# MATHEMATICS

## SECTION - A

**Multiple Choice Questions:** This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

**Choose the correct answer :**

1. If $\sum_{k=1}^{31}\left({}^{31}C_k\right)\left({}^{31}C_{k-1}\right)-\sum_{k=1}^{30}\left({}^{30}C_k\right)\left({}^{30}C_{k-1}\right)=\frac{\alpha(60!)}{(30!)(31!)}$,

   where $\alpha \in \mathbf{R}$, then the value of $16\alpha$ is equal to

   (A) 1411

   (B) 1320

   (C) 1615

   (D) 1855

**Answer (A)**

**Sol.** $\sum_{k=1}^{31}{}^{31}C_k\,{}^{31}C_{k-1}-\sum_{k=1}^{30}{}^{30}C_k\,{}^{30}C_{k-1}$

$$=\sum_{k=1}^{31}{}^{31}C_k\cdot{}^{31}C_{32-k}-\sum_{k=1}^{30}{}^{30}C_k\cdot{}^{30}C_{31-k}$$

$= {}^{62}C_{32} - {}^{60}C_{31}$

$$=\frac{60!}{31!29!}\left(\frac{62\cdot 61}{32\cdot 30}-1\right)=\frac{60!}{31!29!}\frac{2822}{32\cdot 30}$$

$$\alpha=\frac{2822}{32}\ \Rightarrow\ 16\alpha = 1411$$

2. Let a function $f : N \to N$ be defined by

$$f(n)=\begin{cases}2n, & n=2,\ 4,\ 6,\ 8,.....\\ n-1, & n=3,\ 7,\ 11,\ 15,.....\\ \frac{n+1}{2}, & n=1,\ 5,\ 9,\ 13,....\end{cases}$$

   then, $f$ is

   (A) One-one but not onto

   (B) Onto but not one-one

   (C) Neither one-one nor onto

   (D) One-one and onto

**Answer (D)**

**Sol.** When $n$ = 1, 5, 9, 13 then $\frac{n+1}{2}$ will give all odd numbers.

When $n$ = 3, 7, 11, 15 …

$n$ – 1 will be even but not divisible by 4

When $n$ = 2, 4, 6, 8, …

Then $2n$ will give all multiples of 4

So range will be $N$.

And no two values of $n$ give same $y$, so function is one-one and onto.

3. If the system of linear equations

   $2x + 3y - z = -2$

   $x + y + z = 4$

   $x - y + |\lambda|z = 4\lambda - 4$

   where $\lambda \in$ R, has no solution, then

   (A) $\lambda = 7$

   (B) $\lambda = -7$

   (C) $\lambda = 8$

   (D) $\lambda^2 = 1$

**Answer (B)**

**Sol.** $\Delta=\begin{vmatrix}2 & 3 & -1\\ 1 & 1 & 1\\ 1 & -1 & |\lambda|\end{vmatrix}=0\ \Rightarrow\ |\lambda| = 7$

But at $\lambda = 7$, $D_x = D_y = D_z = 0$

$P_1 : 2x + 3y - z = -2$

$P_2 : x + y + z = 4$

$P_3 : x - y + |\lambda|z = 4\lambda - 4$

So clearly $5P_2 - 2P_1 = P_3$, so at $\lambda = 7$, system of equation is having infinite solutions.

So $\lambda = -7$ is correct answer.

4. Let $A$ be a matrix of order 3 × 3 and det ($A$) = 2. Then det (det ($A$) adj (5 adj ($A^3$))) is equal to ______.

   (A) $512 \times 10^6$

   (B) $256 \times 10^6$

   (C) $1024 \times 10^6$

   (D) $256 \times 10^{11}$

**Answer (A)**

**Sol.** $|A| = 2$

$||A|\ \text{adj}(5\ \text{adj}\ A^3)|$

$= |25|A|\ \text{adj}(\text{adj}\ A^3)|$

$= 25^3\ |A|^3 \cdot |\text{adj}\ A^3|^2$

$= 25^3 \cdot 2^3 \cdot |A^3|^4$

$= 25^3 \cdot 2^3 \cdot 2^{12} = 10^6 \cdot 512$

5. The total number of 5-digit numbers, formed by using the digits 1, 2, 3, 5, 6, 7 without repetition, which are multiple of 6, is

(A) 36 (B) 48

(C) 60 (D) 72

**Answer (D)**

**Sol.** Number should be divisible by 6 and it should be even.

Total sum = 1 + 2 + 3 + 5 + 6 + 7 = 24

So number removed should be of type 3.

C-1 : excluding 3 _ _ _ _ 2 ways = 4! × 2 = 48

C-2 : excluding 6 _ _ _ _ 1 way = 4! = 24

Total cases = 48 + 24 = 72

6. Let $A_1$, $A_2$, $A_3$, ... be an increasing geometric progression of positive real numbers. If $A_1A_3A_5A_7 = \frac{1}{1296}$ and $A_2 + A_4 = \frac{7}{36}$, then, the value of $A_6 + A_8 + A_{10}$ is equal to

(A) 33 (B) 37

(C) 43 (D) 47

**Answer (C)**

**Sol.** $\frac{A_4}{r^3} \cdot \frac{A_4}{r} . A_4 r . A_4 r^3 = \frac{1}{1296}$

$A_4 = \frac{1}{6}$

$A_2 = \frac{7}{36} - \frac{1}{6} = \frac{1}{36}$

So $A_6 + A_8 + A_{10} = 1 + 6 + 36$

$= 43$

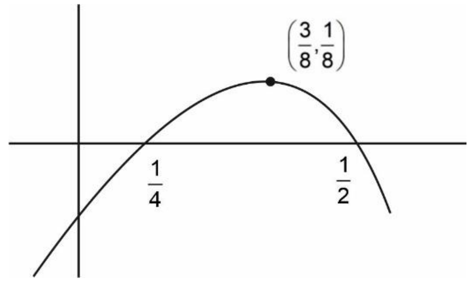
7. Let $[t]$ denote the greatest integer less than or equal to $t$. Then, the value of the integral $\int_0^1 \left[-8x^2 + 6x - 1\right] dx$ is equal to

(A) $-1$ (B) $\frac{-5}{4}$

(C) $\frac{\sqrt{17}-13}{8}$ (D) $\frac{\sqrt{17}-16}{8}$

**Answer (C)**

**Sol.** $\int_0^1 \left[-8x^2 + 6x - 1\right] dx$




$$= \int_0^{\frac{1}{4}} (-1)dx + \int_{\frac{1}{4}}^{\frac{3}{4}} 0dx + \int_{\frac{1}{2}}^{\frac{3}{4}} -1dx + \int_{\frac{3}{4}}^{\frac{3+\sqrt{17}}{8}} -2dx + \int_{\frac{3+\sqrt{17}}{8}}^{1} -3dx$$

$$= -\frac{1}{4} - \frac{1}{4} - 2\left(\frac{3+\sqrt{17}}{8} - \frac{3}{4}\right) - 3\left(1 - \frac{3+\sqrt{17}}{8}\right)$$

$$= \frac{\sqrt{17}-13}{8}$$

8. Let $f : \mathbb{R} \to \mathbb{R}$ be defined as

$$f(x) = \begin{cases} \left[e^x\right], & x < 0 \\ ae^x + [x-1], & 0 \le x < 1 \\ b + \left[\sin(\pi x)\right], & 1 \le x < 2 \\ \left[e^{-x}\right] - c, & x \ge 2 \end{cases}$$

Where $a, b, c \in \mathbb{R}$ and $[t]$ denotes greatest integer less than or equal to $t$. Then, which of the following statements is true?

(A) There exists $a, b, c \in \mathbb{R}$ such that $f$ is continuous on $\mathbb{R}$.

(B) If $f$ is discontinuous at exactly one point, then $a + b + c = 1$

(C) If $f$ is discontinuous at exactly one point, then $a + b + c \neq 1$

(D) $f$ is discontinuous at atleast two points, for any values of $a$, $b$ and $c$

**Answer (C)**

**Sol.** $f(x) = \begin{cases} 0 & x < 0 \\ ae^x - 1 & 0 \le x < 1 \\ b & x = 1 \\ b - 1 & 1 < x < 2 \\ -c & x \ge 2 \end{cases}$

To be continuous at $x = 0$

$a - 1 = 0$

to be continuous at $x = 1$

$ae - 1 = b = b - 1 \Rightarrow$ not possible

to be continuous at $x = 2$

$b - 1 = -c \Rightarrow b + c = 1$

If $a = 1$ and $b + c = 1$ then $f(x)$ is discontinuous at exactly one point

9. The area of the region $S = \left\{(x,\ y) : y^2 \le 8x,\ y \ge \sqrt{2}x, x \ge 1\right\}$ is

(A) $\frac{13\sqrt{2}}{6}$

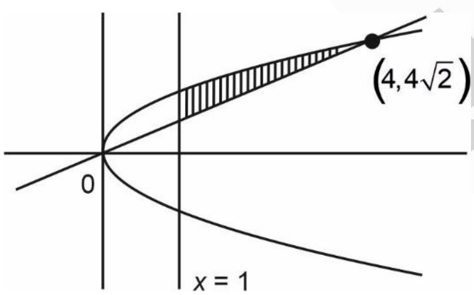
(B) $\frac{11\sqrt{2}}{6}$

(C) $\frac{5\sqrt{2}}{6}$

(D) $\frac{19\sqrt{2}}{6}$

**Answer (B)**

**Sol.**




Required area

$$= \int_1^4 \left(\sqrt{8x} - \sqrt{2}x\right)dx$$

$$= \left.\frac{2\sqrt{8}}{3}x^{\frac{3}{2}} - \frac{x^2}{\sqrt{2}}\right|_1^4$$

$$= \frac{16\sqrt{3}}{3} - \frac{16}{\sqrt{2}} - \frac{2\sqrt{8}}{3} + \frac{1}{\sqrt{2}}$$

$$= \frac{11\sqrt{2}}{6} \text{ sq. units}$$

10. Let the solution curve $y = y(x)$ of the differential equation

$$\left[\frac{x}{\sqrt{x^2 - y^2}} + e^{\frac{y}{x}}\right]x\frac{dy}{dx} = x + \left[\frac{x}{\sqrt{x^2 - y^2}} + e^{\frac{y}{x}}\right]y$$

pass through the points (1, 0) and $(2\alpha, \alpha)$, $\alpha > 0$. Then $\alpha$ is equal to

(A) $\frac{1}{2}\exp\left(\frac{\pi}{6} + \sqrt{e} - 1\right)$ (B) $\frac{1}{2}\exp\left(\frac{\pi}{3} + e - 1\right)$

(C) $\exp\left(\frac{\pi}{6} + \sqrt{e} + 1\right)$ (D) $2\exp\left(\frac{\pi}{3} + \sqrt{e} - 1\right)$

**Answer (A)**

**Sol.** $$\left(\frac{1}{\sqrt{1 - \frac{y^2}{x^2}}} + e^{\frac{y}{x}}\right)\frac{dy}{dx} = 1 + \left(\frac{1}{\sqrt{1 - \frac{y^2}{x^2}}} + e^{\frac{y}{x}}\right)\frac{y}{x}$$

Putting $y = tx$

$$\left(\frac{1}{\sqrt{1 - t^2}} + e^t\right)\left(t + x\frac{dt}{dx}\right) = 1 + \left(\frac{1}{\sqrt{1 - t^2}} + e^t\right)t$$

$$\Rightarrow\ x\left(\frac{1}{\sqrt{1 - t^2}} + e^t\right)\frac{dt}{dx} = 1$$

$\Rightarrow\ \sin^{-1} t + e^t = \ln x + C$

$$\Rightarrow\ \sin^{-1}\left(\frac{y}{x}\right) + e^{y/x} = \ln x + C$$

at $x = 1$, $y = 0$

So, $0 + e^0 = 0 + C \Rightarrow C = 1$

at $(2\alpha, \alpha)$

$$\sin^{-1}\left(\frac{y}{x}\right) + e^{y/x} = \ln x + 1$$

$$\Rightarrow\ \frac{\pi}{6} + e^{\frac{1}{2}} - 1 = \ln(2\alpha)$$

$$\Rightarrow\ \alpha = \frac{1}{2}e^{\left(\frac{\pi}{6} + e^{\frac{1}{2}} - 1\right)}$$

11. Let $y = y(x)$ be the solution of the differential equation $x\left(1 - x^2\right)\frac{dy}{dx} + \left(3x^2y - y - 4x^3\right) = 0,\ x > 1,$ with $y(2) = -2$. Then $y(3)$ is equal to

(A) –18 (B) –12

(C) –6 (D) –3

**Answer (A)**

**Sol.** $\dfrac{dy}{dx}+\dfrac{y(3x^2-1)}{x(1-x^2)}=\dfrac{4x^3}{x(1-x^2)}$

$$IF=e^{\int\frac{3x^2-1}{x-x^3}dx}=e^{-\ln|x^3-x|}=e^{-\ln(x^3-x)}$$

$$=\frac{1}{x^3-x}$$

Solution of D.E. can be given by

$$y.\frac{1}{x^3-x}=\int\frac{4x^3}{x(1-x^2)}.\frac{1}{x(x^2-1)}dx$$

$$\Rightarrow\ \frac{y}{x^3-x}=\int\frac{-4x}{(x^2-1)^2}dx$$

$$\Rightarrow\ \frac{y}{x^3-x}=\frac{2}{(x^2-1)}+c$$

at $x = 2$, $y = -2$

$$\frac{-2}{6}=\frac{2}{3}+c\ \Rightarrow\ c=-1$$

$$\text{at } x=3\Rightarrow\frac{y}{24}=\frac{2}{8}-1\Rightarrow y=-18$$

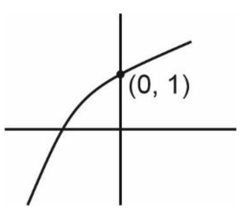
12. The number of real solutions of $x^7 + 5x^3 + 3x + 1 =$ 0 is equal to _______.

(A) 0 (B) 1

(C) 3 (D) 5

**Answer (B)**

**Sol.**




$f'(x)=7x^6+15x^2+3>0\ \forall x\in R$

$f(x)$ is always increasing

So clearly it intersects

x-axis at only one point

13. Let the eccentricity of the hyperbola $H:\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1$ be $\sqrt{\dfrac{5}{2}}$ and length of its latus rectum be $6\sqrt{2}$, If $y = 2x + c$ is a tangent to the hyperbola $H$. then the value of $c^2$ is equal to

(A) 18 (B) 20

(C) 24 (D) 32

**Answer (B)**

**Sol.** $1+\dfrac{b^2}{a^2}=\dfrac{5}{2}\Rightarrow\dfrac{b^2}{a^2}=\dfrac{3}{2}$

$$\frac{2b^2}{a}=6\sqrt{2}\Rightarrow 2.\frac{3}{2}.a=6\sqrt{2}$$

$$\Rightarrow\ a=2\sqrt{2},\ b^2=12$$

$$c^2=a^2m^2-b^2=8.4-12=20$$

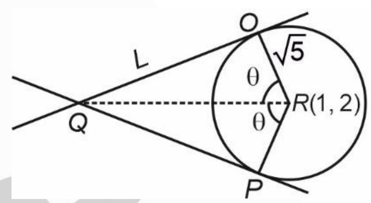
14. If the tangents drawn at the points $O(0, 0)$ and $P\left(1+\sqrt{5},2\right)$ on the circle $x^2 + y^2 - 2x - 4y = 0$ intersect at the point $Q$, then the area of the triangle $OPQ$ is equal to

(A) $\dfrac{3+\sqrt{5}}{2}$ (B) $\dfrac{4+2\sqrt{5}}{2}$

(C) $\dfrac{5+3\sqrt{5}}{2}$ (D) $\dfrac{7+3\sqrt{5}}{2}$

**Answer (C)**

**Sol.**




$\dfrac{\sqrt{5}}{L}=\tan\theta$

$$\tan 2\theta=2\Rightarrow\frac{2\tan\theta}{1-\tan^2\theta}=2$$

$$\tan\theta=\frac{\sqrt{5}-1}{2}\qquad\text{(as }\theta\text{ is acute)}$$

$$\text{Area}=\frac{1}{2}L^2\sin 2\theta=\frac{1}{2}.\frac{5}{\tan^2\theta}.2\sin\theta\cos\theta$$

$$=\frac{5\sin\theta\cos\theta}{\sin^2\theta}.\cos^2\theta$$

$$=5\cot\theta.\cos^2\theta$$

$$=5.\frac{2}{\sqrt{5}-1}.\frac{1}{1+\left(\frac{\sqrt{5}-1}{2}\right)^2}$$

$$=\frac{10}{\sqrt{5}-1}.\frac{4}{4+6-2\sqrt{5}}$$

$$=\frac{40}{2\sqrt{5}(\sqrt{5}-1)^2}=\frac{4\sqrt{5}}{6-2\sqrt{5}}$$

$$=\frac{4\sqrt{5}(6+2\sqrt{5})}{16}$$

$$=\frac{\sqrt{5}(3+\sqrt{5})}{2}$$

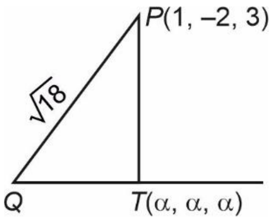
15. If two distinct points $Q$, $R$ lie on the line of intersection of the planes $-x + 2y - z = 0$ and $3x - 5y + 2z = 0$ and $PQ = PR = \sqrt{18}$ where the point $P$ is (1, –2, 3), then the area of the triangle $PQR$ is equal to

(A) $\frac{2}{3}\sqrt{38}$ (B) $\frac{4}{3}\sqrt{38}$

(C) $\frac{8}{3}\sqrt{38}$ (D) $\sqrt{\frac{152}{3}}$

**Answer (B)**

**Sol.**




Line $L$ is $x = y = z$

$\overrightarrow{PQ}.(\hat{i}+\hat{j}+\hat{k}) = 0$

$\Rightarrow (\alpha - 3) + \alpha + 2 + \alpha - 1 = 0$

$\Rightarrow \alpha = \frac{2}{3}$ so, $T = \left(\frac{2}{3}, \frac{2}{3}, \frac{2}{3}\right)$

$PT = \sqrt{\frac{38}{3}}$

$\Rightarrow QT = \frac{4}{\sqrt{3}}$

So, Area $= \left(\frac{1}{2} \times \frac{4}{\sqrt{3}} \times \frac{\sqrt{38}}{\sqrt{3}}\right).2$

$= \frac{4\sqrt{38}}{3}$ sq units

16. The acute angle between the planes $P_1$ and $P_2$, when $P_1$ and $P_2$ are the planes passing through the intersection of the planes $5x + 8y + 13z - 29 = 0$ and $8x - 7y + z - 20 = 0$ and the points (2, 1, 3) and (0, 1, 2), respectively, is

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{4}$

(C) $\frac{\pi}{6}$ (D) $\frac{\pi}{12}$

**Answer (A)**

**Sol.** Family of Plane's equation can be given by

$(5 + 8\lambda)x + (8 - 7\lambda)y + (13 + \lambda)z - (29 + 20\lambda) = 0$

$P_1$ passes through (2, 1, 3)

$\Rightarrow (10 + 16\lambda) + (8 - 7\lambda) + (39 + 3\lambda) - (29 + 20\lambda) = 0$

$\Rightarrow -8\lambda + 28 = 0 \Rightarrow \lambda = \frac{7}{2}$

d.r, s of normal to $P_1$

$\left\langle 33, \frac{-33}{2}, \frac{33}{2}\right\rangle$ or $\left\langle 1, -\frac{1}{2}, \frac{1}{2}\right\rangle$

$P_2$ passes through (0, 1, 2)

$\Rightarrow 8 - 7\lambda + 26 + 2\lambda - (29 + 20\lambda) = 0$

$\Rightarrow 5 - 25\lambda = 0$

$\Rightarrow \lambda = \frac{1}{5}$

d.r, s of normal to $P_2$

$\left\langle \frac{33}{5}, \frac{33}{5}, \frac{66}{5}\right\rangle$ or $\langle 1, 1, 2\rangle$

Angle between normals

$$= \frac{\left(\hat{i} - \frac{1}{2}\hat{j} + \frac{1}{2}\hat{k}\right)\cdot\left(\hat{i} + \hat{j} + 2\hat{k}\right)}{\frac{\sqrt{3}}{2} \qquad \sqrt{6}}$$

$$\cos\theta = \frac{1 - \frac{1}{2} + 1}{3} = \frac{1}{2}$$

$$\theta = \frac{\pi}{3}$$

17. Let the plane $P : \vec{r}\cdot\vec{a} = d$ contain the line of intersection of two planes $\vec{r}\cdot\left(\hat{i} + 3\hat{j} - \hat{k}\right) = 6$ and $\vec{r}\cdot\left(-6\hat{i} + 5\hat{j} - \hat{k}\right) = 7$. If the plane $P$ passes through the point $\left(2, 3, \frac{1}{2}\right)$, then the value of $\frac{|13\,\vec{a}\,|^2}{d^2}$ is equal to

(A) 90

(B) 93

(C) 95

(D) 97

**Answer (B)**

**Sol.** $P_1: x + 3y - z = 6$

$P_2: -6x + 5y - z = 7$

Family of planes passing through line of intersection of $P_1$ and $P_2$ is given by $x(1 - 6\lambda) + y(3 + 5\lambda) + z(-1 - \lambda) - (6 + 7\lambda) = 0$

It passes through $\left(2, 3, \frac{1}{2}\right)$

So, $2(1-6\lambda)+3(3+5\lambda)+\frac{1}{2}(-1-\lambda)-(6+7\lambda)=0$

$\Rightarrow \quad 2-12\lambda+9+15\lambda-\frac{1}{2}-\frac{\lambda}{2}-6-7\lambda=0$

$\Rightarrow \quad \frac{9}{2}-\frac{9\lambda}{2}=0 \Rightarrow \lambda=1$

Required plane is

$-5x + 8y - 2z - 13 = 0$

Or $\vec{r}\cdot(-5\hat{i}+8\hat{j}-2\hat{k})=13$

$\frac{|13\vec{a}|^2}{|d|^2}=\frac{13^2}{(13)^2}\cdot|\vec{a}|^2=93$

18. The probability, that in a randomly selected 3-digit number at least two digits are odd, is

(A) $\frac{19}{36}$ (B) $\frac{15}{36}$

(C) $\frac{13}{36}$ (D) $\frac{23}{36}$

**Answer (A)**

**Sol.** Required cases = Total – all digits even – exactly one digit even

Total = 900 ways

All even $\Rightarrow$ 4 5 5 = 100 ways

One digit odd $\Rightarrow$ odd 5 5 5 = 125 ways

4 odd 5 5 = 100 ways

4 5 odd 5 = 100 ways

Required probability $=\frac{900-425}{900}=\frac{19}{36}$

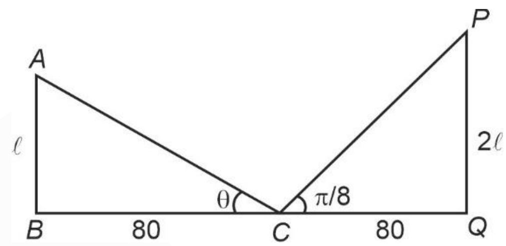
19. Let $AB$ and $PQ$ be two vertical poles, 160 m apart from each other. Let $C$ be the middle point of $B$ and $Q$, which are feet of these two poles. Let $\frac{\pi}{8}$ and $\theta$ be the angles of elevation from $C$ to $P$ and $A$, respectively. If the height of pole $PQ$ is twice the height of pole $AB$, then $\tan^2\theta$ is equal to

(A) $\frac{3-2\sqrt{2}}{2}$ (B) $\frac{3+\sqrt{2}}{2}$

(C) $\frac{3-2\sqrt{2}}{4}$ (D) $\frac{3-\sqrt{2}}{4}$

**Answer (C)**

**Sol.**




$\frac{\ell}{80}=\tan\theta \quad \ldots(i)$

$\frac{2\ell}{80}=\tan\frac{\pi}{8} \quad \ldots(ii)$

From (i) and (ii)

$\frac{1}{2}=\frac{\tan\theta}{\tan\frac{\pi}{8}} \Rightarrow \tan^2\theta=\frac{1}{4}\tan^2\frac{\pi}{8}$

$\Rightarrow \quad \tan^2\theta=\frac{\sqrt{2}-1}{4\left(\sqrt{2}+1\right)}=\frac{3-2\sqrt{2}}{4}$

20. Let $p$, $q$, $r$ be three logical statements. Consider the compound statements

$S_1$ : $((\sim p) \vee q) \vee ((\sim p) \vee r)$ and

$S_2$ : $p \rightarrow (q \vee r)$

Then, which of the following is **NOT** true?

(A) If $S_2$ is True, then $S_1$ is True

(B) If $S_2$ is False, then $S_1$ is False

(C) If $S_2$ is False, then $S_1$ is True

(D) If $S_1$ is False, then $S_2$ is False

**Answer (C)**

**Sol.** $S_1$: $(\sim p \vee q) \vee (\sim p \vee r)$

$\cong (\sim p \vee q \vee r)$

$S_2$: $\sim p \vee (q \vee r)$

Both are same

So, option (C) is incorrect.

## SECTION - B

**Numerical Value Type Questions:** This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. Let $R_1$ and $R_2$ be relations on the set {1, 2, ....., 50} such that

   $R_1 = \{(p, p^n) : p$ is a prime and $n \geq 0$ is an integer$\}$ and
   $R_2 = \{(p, p^n) : p$ is a prime and $n = 0$ or $1\}$.

   Then, the number of elements in $R_1 - R_2$ is ______.

**Answer (8)**

**Sol.** $R_1 - R_2 = \{(2, 2^2), (2, 2^3), (2, 2^4), (2, 2^5), (3, 3^2) (3, 3^3), (5, 5^2), (7, 7^2)\}$

So number of elements = 8

2. The number of real solutions of the equation $e^{4x} + 4e^{3x} - 58e^{2x} + 4e^x + 1 = 0$ is _____.

**Answer (2)**

**Sol.** Dividing by $e^{2x}$

$e^{2x} + 4e^x - 58 + 4e^{-x} + e^{-2x} = 0$

$\Rightarrow (e^x + e^{-x})^2 + 4(e^x + e^{-x}) - 60 = 0$

Let $e^x + e^{-x} = t \in [2, \infty)$

$\Rightarrow t^2 + 4t - 60 = 0$

$\Rightarrow t = 6$ is only possible solution

$e^x + e^{-x} = 6 \Rightarrow e^{2x} - 6e^x + 1 = 0$

Let $e^x = p$,

$p^2 - 6p + 1 = 0$

$$\Rightarrow p = \frac{3+\sqrt{5}}{2} \text{ OR } \frac{3-\sqrt{5}}{2}$$

$$\text{So } x = \ln\left(\frac{3+\sqrt{5}}{2}\right) \text{ OR } \ln\left(\frac{3-\sqrt{5}}{2}\right)$$

3. The mean and standard deviation of 15 observations are found to be 8 and 3 respectively. On rechecking it was found that, in the observations, 20 was misread as 5. Then, the correct variance is equal to _______.

**Answer (17)**

**Sol.** $\dfrac{\Sigma x_i^2}{15} - 8^2 = 9 \Rightarrow \Sigma x_i^2 = 15 \times 73 = 1095$

Let $\bar{x}_c$ be corrected mean $\bar{x}_c = 9$

$\Sigma x_c^2 = 1095 - 25 + 400 = 1470$

Correct variance $= \dfrac{1470}{15} - (9)^2 = 98 - 81 = 17$

4. If $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$, $\vec{b} = 3\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ are coplanar vectors and $\vec{a}\cdot\vec{c} = 5$, $\vec{b} \perp \vec{c}$, then $122(c_1 + c_2 + c_3)$ is equal to __________.

**Answer (150)**

**Sol.** $2C_1 + C_2 + 3C_3 = 5$ ...(i)

$3C_1 + 3C_2 + C_3 = 0$ ...(ii)

$$\left[\vec{a}\vec{b}\vec{c}\right] = \begin{vmatrix} 2 & 1 & 3 \\ 3 & 3 & 1 \\ C_1 & C_2 & C_3 \end{vmatrix}$$

$= 2(3C_3 - C_2) - 1(3C_3 - C_1) + 3(3C_2 - 3C_1)$

$= 3C_3 + 7C_2 - 8C_1$

$\Rightarrow 8C_1 - 7C_2 - 3C_3 = 0$ ...(iii)

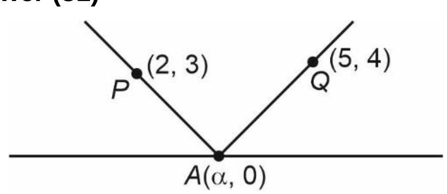
$$C_1 = \frac{10}{122}, C_2 = \frac{-85}{122}, C_3 = \frac{225}{122}$$

So $122(C_1 + C_2 + C_3) = 150$

5. A ray of light passing through the point $P(2, 3)$ reflects on the $x$-axis at point $A$ and the reflected ray passes through the point $Q(5, 4)$. Let $R$ be the point that divides the line segment $AQ$ internally into the ratio 2 : 1. Let the co-ordinates of the foot of the perpendicular $M$ from $R$ on the bisector of the angle $PAQ$ be $(\alpha, \beta)$. Then, the value of $7\alpha + 3\beta$ is equal to ___________.

**Answer (31)**

**Sol.**




$$\frac{4}{5-\alpha} = \frac{3}{\alpha - 2} \Rightarrow 4\alpha - 8 = 15 - 3\alpha$$

$$\alpha = \frac{23}{7}$$

$$A = \left(\frac{23}{7}, 0\right) \quad Q = (5, 4)$$

$$R = \left( \frac{10 + \frac{23}{7}}{3}, \frac{8}{3} \right)$$

$$= \left( \frac{31}{7}, \frac{8}{3} \right)$$

Bisector of angle $PAQ$ is $X = \frac{23}{7}$

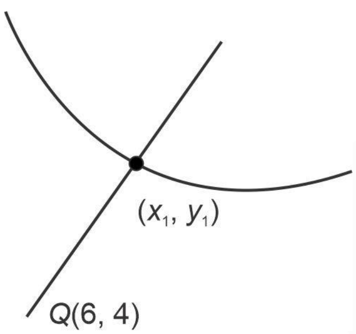
$$\Rightarrow \quad M = \left( \frac{23}{7}, \frac{8}{3} \right)$$

So, $7\alpha + 3\beta = 31$

6. Let $l$ be a line which is normal to the curve $y = 2x^2 + x + 2$ at a point $P$ on the curve. If the point $Q(6, 4)$ lies on the line $l$ and $O$ is origin, then the area of the triangle $OPQ$ is equal to ___________.

**Answer (13)**

**Sol.**




$$\frac{y_1 - 4}{x_1 - 6} = -\frac{1}{4x_1 + 1}$$

$$\Rightarrow \quad \frac{2x_1^2 + x_1 - 2}{x_1 - 6} = -\frac{1}{4x_1 + 1}$$

$$\Rightarrow \quad 6 - x_1 = 8x_1^3 + 6x_1^2 - 7x_1 - 2$$

$$\Rightarrow \quad 8x_1^3 + 6x_1^2 - 6x_1 - 8 = 0$$

So $x_1 = 1 \Rightarrow y_1 = 5$

$$\text{Area} = \left| \frac{1}{2} \begin{vmatrix} 0 & 0 & 1 \\ 6 & 4 & 1 \\ 1 & 5 & 1 \end{vmatrix} \right| = 13.$$

7. Let $A = \{1, a_1, a_2 \ldots a_{18}, 77\}$ be a set of integers with $1 < a_1 < a_2 < \ldots < a_{18} < 77$. Let the set $A + A = \{x + y : x, y \in A\}$ contain exactly 39 elements. Then, the value of $a_1 + a_2 + \ldots + a_{18}$ is equal to _____.

**Answer (702)**

**Sol.** If we write the elements of $A + A$, we can certainly find 39 distinct elements as $1 + 1, 1 + a_1, 1 + a_2, \ldots 1 + a_{18}, 1 + 77, a_1 + 77, a_2 + 77, \ldots a_{18} + 77, 77 + 77$.

It means all other sums are already present in these 39 values, which is only possible in case when all numbers are in A.P.

Let the common difference be '$d$'.

$77 = 1 + 19d \Rightarrow d = 4$

$$\text{So, } \sum_{i=1}^{18} a_1 = \frac{18}{2}[2a_1 + 17d] = 9[10 + 68] = 702$$

8. The number of positive integers $k$ such that the constant term in the binomial expansion of $\left(2x^3 + \frac{3}{x^k}\right)^{12}, x \neq 0$ is $2^8 \cdot \ell$, where $\ell$ is an odd integer, is______.

**Answer (2)**

**Sol.** $T_{r+1} = {}^{12}C_r \left(2x^3\right)^{12-r} \left(\frac{3}{x^k}\right)^r$

$= {}^{12}C_r\, 2^{12-r} 3^r\ x^{36-3r-kr}$

For constant term $36 - 3r - kr = 0$

$$r = \frac{36}{3+k}$$

So, $k$ can be 1, 3, 6, 9, 15, 33

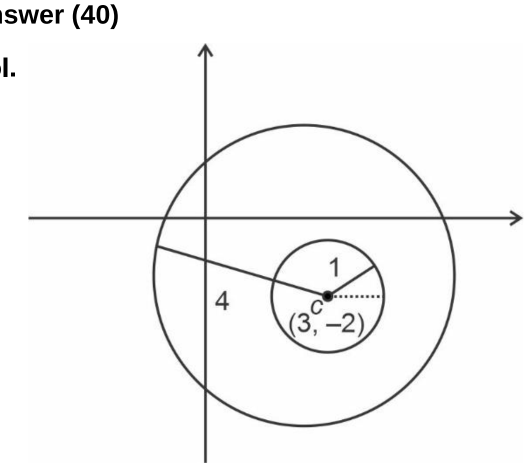
In order to get $2^8$, check by putting values of $k$ and corresponding in general term. By checking, it is possible only where $k = 3$ or 6

9. The number of elements in the set

$\{z = a + ib \in \mathbb{C} : a, b \in \mathbb{Z} \text{ and } 1 < |z - 3 + 2i| < 4\}$ is __________.

**Answer (40)**

**Sol.**




at line $y = -2$, we have $(5, -2)$ $(6, -2)$ $(1, -2)$ $(0, -2)$

$\Rightarrow$ 4 points

at line $y = -1$, we have (4, –1) (5, –1) (6, –1) (2, –1) (1, –1) (0, –1) $\Rightarrow$ 6 points

at line $y = 0$, we have (0, 0) (1, 0) (2, 0) (3, 0) (4, 0) (5, 0) (6, 0) $\Rightarrow$ 7 points

at line $y = 1$, we have (1, 1), (2, 1), (3, 1), (4, 1), (5, 1) i.e. 5 points

symmetrically

at line $y = -5$, we have 5 points

at line $y = -4$, we have 7 points

at line $y = -3$, we have 6 points

So Total integral points = 2(5 + 7 + 6) + 4

= 40

10. Let the lines $y + 2x = \sqrt{11} + 7\sqrt{7}$ and $2y + x = 2\sqrt{11} + 6\sqrt{7}$ be normal to a circle $C : (x - h)^2 + (y - k)^2 = r^2$. If the line $\sqrt{11}y - 3x = \frac{5\sqrt{77}}{3} + 11$ is tangent to the circle $C$, then the value of $(5h - 8k)^2 + 5r^2$ is equal to ________.

**Answer (816)**

**Sol.** $L_1 : \; y + 2x = \sqrt{11} + 7\sqrt{7}$

$L_2 : \; 2y + x = 2\sqrt{11} + 6\sqrt{7}$

Point of intersection of these two lines is centre of circle i.e. $\left(\frac{8}{3}\sqrt{7}, \sqrt{11} + \frac{5}{3}\sqrt{7}\right)$

$\perp^r$ from centre to line $3x - \sqrt{11}y + \left(\frac{5\sqrt{77}}{3} + 11\right) = 0$

is radius of circle

$$\Rightarrow \quad r = \left|\frac{8\sqrt{7} - 11 - \frac{5}{3}\sqrt{77} + \frac{5\sqrt{77}}{3} + 11}{\sqrt{20}}\right|$$

$$= \left|\sqrt[4]{\frac{7}{5}}\right| = \sqrt[4]{\frac{7}{5}} \text{ units}$$

So $(5h - 8K)^2 + 5r^2$

$$= \left(\frac{40}{3}\sqrt{7} - 8\sqrt{11} - \frac{40}{3}\sqrt{7}\right)^2 + 5.16.\frac{7}{5}$$

= 64 × 11 + 112 = 816.

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