Regd. Office : Aakash Tower, 8, Pusa Road, New Delhi-110005, Ph.011-47623456

Mathematics JEE Solutions 2022

Mathematics

1. The value of $\cot\left(\sum_{n=1}^{50}\tan^{-1}\left(\frac{1}{1+n+n^2}\right)\right)$ is equal to

(1) $\frac{25}{26}$ (2) $\frac{3}{25}$

(3) $\frac{26}{25}$ (4) $\frac{3}{26}$

Sol. Answer (3)

$$\cot\left(\sum_{n=1}^{50}\tan^{-1}\left(\frac{1}{1+n+n^2}\right)\right)$$

$$=\cot\left(\sum_{n=1}^{50}\tan^{-1}\left(\frac{(n+1)-n}{1+(n+1)\cdot n}\right)\right)$$

$$=\cot\left(\sum_{n=1}^{50}\tan^{-1}(n+1)-\tan(n)\right)$$

$$=\cot(\tan^{-1}(2)-\tan^{-1}(1))+(\tan^{-1}(3)-\tan^{-1}(2))$$

$$+\cdots(\tan^{-1}(51)-\tan^{-1}(50))$$

$$=\cot(\tan^{-1}(51)-\tan^{-1}(1))$$

$$=\cot\left(\tan^{-1}\frac{(51-1)}{1+51}\right)=\cot\left(\tan^{-1}\frac{50}{52}\right)$$

$$=\cot\left(\cot^{-1}\frac{52}{50}\right)$$

$$\frac{52}{50}=\frac{26}{25}$$

2. If $s=2+\frac{6}{7}+\frac{12}{7^2}+\frac{20}{7^3}+\cdots\infty$ then find $4s$.

(1) $\left(\frac{7}{2}\right)^2$ (2) $\left(\frac{7}{3}\right)^3$

(3) $\frac{7}{3}$ (4) $\left(\frac{7}{3}\right)^4$

Sol. Answer (2)

$$s=2+\frac{6}{7}+\frac{12}{7^2}+\frac{20}{7^3}+\cdots\infty$$

$$\Rightarrow \frac{s}{7}=\frac{2}{7}+\frac{6}{7^2}+\frac{12}{7^3}+\cdots\infty$$

Subtracting $\frac{6s}{7}=2+\frac{4}{7}+\frac{6}{7^2}+\frac{8}{7^3}+\cdots\infty$

$$\frac{6s}{49}=\frac{2}{7}+\frac{4}{7^2}+\frac{6}{7^3}+\cdots\infty$$

$$\frac{36s}{49}=2+\frac{2}{7}+\frac{2}{7^2}+\cdots\infty \text{ \{infinite g.p\}}$$

$$\Rightarrow \frac{36s}{49}=\frac{2}{1-\frac{1}{7}}=\frac{14}{6}$$

Hence $s=\left(\frac{7}{3}\right)\times\frac{49}{36}$ and $4s=\left(\frac{7}{3}\right)^3$

3. The number of complex numbers z satisfying $|z-(4+3i)|=2$ and $|z|+|z-4|=6$ is

(1) 1 (2) 2

(3) 3 (4) 4

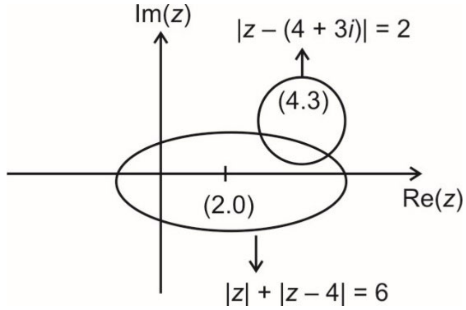
Sol. Answer (2)

$|z-(4+3i)|=2$ and $|z|+|z-4|=6$

$|z-(4+3i)|=2$ represents a circle with centre (4, 3)

$|z|+|z-4|=6$ represents an ellipse with foci (0, 0) and (4, 0)

and length of major axis as 6 units.



In the coordinate form,

$$|z-(4+3i)|=2 \;\Rightarrow\; (x-4)^2+(y-3)^2=4$$

$$|z|+|z-4|=6 \;\Rightarrow\; \frac{(x-2)^2}{3^2}+\frac{(y-0)^2}{5}=1$$

Drawing the two diagrams, we can verify that the two curves intersect at two distinct points.

4. If f is differentiable function such that $\int_{\cos x}^{1} t^2 f(t)\,dt = \sin^3 x+\cos x$. Then the value of $\frac{1}{\sqrt{3}}f'\left(\frac{1}{\sqrt{3}}\right)$

(1) $6-\frac{21}{\sqrt{2}}$ (2) $6+\frac{9}{\sqrt{2}}$

(3) $6-\frac{9}{\sqrt{2}}$ (4) $3-\sqrt{21}$

Sol. Answer (3)

$$\because \int_{\cos x}^{1} t^2 f(t)\,dt = \sin^3 x+\cos x \quad \ldots(1)$$

On differentiating both sides w.r.t. x we get

$$\sin x.\cos^2 x f(\cos x) = 3\sin^2 x\cdot\cos x-\sin x$$

$$\therefore\; \cos^2 x\cdot f(\cos x) = 3\sin x\cdot\cos x-1 \;\ldots(2)$$

When $\cos x=\frac{1}{\sqrt{3}}$ then $\sin x=\frac{\sqrt{2}}{\sqrt{3}}$, if $x\in\left(0,\frac{\pi}{2}\right)$.

$$\therefore\; f\left(\frac{1}{\sqrt{3}}\right)=3\left(\sqrt{2}-1\right) \quad \ldots(3)$$

Again on differentiating both sides of Eq. (2) w.r.t. x we get:

$$-2\sin x\cdot\cos x\cdot f(\cos x)+\cos^2 x\cdot f'(\cos x)(-\sin x) = 3\cos 2x$$

On replacing the values we get:

$$-2\sqrt{2}\left(\sqrt{2}-1\right)-\frac{\sqrt{2}}{3\sqrt{3}}f'\left(\frac{1}{\sqrt{3}}\right)=-1$$

$$\therefore\; \frac{1}{\sqrt{3}}f'\left(\frac{1}{\sqrt{3}}\right)=\frac{3}{\sqrt{2}}\left(2\sqrt{2}-3\right) \;=6-\frac{9}{\sqrt{2}}$$

5. Which of the following is a tautology?

(1) $(\sim p\wedge q)\vee(p\vee\sim p)$

(2) $(p\to q)\vee q$

(3) $(p\leftrightarrow q)\vee(p\wedge q)$

(4) $p\vee(p\leftrightarrow q)$

Sol. Answer (1)

$(\sim p\wedge q)\vee(p\vee\sim p)$

$\equiv(\sim p\wedge q)\vee T$ ($\because$ $p\vee\sim p$ is always tautology)

$\equiv T$

6. The equation of parabola whose vertex is (5, 4) and equation of directrix is $3x + y - 29 = 0$ is $x^2 + ay^2 + bxy + cx + dy + e = 0$. The value of $(a + b + c + d + e)$ is

(1) 711 (2) –711

(3) 567 (4) –576

Sol. Answer (4)

Let focus at $s(\alpha, \beta)$ by

Foot of perpendicular formula

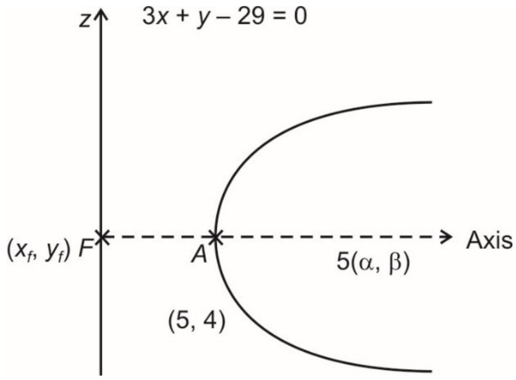
$$\frac{x_f-5}{3}=\frac{y_f-4}{1}=\frac{-(15+4-29)}{10}=1$$

$\Rightarrow\; x_f=8$ and $y_f=5\Rightarrow F(8,5)$

Now by mid point formula

$$\Rightarrow\; \frac{\alpha+8}{2}=5 \text{ and } \frac{\beta+5}{2}=4$$

$$\Rightarrow\; \boxed{\alpha=2 \text{ and } \beta=3}$$



Thus focus s lies at (2, 3) by definition of parabola

$$(x-2)^2+(y-3)^2=\frac{(3x+y-29)^2}{10}$$

$$10(x^2+y^2-4x-6y+13)$$
$$=9x^2+y^2+841+6xy-58y-174x$$

Here final equation of parabola

$$\equiv \boxed{x^2+9y^2-6xy+134x-2y-711=0}$$

$\Rightarrow$ a = 9, b = –6, c = 134, d = –2 and e = –711

$$\Rightarrow \boxed{a+b+c+d+e=-576}$$

7. The shortest distance between the lines $\frac{x-1}{4}=\frac{y-2}{2}=\frac{z-3}{3}$ and $\frac{x-5}{5}=\frac{y-3}{6}=\frac{z-2}{7}$ is

(1) $\sqrt{43}$ (2) $\frac{43}{\sqrt{381}}$

(3) $\frac{43}{\sqrt{391}}$ (4) $\sqrt{381}$

Sol. Answer (2)

In vector from

$$l_1 \equiv \left(\hat{i}+2\hat{j}+3\hat{k}\right)+\lambda\left(4\hat{i}+2\hat{j}+3\hat{k}\right)$$

$$l_2 \equiv \left(5\hat{i}+3\hat{j}+2\hat{k}\right)+\mu\left(5\hat{i}+6\hat{j}+7\hat{k}\right)$$

Clearly $\vec{a}_1=\hat{i}+2\hat{j}+3\hat{k}, \vec{a}_2=5\hat{i}+3\hat{j}+2\hat{k}$

$\vec{b}_1=4\hat{i}+2\hat{j}+3\hat{k}$ and $\vec{b}_2=5\hat{i}+6\hat{j}+7\hat{k}$

Shortest distance = $\left|\frac{(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2)}{\left|\vec{b}_1\times\vec{b}_2\right|}\right|$

Now $\vec{a}_2-\vec{a}_1=4\hat{i}+\hat{j}-\hat{k}$

$$\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 4 & 2 & 3\\ 5 & 6 & 7\end{vmatrix}=-4\hat{i}-13\hat{j}+14\hat{k}$$

by putting all in formula

$$\text{S.D}=\left|\frac{-16-13-14}{\sqrt{16+169+196}}\right|=\frac{43}{\sqrt{381}}$$

8. The value of $\int_0^1 \frac{dx}{7^{\left[\frac{1}{x}\right]}}$ is (where [·] denotes the greatest integer function)

(1) $1-6\ln\left(\frac{6}{7}\right)$ (2) $1+6\ln\left(\frac{6}{7}\right)$

(3) $1-7\ln\left(\frac{6}{7}\right)$ (4) $1+7\ln\left(\frac{6}{7}\right)$

Sol. Answer (2)

$I=\int_0^1 \frac{dx}{7^{[1/x]}}$ Let $x=\frac{1}{t}$

$$\Rightarrow dx=-\frac{1}{t^2}dt$$

$$\Rightarrow I=\int_1^{\infty}\frac{dt}{7^{[t]}t^2}$$

Now Let $I_k=\int_k^{k+1}\frac{dt}{7^{[t]}t^2}$ k = 1,2,3,…

$$\Rightarrow I_k=\int\frac{dt}{7^k t^2}=\frac{1}{7^k}\left[-\frac{1}{t}\right]_k^{k+1}=\frac{1}{7^k}\left[\frac{1}{k}-\frac{1}{k+1}\right]$$

$$I_k=\frac{1}{k\cdot 7^k}-\frac{7}{(k+1)\cdot 7^{k+1}}$$

Now $I=I_1+I_2+I_3+\ldots\infty$

$$=\left(\frac{1}{1\cdot 7}+\frac{1}{2\cdot 7^2}+\frac{1}{3\cdot 7^3}+\ldots\infty\right)-7\left(\frac{1}{2\cdot 7^2}+\frac{1}{3\cdot 7^3}+\ldots\infty\right)$$

$$=-\log\left(1-\frac{1}{7}\right)+7\left[\log\left(1-\frac{1}{7}\right)+\frac{1}{7}\right]$$

$$=-\log\frac{6}{7}+7\log\frac{6}{7}+1=1+6\log\frac{6}{7}$$

9. If a_1, a_2, … and b_1, b_2, … are two A.P's and a_1 = 2, a_{10} = 3 if a_1b_1 = 1 = $a_{10}b_{10}$. Find a_4b_4

(1) $\frac{27}{28}$ (2) $\frac{28}{27}$

(3) $\frac{9}{16}$ (4) $\frac{16}{9}$

Sol. Answer (2)

Let $a_1, a_2, a_3 ..., a_{10}$ are in AP with common difference d_1.

So, $9d_1 = a_{10} - a_1 = 1$

$\Rightarrow d_1 = \frac{1}{9}$

Hence, $a_4 = a_1 + 3d_1 = 2 + \frac{3}{9} = \frac{7}{3}$

Let $b_1, b_2, b_3 ..., b_{10}$ ae in AP with common difference d_2. Here $b_1 = \frac{1}{2}$ and $b_{10} = \frac{1}{3}$

So, $9d_2 = b_{10} - b_1 = -\frac{1}{6}$

$\Rightarrow d_2 = -\frac{1}{54}$

$b_4 = b_1 + 3d_2 = \frac{1}{2} - \frac{1}{18} = \frac{4}{9}$

So, $a_4 \cdot b_4 = \frac{28}{27}$

10. *A* and *B* are two 3 × 3 matrices such that *AB* = I, $|A| = \frac{1}{8}$, then find $|adj(B.adj2A)|$.

(1) 128 (2) 32

(3) 64 (4) 102

Sol. Answer (3)

Given : $AB = I$, $|A| = \frac{1}{8}$, *A* & *B* are square matrices of order 3.

To find $|adj(B(adj2A))|$

$$|AB| = 1$$
$$|A||B| = 1$$
$$|B| = 8$$

We know $adj(KA) = k^{n-1}adj(A)$, *n* is order of matrix *A.*

$\therefore |adj(B.adj2A)|$

$= |B|^2 |adj2A|^2$

$= |B|^2 |2^2 adjA|^2$

$= |B|^2 \times 2^{12} \times |A|^4$

$= 2^{12} \times \left(\frac{1}{8}\right)^4 \times 8^2$

$= 8^4 \times \frac{1}{8^2} = 64$

11. If the curve satisfying the differential equation $(\tan^{-1} y + x)dx = (1 + y^2)dx$ passes through (1, 0) then find *x* at *y* = 1

(1) $\left(\frac{\pi}{4} - 1\right)$

(2) $-e^{-\pi/4} + \frac{4}{\pi}\left(e^{-\pi/4} + 1\right)$

(3) $\left(\frac{\pi}{4} - 1\right) + 2e^{-\pi/4}$

(4) $\left(\frac{\pi}{4} + 1\right) + 2e^{\pi/4}$

Sol. Answer (2)

$(\tan^{-1} y + x)\, dy = (1 + y^2)dx$

$\Rightarrow \frac{dx}{dy} - \frac{x}{1 + y^2} = \frac{\tan^{-1} y}{1 + y^2}$

I.F $= e^{-\int \frac{1}{1+y^2} dy} = e^{-\tan^{-1} y}$

Solution is $x \times e^{-\tan^{-1} y} = \int e^{-\tan^{-1} y} \frac{(\tan^{-1} y)}{1 + y^2} dx$

$= \int e^{-t}\, t\, dt$ Put $\tan^{-1} y = t$

$= -te^{-t} - e^{-t} + c$

$\Rightarrow x \tan^{-1} y = -e^{-\tan^{-1} y}(\tan^{-1} y + 1) + c$

Sub (1, 0) $\Rightarrow$ 0 = –1 + *c* $\Rightarrow$ *c* = 1

for $y = 1 \Rightarrow x \times \frac{\pi}{4} = -e^{\pi/4}\left(\frac{\pi}{4} + 1\right) + 1$

$x = -e^{-\pi/4} + \frac{4}{\pi}\left(e^{-\pi/4} + 1\right)$

12. sin 36° is root of equation whose coefficients are rational is given by

(1) $16x^4 + 20x^2 - 5 = 0$

(2) $16x^4 + 20x^2 + 5 = 0$

(3) $16x^4 - 20x^2 + 5 = 0$

(4) $8x^4 - 20x^2 + 5 = 0$

Sol. Answer (3)

$\sin 36° = x = \frac{\sqrt{10 - 2\sqrt{5}}}{4}$

$16x^2 = 10 - 2\sqrt{5}$

$(16x^2 - 10)^2 = 20$

$(8x^2 - 5)^2 = 5$

$64x^4 + 25 - 80x^2 - 5 = 0$

$64x^2 - 80x^2 + 20 = 0$

$16x^4 - 20x^2 + 5 = 0$

13. Let A be a 2 × 2 matrix, whose entries are taken from the set {0, 1, 2, 3, 4, 5, 6} such that sum of all entries is a prime number between 2 and 6 (both excluded). Find number of possible

Sol. Answer (76.00)

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$a, b, c, d \in \{0, 1, 2, 3, 4, 5, 6\}$

ATQ, $a + b + c + d = P$ $\quad P \in (2, 6)$

as P is a prime no so P can be 3, 5

Case 1: $a + b + c + d = 3$

One number be 3, other are '0' = ${}^4C_1$ = 4 ways

One number be 2, one be '1' and other two are zero

${}^4C_1 \times {}^3C_1 \times {}^2C_2$ = 12 ways

3 number be one, one number be zero

= ${}^4C_1$ = 4 ways

So total ways = 4 + 4 + 12 = 20 ways

Case 2: $a + b + c + d = 5$

(5, 0, 0, 0), (4, 1, 0, 0), (2, 3, 0, 0)

(1, 2, 2, 0), (3, 1, 1, 0), (2, 1, 1, 1)

Possible arrangement

$$= \frac{4!}{3!} + \frac{4!}{2!} + \frac{4!}{2!} + \frac{4!}{2!} + \frac{4!}{2!} + \frac{4!}{3!}$$

= 4 + 12 + 12 + 12 + 12 + 4 = 56 ways

Total required ways = 56 + 20 = 76 ways

14. Consider element 4, 5, 6, 6, 7, 8, x, y. If mean = 6 and variance = $\frac{9}{4}$. Find $x^4 + y^2$.

Sol. Answer (320)

Elements $\rightarrow$ 4,5,6,6, 7,8, x,y ($x < y$)

To find $x^4 + y^2$

Mean = 6

Variance = $\frac{9}{4}$

mean = $\frac{4+5+6+6+7+8+x+y}{8} = 6$

$\Rightarrow 36 + x + y = 48$

$\Rightarrow x + y = 12 \quad \ldots(1)$

Also,

Variance = $\frac{\sum x_i^2}{n}$ -(mean)^2 = $\frac{4^2+5^2+2\times 6^2+7^2+8^2+x^2+y^2}{8} - (6)^2$

$$\frac{9}{4} = \frac{226 + x^2 + y^2}{8} - 36$$

$$\Rightarrow 226 + x^2 + y^2 = 8\left(\frac{9}{4} + 36\right) = 8\left(\frac{153}{4}\right) = 306$$

$x^2 + y^2 = 80$

$\Rightarrow (x+y)^2 - 2xy = 80$

$\Rightarrow 2xy = 144 \cdot 80 = 64$

$xy = 32$

$\therefore x + y = 12,\ xy = 32$

$\Rightarrow x = 4,\ y = 8\ (\because x < y)$

$\Rightarrow x^4 + y^2 = 8^2 + 4^4 = 64 + 256 = 320$

15. $f(x) = [1+x] + \dfrac{\alpha^{2[x]+\{x\}} + [x] - 1}{2[x] + \{x\}}$

If $\lim\limits_{x \to 0^-} f(x) = \alpha - \frac{4}{3}$. What is the integral value of α

Sol. Answer (03.00)

If $x \to 0^-$

$\Rightarrow [x] = -1\ \&\ \{x\} = x + 1$

$$\lim_{x \to 0^-} [1+x] + \frac{\alpha^{2(-1)+x+1} + (-1) - 1}{2(-1) + x + 1}$$

$$= 1 + (-1) + \frac{\alpha^{x-1} - 2}{x - 1}$$

$$\Rightarrow \lim_{x \to 0^-} \frac{\alpha^{x-1} - 2}{x - 1}$$

$$\Rightarrow \frac{\alpha^{-1} - 2}{-1} = \alpha - \frac{4}{3}$$

$$\Rightarrow \frac{1}{\alpha} - 2 = \frac{4}{3} - \alpha$$

$$\Rightarrow 3 - 6\alpha = 4\alpha - 3\alpha^2$$

$\Rightarrow 3\alpha^2 - 10\alpha + 3 = 0$ where $\alpha = 3$ or $\alpha = \frac{2}{3}$

Hence integral value is $\alpha = 3$.

16. $f(x) = \int_0^{x^2} \frac{t^2 - 5t + 4}{2 + e^t} dt$

Find the number of points of Local Max. & Local Min.

Sol. Answer (05.00)

$f'(x) = \left(\frac{x^4 - 5x^2 + 4}{2 + e^{x^2}}\right)(2x) = 0$

$\Rightarrow x\left(x^2 - 1\right)\left(x^2 - 4\right) = 0$

$\Rightarrow x(x-1)(x+1)(x-2)(x+2) = 0$

$\therefore$ There are total 5 points of lol max. and min.

17. $y(x) = (x^x)^x$ then the value of $\frac{d^2y}{dx^2} + 20$ at $x = 1$ is

Sol. Answer (24.00)

$y(x) = (x^x)^x$ at $x = 1 \Rightarrow y = 1$

$\ell ny = x^2 \ell nx$

$\frac{1}{y}y' = x^2\frac{1}{x} + 2x \cdot \ln x$

$y' = y(x + 2x\ln x) \qquad \ldots(1)$

$(y')_{x=1} = 1(1+0) = 1$

$y'' = y\left(1 + \frac{2x}{x} + 2\ln x\right) + (x + 2x\ln x)\cdot y'$

$(y'')_{x=1} = 1(1+2+0) + (1\times 1) = 4$

So, $\frac{d^2y}{dx^2} + 20 = 24$

❑ ❑ ❑