. Answer (2)

In GP. $a_n = a_1 r^{n-1}$

Given $a_1 r + a_1 r^3 = 2a_1 r^2 + 1$ …(1)

And $a_1 r + a_1 r^2 = 2a_1 r^3$ …(2)

From (1), (2) $1 + r = 2r^2$

$\Rightarrow r = 1$ or $\frac{-1}{2}$

If $r = 1$ in (1) $\Rightarrow 2a_1 = 2a_1 + 1 \Rightarrow \boxed{1=0}$ (not possible)

$\Rightarrow \boxed{r = -\frac{1}{2}} \Rightarrow$ put in (1)

$\Rightarrow \boxed{a_1 = -\frac{8}{9}}$

$\Rightarrow a_2 + a_3 + 2a_4$

$= a_1 r(1 + r + 2r^2)$

$= \left(\frac{-8}{9}\right)\left(-\frac{1}{2}\right)\left(1 - \frac{1}{2} + \frac{1}{2}\right) = \frac{4}{9}$

15. If slope of common tangent of circle $x^2 + y^2 = 12$ and ellipse $\frac{x^2}{9} + \frac{y^2}{16} = 1$ is m then 12 m^2 is

(1) 16 (2) 3

(3) 4 (4) $\frac{4}{3}$

Sol. Answer (1)

Let the slope of common tangent be m equation of tangent to $x^2 + y^2 = 12$

$y = mx \pm \sqrt{12 + 12m^2}$

Equation of tangent to ellipse $\frac{x^2}{9} + \frac{y^2}{16} = 1$

$y = mx \pm \sqrt{9m^2 + 16}$

Both equation are identical

$\Rightarrow\ 9m^2 + 16 = 12 + 12m^2$

$3m^2 = 4$

$\Rightarrow\ m^2 = \frac{4}{3}$

$\therefore\ 12m^2 = 12 \cdot \frac{4}{3} = 16$

16. $\left[{}^{40}C_0 + {}^{41}C_1 + {}^{42}C_2 + \cdots + {}^{60}C_{20}\right] = \frac{m}{n}\ {}^{60}C_{20}$ where m & n are co-prime, then $m + n$ is equal to

Sol. Answer (102)

${}^{40}C_0 + {}^{41}C_1 + {}^{42}C_2 + \cdots + {}^{60}C_{20}$

$\Rightarrow {}^{40}C_{40} + {}^{41}C_{40} + {}^{42}C_{40} + \cdots\ {}^{60}C_{40}$ ...(1)

${}^{r}C_r + {}^{r+1}C_r + {}^{r+2}C_r + \cdots\ {}^{r+n}C_r = {}^{n+r+1}C_{r+1}$

Equation (1) becomes

${}^{40}C_r + {}^{41}C_{40} + \cdots\ {}^{60}C_{40} = {}^{61}C_{41}$

$= \frac{61}{41} \cdot {}^{60}C_{40}$

$= \frac{61}{41} \cdot {}^{60}C_{20}$

$m = 61$

$n = 41$

m + n = 61 + 41 = 102

17. If L is tangent to hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ and L_2 a straight line passing through (0, 0) and perpendicular to L_1. If the locus of point of intersection of L_1 & L_2 is $\left(x^2 + y^2\right)^2 = \alpha x^2 + \beta y^2$, then $\alpha + \beta$ is equal to

Sol. Answer (5)

$\frac{x^2}{9} - \frac{y^2}{4} = 1$ ...(1)

Equation of tangent to (1) is

$L_1 : y = mx + \sqrt{9m^2 - 4}$ ...(2)

Now $L_2 : y = -\frac{x}{m}$ ...(3)

Eliminate m from (2) & (3)

$\Rightarrow y = \frac{-x^2}{y} + \sqrt{\frac{9x^2}{y^2} - 4}$

$\Rightarrow \frac{\left(y^2 + x^2\right)}{y} = \sqrt{\frac{9x^2}{y^2} - 4}$

$\Rightarrow \left(x^2 + y^2\right)^2 = \left(9x^2 - 4y^2\right)$

Compare it with

$\left(x^2 + y^2\right)^2 = \alpha x^2 + \beta y^2$

$\alpha = 9,\ \beta = -4$

$\Rightarrow \alpha + \beta = 5$

18. If p and q are real number and $p + q = 3$, $p^4 + q^4 = 369$. Find $\left(\frac{1}{p} + \frac{1}{q}\right)^{-2}$.

Sol. Answer (4)

$p + q = 3$

$\Rightarrow p^2 + q^2 + 2pq = 9$

$\Rightarrow \left(p^2 + q^2\right) = (9 - 2pq)^2$

$\Rightarrow p^4 + q^4 + 2p^2q^2 = (9 - 2pq)^2$

$\Rightarrow 369 + 2p^2q^2 = 81 + 4p^2q^2 - 36pq$

$\Rightarrow 2p^2q^2 - 36pq - 288 = 0$

$\Rightarrow p^2q^2 - 18pq - 144 = 0$

$\Rightarrow pq = -6\ or\ 24$

If $pq = 24$,

$p^2+q^2=9-2(pq)=-ve$ (not possible)

Hence, $pq = -6$

Now $\left(\frac{1}{p}+\frac{1}{q}\right)^{-2}=\left(\frac{pq}{p+q}\right)^2=\left(\frac{-6}{3}\right)^2=4$

19. If $z^2+z+1=0, z\in C$ $\left|\sum_{k=1}^{15}\left(z^k+\frac{1}{z^k}\right)^2\right|$ is equal to

Sol. Answer (30)

$z^2+z+1=0 \Rightarrow z=\omega$ or ω^2

$$\sum_{k=1}^{15}\left(z^k+\frac{1}{z^k}\right)^2=\left(\omega+\frac{1}{\omega}\right)^2+\left(\omega^2+\frac{1}{\omega^2}\right)^2+\left(\omega^3+\frac{1}{\omega^3}\right)^2+\cdots+\left(\omega^{15}+\frac{1}{\omega^{15}}\right)^2$$

$$=\left(\omega+\omega^2\right)^2+\left(\omega^2+\omega\right)^2+(1+1)^2+\cdots+(1+1)^2$$

$$=(-1)^2+(-1)^2+(2)^2+(-1)^2+(-1)^2+(2)^2+\cdots+(2)^2$$

$$=5\times 2^2+10(-1)^2=20+10=30$$

20. A six digit number is formed randomly using digits 1 and 8. If the probability of this number is divisible by 21 is P, then $96P$ is equal to

Sol. Answer (33)

To make divisible by 3

Sum must be divisible by 3

$\Rightarrow$ this can happen only if no. of 1's = no. of 8's or Every digit is 1 or every digit is 8

Now to make divisible by 7

|2(last digit) – (remaining number)| = $7k_1$, $k\in Z$

This is only possible if

(i) All are 1s	111111	$\to 1$
or		
(ii) All are 8 s	888888	$\to 1$
or		
(iii) 3 1s and 3 8s		$\to \frac{6!}{3!3!}$

Hence total possibilities = 2^6

And total no. divisible by 21 = no.s divisible by 3 & 7 = $=\frac{6!}{3!3!}+2=22$

$\Rightarrow P=\frac{22}{2^6}$

$96P=33$

21. The mean of 50 observations is 15 and standard deviation is 2. However one observation was wrongly recorded. The sum of the correct and incorrect observation is 70. If the mean of the corrected set of observation is 16, then the variance is

Sol. Answer (43)

$\bar{x}=\frac{\sum x_i}{50}=15$

$\Rightarrow \sum x_i = 750$ ...(1)

$x^2=\frac{\sum (x_i)^2}{50}-(15)^2=4$

$\sum (x_i)^2=11450$

New $\bar{x}=16$

$\Rightarrow \sum x_{i\,new}=16\times 50=800$ ...(2)

Let a be the incorrect observation

$\therefore$ a + 50 be the correct observation $(\because (2)-(1)=50)$

$a + a + 50 = 70$

$\Rightarrow a = 10$

Correct observation = 60

$\therefore$ New variance = $\frac{11450-10^2+60^2}{50}-(16)^2$

$=\frac{14950}{50}-256$

= 299 – 256 = 43

22. Find number of three digits numbers n such that GCD(n, 36) = 2.

Sol. Answer (150)

$36=2^2\times 3^2$

$\Rightarrow$ To get $gcd(n, 36) = 2$

Power of 2 in n must be exactly 1.

$\Rightarrow$ No. is divisible by 2 but not divisible by 4 and not divisible by 3 also.

So by Inclusion exclusion total sum numbers

= (divisible by 2) – (divisible by 4) – (divisible by 3) + (divisible by 12)

= 451 – 226 – 150 + 75 = 150

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