Aakash + BYJU'S

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Mathematics JEE Solutions 2022

Mathematics

1. A matrix 'A' of order 3 × 3 and det(A) = 2, then det(det(adj adj A)^3) is equal to

(1) 2^{23} (2) 2^{13}

(3) 2^{15} (4) 2^{12}

Sol. Answer (3)

$$\left||A|(\text{adj }(\text{adj } A))^3\right| = |A|^3\,|\text{adj}(\text{adj } A)|^3$$

$$(\because\ |kA| = k^n|A|)$$

$$= |A|^3\left(|A|^4\right)^3 \quad \left(\because\ |\text{adj}(\text{adj } A)| = |A|^{(n-1)^2}\right)$$

$$= |A|^{15} = 2^{15}$$

2. Number of real solution of the equation $4x^7 + 3x^3 + 5x + 1 = 0$ is/are

(1) 0 (2) 1

(3) 2 (4) 3

Sol. Answer (2)

$$f'(x) = 28x^6 + 9x^2 + 5$$

$$x^6 \geq 0,\ x^2 \geq 0 \text{ and } 5 > 0$$

$\Rightarrow f'(x) > 0 \Rightarrow f(x)$ is increasing and f is 7th degree polynomial

Hence it can only intersect x-axis once

Thus $f(x) = 0$ has exactly one real solution

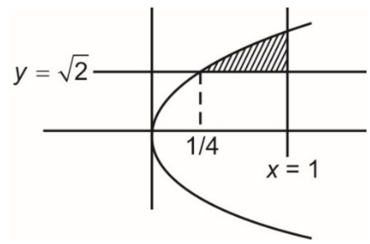
3. Area of the region $\{(x, y) : y^2 \leq 8x,\ y \geq \sqrt{2},\ x \leq 1\}$

(1) $\frac{7}{3\sqrt{2}}$ (2) $\frac{3}{2\sqrt{2}}$

(3) $\frac{6}{\sqrt{2}}$ (4) $\frac{5}{6\sqrt{2}}$

Sol. Answer (4)

$$y^2 \leq 8x, \quad y \geq \sqrt{2} \quad x \leq 1$$



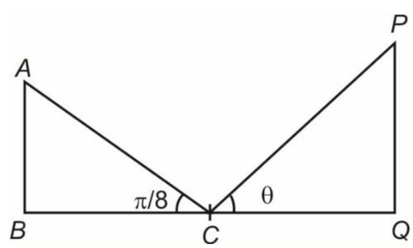
$$\text{Area} = \int_{1/4}^{1}\left(2\sqrt{2}x^{1/2} - \sqrt{2}\right)dx$$

$$= \left[2\sqrt{2} \times \frac{2}{3}x^{3/2} - \sqrt{2}x\right]_{1/4}^{1}$$

$$= \frac{4\sqrt{2}}{3}\left(1 - \frac{1}{8}\right) - \sqrt{2}\cdot\frac{3}{4}$$

$$= \frac{4\sqrt{2}}{3}\cdot\frac{7}{8} - \frac{3\sqrt{2}}{4} = \frac{5}{6\sqrt{2}}$$

4.



($PQ = 2AB$)

Two poles 160 m apart and C is the mid-point, then find $\tan^2\theta$.

(1) $2\left(\sqrt{2}+1\right)$ (2) $2\left(\sqrt{2}-1\right)$

(3) $4\left(3+2\sqrt{2}\right)$ (4) $4\left(3-2\sqrt{2}\right)$

Sol. Answer (4)

If C is mid-point

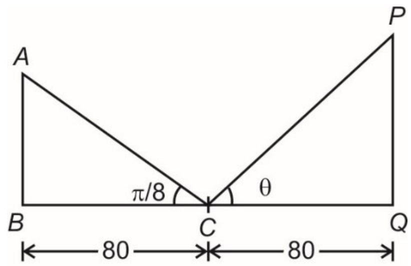
Then $BC = CQ = 80$ m

In ΔABC

$\frac{AB}{BC} = \tan\frac{\pi}{8}$ ...(1)

In ΔPQC

$\frac{PQ}{QC} = \tan\theta$ ...(2)



Dividing (1) and (2)

$\Rightarrow \frac{AB}{PQ} \times \frac{QC}{BC} = \frac{\tan\frac{\pi}{8}}{\tan\theta}$ $\{QC = BC$ and $PQ = 2AB\}$

$\Rightarrow \frac{\tan\frac{\pi}{8}}{\tan\theta} = \frac{1}{2} \Rightarrow \tan\theta = 2\tan\frac{\pi}{8}$

Here $\tan^2\theta = 4\tan^2\frac{\pi}{8} = 4(\sqrt{2}-1)^2 = 4(3-2\sqrt{2})$

5. Eccentricity of hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $\frac{\sqrt{5}}{2}$ and length of latus rectum is $\frac{\sqrt{3}}{2}$. If tangent is $y = 2x + c$, then the value of c is ______.

(1) $\pm\frac{3\sqrt{5}}{2}$ (2) $\pm\frac{5\sqrt{3}}{2}$

(3) $\pm\frac{3\sqrt{2}}{5}$ (4) $\pm\frac{5\sqrt{2}}{3}$

Sol. Answer (1)

$e^2 = 1 + \frac{b^2}{a^2}$

$\frac{5}{4} = 1 + \frac{b^2}{a^2} \Rightarrow \frac{b^2}{a^2} = \frac{1}{4}$ ...(1)

$\therefore$ L.R = $\frac{\sqrt{3}}{2} \Rightarrow \frac{2b^2}{a} = \frac{\sqrt{3}}{2}$

$\frac{4b^4}{a^2} = \frac{3}{4}$

$\Rightarrow \frac{4b^4}{4b^2} = \frac{3}{4}$

$\therefore b^2 = \frac{3}{4}$ $a^2 = 4b^2$

$a^2 = 3$

Now $a^2 = 3$ $b^2 = \frac{3}{4}$

$y = 2x + c$ in tangent

$c^2 = a^2m^2 - b^2$

$c^2 = 3 \cdot 4 - \frac{3}{4} \Rightarrow c^2 = \frac{45}{4}$

$c = \pm\frac{3\sqrt{5}}{2}$

6. $\sum_{k=1}^{31} {}^{31}C_k \cdot {}^{31}C_{k-1} - \sum_{k=1}^{30} {}^{30}C_k \cdot {}^{30}C_{k-1}$ is equal to

(1) ${}^{62}C_{32} - {}^{59}C_{31}$ (2) ${}^{62}C_{32} - {}^{60}C_{31}$

(3) ${}^{62}C_{31} - 1$ (4) ${}^{62}C_{32}$

Sol. Answer (2)

$\sum_{k=1}^{31} {}^{31}C_k \times {}^{31}C_{k-1} = {}^{31}C_0 \times {}^{31}C_1 + {}^{31}C_1 \times {}^{31}C_2 + \cdots + {}^{31}C_{30}\ {}^{31}C_{31}$

$=$ coefficient x in $\left(1+\frac{1}{x}\right)^{31}(1+x)^{31}$

$=$ coefficient x in $\frac{(1+x)^{62}}{x^{31}} = {}^{62}C_{32}$

Similarly $\sum_{k=1}^{30} {}^{30}C_k \times {}^{30}C_{k-1} = {}^{60}C_{31}$

$\therefore$ Answer ${}^{62}C_{32} - {}^{60}C_{31}$

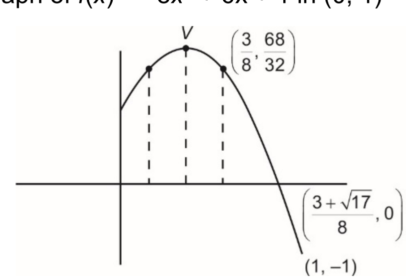
7. If $\int_0^1 \left[-8x^2 + 6x + 1\right] dx = t$ where [·] notes G.I.F then t equal to

(1) $\frac{\sqrt{17}-3}{8}$ (2) $\frac{3+\sqrt{17}}{8}$

(3) $-\frac{5}{4}$ (4) –1

Sol. Answer (2)

Graph of $f(x) = -8x^2 + 6x + 1$ in (0, 1)



$\rightarrow$ Clearly positive roots is

$\frac{3+\sqrt{17}}{8}$ {by quadratic formula}

$\rightarrow$ for vertex $\Rightarrow \frac{d}{dx}(-8x^2+6x+1)=0$

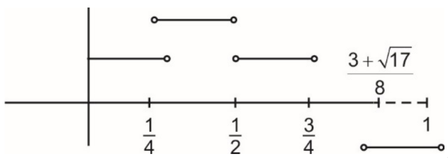
$$\Rightarrow\ x=\frac{3}{8} \text{ and } y=\frac{68}{32}=\frac{17}{8}$$

$\rightarrow -8x^2 + 6x + 1 = 2$ at $x=\frac{1}{4}$ and $x=\frac{1}{2}$

$\rightarrow -8x^2 + 6x + 1 = 1$ at $x = 0$ and $x = \frac{3}{4}$

So $\int_0^1 \left[-8x^2+6x+1\right] = \int_0^{1/4} dx + \int_{1/4}^{1/2} 2dx + \int_{1/2}^{3/4} 1dx$

$$+\int_{3/4}^{(3+\sqrt{17})/8} 0\,dx + \int_{\frac{3+\sqrt{17}}{8}}^{1} (-1)\,dx$$



$=\frac{1}{4}+\frac{1}{2}+\frac{1}{4}-1+\frac{3+\sqrt{17}}{8}$

$= \boxed{=\frac{3+\sqrt{17}}{8}}$

8. Find the number of 5 digit numbers formed by choosing digits form 1, 2, 3, 5, 6, 7 without repetition, such that it is divisible by 6

(1) 72 (2) 48

(3) 36 (4) 24

Sol. Answer (1)

1 + 2 + 3 + 5 + 6 + 7 = 24

Case 1: 1 + 2 + 5 + 6 + 7 = 21

$\underbrace{-\ -\ -\ -}_{4!}$ $\boxed{\text{2 or 5}}$ (2 ways)

Number of such no.s = 2 × 4! = 48

Case 2: 1 + 2 + 3 + 5 + 7 = 18

$\underbrace{-\ -\ -\ -}_{4!}$ $\boxed{2}$

Number of such numbers = 4! = 24

Total = 48 + 24 = 72

9. Let $f : N \rightarrow N$ be defined as

$$f(n)=\begin{cases} 2n & \text{if } n=2,4,6,\ldots \\ n-1 & \text{if } n=3,7,11,\ldots \\ \frac{n+1}{2} & \text{if } n=1,5,9,\ldots \end{cases}$$

Then $f(n)$ is

(1) One-one and onto

(2) One-one but not onto

(3) Onto but not one-one

(4) Neither one-one nor onto

Sol. Answer (1)

$$f(n)=\begin{cases} 2n & \text{if } n=2,4,6,\ldots \\ n-1 & \text{if } n=3,7,11,\ldots \\ \frac{n+1}{2} & \text{if } n=1,5,9,\ldots \end{cases}$$

When n is even i.e. $n = 2m$

$\Rightarrow f(n)=2n$ i.e. $f(2m)=4m \quad m\in N$

When n is of the form $4m - 1$, then

$f(4m-1)=4m-1-1=2(2m-1)$

When n is of the form $4m + 1$, then

$f(4m+1)=\frac{4m+1+1}{2}=(2m+1)$

Clearly $f(n)$ has all values of N

$\Rightarrow f(n)$ is onto

And $f(n)$ has unique value for each $n \Rightarrow f(n)$ is one-one also.

Hence $f(n)$ is one-one and onto.

10. If $A_1, A_2, A_3, \ldots$ are in increasing G.P and $A_1A_3A_5A_7 = \frac{1}{1296}$ and $A_2 + A_4 = \frac{7}{36}$, then find the value of $A_6 + A_8 + A_{10}$

(1) 33 (2) 37

(3) 43 (4) 47

Sol. Answer (3)

$A_1=a, A_2=ar, A_3=ar^2, \ldots$

$a\cdot ar^2\cdot ar^4\cdot ar^6=\frac{1}{1296}$

$\Rightarrow a^4r^{12}=\frac{1}{1296}$

$\Rightarrow ar^3=\frac{1}{6}$ …(1)

$A_2+A_4=\frac{7}{36}$

$\Rightarrow ar+ar^3=\frac{7}{36}$

$\Rightarrow ar=\frac{7}{36}-\frac{1}{6}$

$\Rightarrow ar = \dfrac{1}{36}$ ...(2)

Dividing (1) by (2)

$\dfrac{ar^3}{ar} = 6$

$\Rightarrow r^2 = 6$

$\Rightarrow A_6 + A_8 + A_{10} = ar^5 + ar^7 + ar^9$

$= ar\left(r^4 + r^6 + r^8\right)$

$= \dfrac{1}{36}\left(36 + 216 + 36^2\right)$ = 43

11. **S-I:** $p \to (\varepsilon \vee q)$

S-II: $(\sim p \vee q) \vee (\sim P \vee \varepsilon)$

Which of the following is incorrect?

(1) If S-I is true then S-II is true

(2) If S-I is false then S-II is true

(3) If S-I is true then S-II is false

(4) None of these

Sol. Answer (1)

S-I:

$p \to (\varepsilon \vee q) \equiv \sim p \vee (\varepsilon \vee q)$ $(\because x \to y \equiv \sim x \vee y)$

$\equiv (\sim p \vee \varepsilon) \vee (\sim p \vee q) \Rightarrow$ S-II

$\Rightarrow$ S-I and S-II are equivalent

12. If a curve satisfies the differential equation $\left(e^{y/x} + \dfrac{x}{\sqrt{x^2 - y^2}}\right)(xdy - ydx) = xdx$ and passes through (1, 0) and $(2\alpha, \alpha)$. Find the value of α.

(1) $\dfrac{e^{\pi/6}}{2}$ (2) $\dfrac{e^{\sqrt{e}}}{2}$

(3) $\dfrac{e^{\sqrt{e}+\frac{\pi}{6}-1}}{2}$ (4) $\dfrac{e^{\sqrt{e}-\frac{\pi}{6}-1}}{2}$

Sol. Answer (3)

$\left(e^{\frac{y}{x}} + \dfrac{x}{\sqrt{x^2 - y^2}}\right)(xdy - ydx) = xdx$

$\Rightarrow \left(e^{\frac{y}{x}} + \dfrac{1}{\sqrt{1-\left(\frac{y}{x}\right)^2}}\right)\dfrac{(xdy - ydx)}{x^2} = \int \dfrac{dx}{x}$

$\int \left(e^{\frac{y}{x}} + \dfrac{1}{\sqrt{1-\left(\frac{y}{x}\right)^2}}\right) d\left(\dfrac{y}{x}\right) = \int \dfrac{dx}{x}$

$\dfrac{y}{x} = t$

$\int \left(e^t + \dfrac{1}{\sqrt{1-t^2}}\right) dt = \ln x + c$

$e^t + \sin^{-1}(t) = \ln x + c$

$e^{\frac{y}{x}} + \sin^{-1}\left(\dfrac{y}{x}\right) = \ln x + c$

Put x = 1 y = 0

1 + 0 = 0 + c $\Rightarrow c = 1$

Put $x = 2\alpha$ $y = \alpha$

$e^{\frac{1}{2}} + \dfrac{\pi}{6} = \ln(2\alpha) + 1$

$\Rightarrow \alpha = \dfrac{1}{2} e^{\sqrt{e}+\frac{\pi}{6}-1}$

13. Number of three digit numbers having atleast two digit odd is

Sol. Answer (475)

Case I: All digits are odd is possible in 5 × 5 × 5 ways = 125

Case II: Tow digits are odd

$\overline{\text{odd}}$	$\overline{\text{odd}}$	$\overline{\text{even}}$	$\to$	$5 \times 5 \times 5 = 125$
odd	even	odd	$\to$	$5 \times 5 \times 5 = 125$
even	odd	odd	$\to$	$4 \times 5 \times 5 = 125$

$\downarrow$

(can not be zero)

Number of numbers = 125 + 125 + 125 + 100 = 475

14. The term independent of x in $\left(2x^3 + \dfrac{3}{x^k}\right)^{12}$ is equal to $2^8 \cdot l$ where $l \in$ odd natural numbers, then find the number of values of k.

Sol. Answer (2)

General term $t_{r+1} = {}^{12}C_r \left(2x^3\right)^{12-r}\left(\dfrac{3}{x^k}\right)^r$

$= {}^{12}C_r 2^{12-r} 3^r (x)^{36-3r-kr}$

For Independent term $36 - 3r - kr = 0$

$\Rightarrow r = \dfrac{36}{3+k}$, but $r \in \{0, 1, 2, \ldots 12\}$

$\Rightarrow k \in \{0,1,3,6,9\}$

$\Rightarrow r \in \{12,9,6,4,3\}$

For ${}^{12}C_r 2^{12-r} 3^r = 2^8 \cdot I$

$\Rightarrow r = 6$ or 4

Thus 2 values of k are there.

15. Mean and standard deviation of 15 observations is 8 and 3 respectively. While calculations observation 20 is misread as 5. What is the correct variance?

Sol. Answer (17)

Mean $\overline{X} = 8$ and variance $\sigma^2 = 9$

$\overline{X} = \frac{\sum x_i}{15} \Rightarrow \sum x_i = 120$

And $\sigma^2 = \frac{\sum x_i^2}{n} - \left(\overline{X}\right)^2 \Rightarrow 9 = \frac{\sum x_i^2}{15} - 64$

$\Rightarrow \sum x_i^2 = 15 \times 73 = 1095$

Now correct

$\sum x_i = 120 - 5 + 20 = 135$ and

$\sum x_i^2 = 1095 + 20^2 - 5^2 = 1470$

And correct mean = $\frac{\sum x_i}{15} = \frac{135}{15} = 9$

Correct variance

$= \frac{\sum x_i^2}{n} - \left(\overline{x}\right)^2 = \frac{1470}{15} - 9^2 = 98 - 81 = 17$

16. If $x\left(1-x^2\right)\frac{dy}{dx} = 4x^3 - 3x^2y + y$ and $y(2) = -2$, then $y(3)$ is equal to (here $x > 1$).

Sol. Answer (–18)

$x\left(1-x^2\right)y'' = 4x^3 - 3x^2y + y$

$\Rightarrow y' + y\left(\frac{3x^2-1}{x\left(1-x^2\right)}\right) = \frac{4x^3}{x\left(1-x^2\right)}$

$\Rightarrow IF = e^{\int \frac{3x^2-1}{x\left(1-x^2\right)}dx} = e^{-\ln\left(x^3-x\right)} = \left(\frac{1}{x^3-x}\right)$

$\Rightarrow y\left(\frac{1}{x^3-x}\right) = \int \frac{4x^3}{(1-x^2)x} \times \frac{1}{\left(x^3-x\right)}dx$

$\Rightarrow y\left(\frac{1}{x^3-x}\right) = \int \frac{-4x}{\left(x^2-1\right)^2}dx$

$\Rightarrow y\left(\frac{1}{x^3-x}\right) = \frac{2}{x^2-1} + k$

$y(2) = -2$

$\Rightarrow -2\left(\frac{1}{6}\right) = \frac{2}{3} + k$

$\Rightarrow k = -1$

Hence $y(3)\left(\frac{1}{24}\right) = \left(\frac{2}{8}\right) + (-1)$

$\Rightarrow y(3) = -18$

17. Relation defined on Set $A = \{1,2,3,...,50\}$

Let $R_1 = \left\{\left(p, p^n\right):\ p \text{ is a prime number, } n \in Z\right\}$

$R_2 = \left\{\left(p, p^n\right):\ p \text{ is a prime number, } n = 0,1\right\}$

Find $n(R_1) - n(R_2)$.

Sol. Answer (8)

$R_2 = \{(p,1),(p,p)\}$

$R_1 = \left\{(p,1),(p,p),\left(p,\frac{1}{p}\right),\left(p,p^2\right),\left(p,\frac{1}{p^2}\right),....\right\}$

$n(R_1) - n(R_2)$ gives the elements $\left(p,p^2\right),\left(p,p^3\right),....$ as these relations are defined on A & $P^n \le 50$

$\left(p,p^n\right) \in A \times A$ for

$p = 2,\ n = 2,3,4,5$
$p = 3,\ n = 2,3$
$p = 5,\ n = 2$
$p = 7,\ n = 2$

$\therefore n(R_1) = n(R_2) = 8$

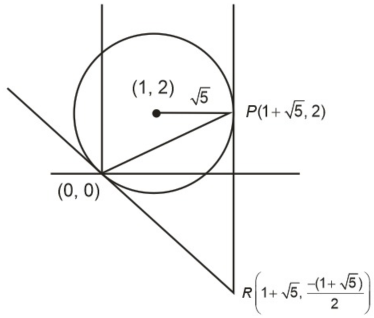
18. Consider the circle $x^2 + y^2 - 2x - 4y = 0$. If O represents origin, point P is $\left(1+\sqrt{5},2\right)$ and R is the intersection of the tangents drawn at P and O, then find the area of ΔOPR

Sol. Answer ($\frac{\sqrt{5}}{2}\left(3+\sqrt{5}\right)$)

$x^2 + y^2 - 2x - 4y = 0 \quad c(1, 2) \quad r = \sqrt{5}$

Equation of tangent at $P\left(1+\sqrt{5},2\right)$ is

$x = 1 + \sqrt{5}$ …(1)



Equation of tangent at $O(0, 0)$ is

$2x + 4y = 0$

$\Rightarrow x + 2y = 0 \qquad \ldots(2)$

Point of intersection of (1) & (2) is

$R\left(1+\sqrt{5}, \dfrac{-\left(1+\sqrt{5}\right)}{2}\right)$

Now area of $\Delta OPR = \dfrac{1}{2}\cdot\left(1+\sqrt{5}\right)\times\left(2+\dfrac{1+\sqrt{5}}{2}\right)$

$= \dfrac{1}{2}\left(1+\sqrt{5}\right)\times\dfrac{\sqrt{5}}{2}\left(1+\sqrt{5}\right) = \dfrac{\sqrt{5}}{4}\left(1+5+2\sqrt{5}\right)$

$= \dfrac{\sqrt{5}}{4}\left(6+2\sqrt{5}\right) = \dfrac{\sqrt{5}}{2}\left(3+\sqrt{5}\right)$

19. Find acute angle between two planes $P1$ and $P2$ passing through the intersection of $5x + 8y + 3z - 20 = 0$ and $8x - 6y + z - 29 = 0$ and also passing through the points (2, 3, 1) and (0, 2, 1), respectively

Sol. Answer (0.98)

General plane would be $P_1 + \lambda P_2$

$\Rightarrow (5x + 8y + 3z - 20) + \lambda(8x - 7y + z - 29) = 0$

If it pass through (2, 3, 1)

then,

$(10 + 24 + 3 - 20) + \lambda_1(16 - 21 + 1 - 29) = 0$

$\lambda_1 = \dfrac{17}{33}$

If the plane passes through (0, 2, 1)

Then,

$(0 + 16 + 3 - 20) + \lambda(0 - 14 + 1 - 29) = 0$

$\Rightarrow \lambda_2 = \dfrac{-1}{42}$

Hence plane are :

$\equiv x\left(5+\dfrac{136}{33}\right) + y\left(8-\dfrac{119}{33}\right) + z\left(3+\dfrac{17}{33}\right) = 20 + 29\times\dfrac{17}{33}$

$\equiv x(301) + y(145) + z(116) =$ some constant

And $x(202) + y(49) + z(125) =$ constant

Hence,

$\theta = \cos^{-1}\left\{\dfrac{82,407}{\sqrt{125082}\times\sqrt{58427}}\right\} = \cos^{-1}(0.98)$

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