Date: 28/08/2022

Aakash BYJU'S

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Time : 3 hrs. **Max. Marks: 180**

# Answers & Solutions

*for*

# JEE (Advanced)-2022 (Paper-2)

## PART-I : PHYSICS

**SECTION – 1 (Maximum marks : 24)**

- This section contains **EIGHT (08)** questions.
- The answer to each question is a **SINGLE DIGIT INTEGER ranging from 0 TO 9, BOTH INCLUSIVE**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

| | | | |
|---|---|---|---|
| *Full Marks* | : | +3 | If **ONLY** the correct integer is entered; |
| *Zero Marks* | : | 0 | If the question is unanswered; |
| *Negative Marks* | : | −1 | In all other cases. |

1. A particle of mass 1 kg is subjected to a force which depends on the position as $\vec{F} = -k(x\hat{i} + y\hat{j})$ kg ms$^{-2}$ with $k$ = 1 kg s$^{-2}$. At time $t$ = 0, the particle's position $\vec{r} = \left(\frac{1}{\sqrt{2}}\hat{i} + \sqrt{2}\hat{j}\right)$ m and its velocity $\vec{v} = \left(-\sqrt{2}\hat{i} + \sqrt{2}\hat{j} + \frac{2}{\pi}\hat{k}\right)$ ms$^{-1}$. Let $v_x$ and $v_y$ denote the $x$ and the $y$ components of the particle's velocity, respectively. **Ignore gravity.** When $z$ = 0.5 m, the value of $(xv_y - yv_x)$ is _______ m$^2$s$^{-1}$.

**Answer (3)**

**Sol.** $F_x = -x = ma_x$.

So $a_x = \frac{d^2x}{dt^2} = -x$

$\Rightarrow x = A_x \sin(\omega t + \phi_x)$  ($\omega$ = 1 rad/s) and $v_x = A_x\omega\cos(\omega t + \phi_x)$

at $t = 0$, $x = \frac{1}{\sqrt{2}}$ m and $v_x = -\sqrt{2}$ m/s

So $\frac{1}{\sqrt{2}} = A_x \sin\phi_x$

and $-\sqrt{2} = A_x \cos\phi_x$

$\Rightarrow \tan\phi_x = -\frac{1}{2}$ … (1)

and $A_x = \sqrt{\frac{5}{2}}$ m … (2)

Similarly

$F_y = -y = ma_y$.

$\Rightarrow \frac{d^2y}{dt^2} = -y$

So, $y = A_y \sin(\omega t + \phi_y)$ ($\omega$ = 1 rad/s)

and $v_y = A_y\omega\cos(\omega t + \phi_y)$

at $t = 0$ $y = \sqrt{2}$ m and $v_y = \sqrt{2}$ m/s

So $\sqrt{2} = A_y \sin\phi$

and $\sqrt{2} = A_y \cos\phi$

$\Rightarrow \phi = \frac{\pi}{4}$ and $A_y = 2$ (3 and 4).

So, $(xv_y - yv_x) = \sqrt{\frac{5}{2}} \sin(\omega t + \phi_x) \times 2\cos(\omega t + \phi_y) - 2\sin(\omega t + \phi_y) \times \sqrt{\frac{5}{2}} \cos(\omega t + \phi_x)$

$= \sqrt{\frac{5}{2}} \times 2\,(\sin(\omega t + \phi_x)\cos(\omega t + \phi_y) - \sin(\omega t + \phi_y) \times \cos(\omega t + \phi_x))$

$= \sqrt{10}\sin(\phi_x - \phi_y)$

$= \sqrt{10}(\sin\phi_x \cos\phi_y - \cos\phi_x \sin\phi_y)$

$= \sqrt{10}\left(\frac{1}{\sqrt{5}} \times \frac{1}{\sqrt{2}} - \left(-\frac{2}{\sqrt{5}}\right) \times \frac{1}{\sqrt{2}}\right)$

$= 3$

2. In a radioactive decay chain reaction, ${}^{230}_{90}\text{Th}$ nucleus decays into ${}^{214}_{84}\text{Po}$ nucleus. The ratio of the number of $\alpha$ to number of $\beta^-$ particles emitted in this process is _______.

**Answer (2)**

**Sol.** Let number of $\alpha$ particles are $n_\alpha$ and $\beta$ particles are $n_\beta$ so

$4n_\alpha = 230 - 214$

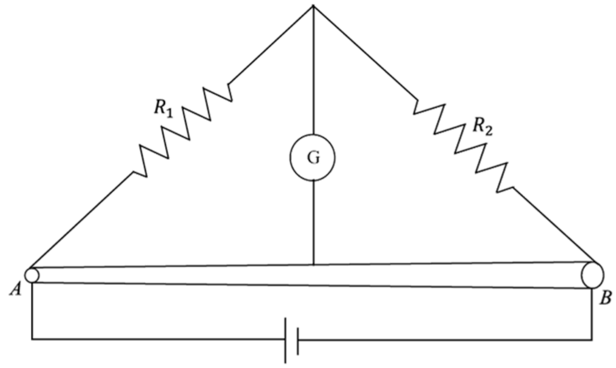
$\Rightarrow n_\alpha = 4$

$n_\beta = 84 - (90 - 2n_\alpha)$

$n_\beta = 2$

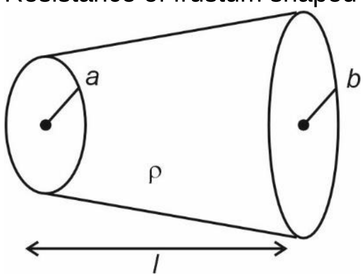
So $\frac{n_\alpha}{n_\beta} = 2$

3. Two resistances $R_1$ = X Ω and $R_2$ = 1 Ω are connected to a wire *AB* of uniform resistivity, as shown in the figure. The radius of the wire varies linearly along its axis from 0.2 mm at *A* to 1 mm at *B*. A galvanometer (*G*) connected to the center of the wire, 50 cm from each end along its axis, shows zero deflection when *A* and *B* are connected to a battery. The value of *X* is _______.



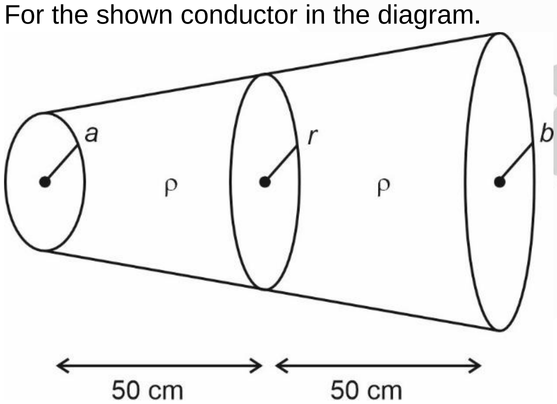

**Answer (5)**

**Sol.** Resistance of frustum shaped conductor shown is




$$R = \rho\frac{l}{\pi ab}$$

For the shown conductor in the diagram.




$$r = \frac{a+b}{2} = \frac{0.2+1}{2} = 0.6$$

thus, the resistance of left half is $P = \frac{\rho\times0.5\times10^6}{\pi\times0.2\times0.6}$

and the resistance of right half is $Q = \frac{\rho\times0.5\times10^6}{\pi\times0.6\times1}$

for Wheatstone to be balanced

$$\frac{R_1}{P} = \frac{R_2}{Q}$$

$$\frac{X\ \pi\times0.2\times0.6}{\rho\times0.5\times10^6} = \frac{1\pi\times0.6\times1}{\rho\times0.5\times10^6}$$

$\Rightarrow\ X = 5$

4. In a particular system of units, a physical quantity can be expressed in terms of the electric charge $e$, electron mass $m_e$, Planck's constant $h$, and Coulomb's constant $k = \frac{1}{4\pi\varepsilon_0}$, where $\varepsilon_0$ is the permittivity of vacuum. In terms of these physical constants, the dimension of the magnetic field is $[B] = [e]^\alpha [m_e]^\beta [h]^\gamma [k]^\delta$. The value of $\alpha + \beta + \gamma + \delta$ is ______.

**Answer (4)**

**Sol.** $[B] = [e]^\alpha [m_e]^\beta [h^\gamma] [k]^\delta$

$[M^1T^{-2}I^{-1}] = [IT]^\alpha [M]^\beta [ML^2T^{-1}]^\gamma [ML^3T^{-4}]^\delta$

So, $\beta + \gamma + \delta = 1$ ...(i)

$2\gamma + 3\delta = 0$ ...(ii)

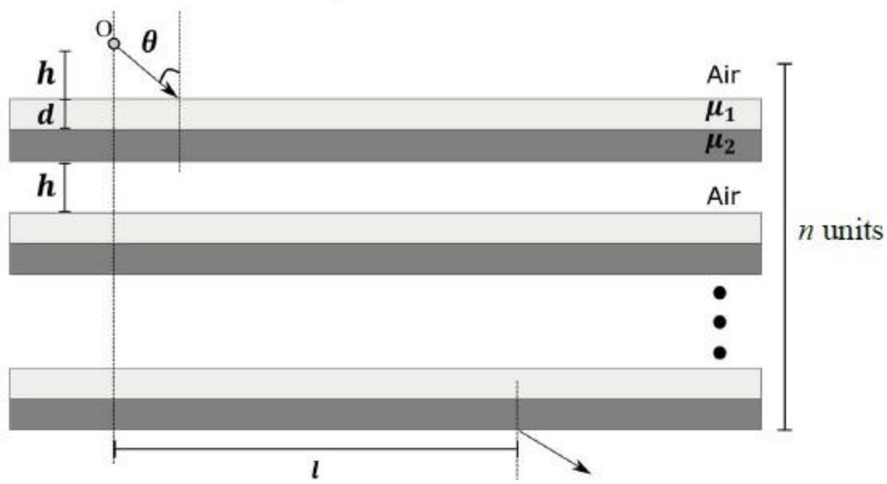
$\alpha - \gamma - 4\delta = -2$ ...(iii)

$\alpha - 2\delta = -1$ ...(iv)

On solving

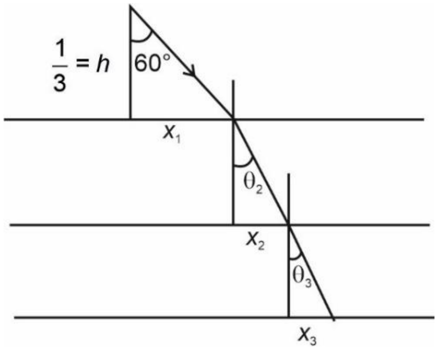
so, $\alpha + \beta + \gamma + \delta = 4$

5. Consider a configuration of $n$ identical units, each consisting of three layers. The first layer is a column of air of height $h = \frac{1}{3}$ cm, and the second and third layers are of equal thickness $d = \frac{\sqrt{3}-1}{2}$ cm, and refractive indices $\mu_1 = \sqrt{\frac{3}{2}}$ and $\mu_2 = \sqrt{3}$, respectively. A light source $O$ is placed on the top of the first unit, as shown in the figure. A ray of light from $O$ is incident on the second layer of the first unit at an angle of $\theta = 60°$ to the normal. For a specific value of $n$, the ray of light emerges from the bottom of the configuration at a distance $l = \frac{8}{\sqrt{3}}$ cm, as shown in the figure. The value of $n$ is _______.




**Answer (4)**

**Sol.**




$$x_1 = \frac{1}{3} \times \tan 60° = \frac{1}{\sqrt{3}} \text{ cm}$$

and, $1 \times \frac{\sqrt{3}}{2} = \sqrt{\frac{3}{2}} \times \sin\theta_2$

$\Rightarrow \theta_2 = 45°$

$\Rightarrow x_2 = d$

and, $1 \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2} \times \sin\theta_2$

$\Rightarrow \theta_3 = 30°$

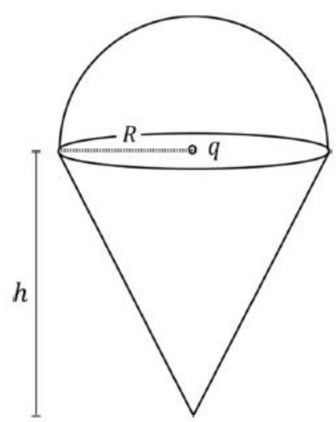
$\Rightarrow x_3 = \frac{d}{\sqrt{3}}$

$$\therefore \quad x_1 + x_2 + x_3 = \frac{1}{\sqrt{3}} + \frac{\left(\sqrt{3}-1\right)}{2}\left(1 + \frac{1}{\sqrt{3}}\right)$$

$$= \frac{2}{\sqrt{3}} \text{ cm}$$

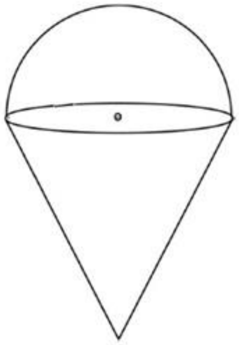
$$\therefore \quad n = \frac{l}{x_1 + x_2 + x_3} = \frac{8/\sqrt{3}}{2/\sqrt{3}} = 4$$

6. A charge $q$ is surrounded by a closed surface consisting of an inverted cone of height $h$ and base radius $R$, and a hemisphere of radius $R$ as shown in the figure. The electric flux through the conical surface is $\frac{nq}{6\varepsilon_0}$ (in SI units). The value of $n$ is _________.




**Answer (3)**

**Sol.**



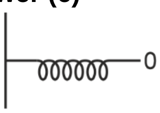
$\phi$ through cone $= \dfrac{q}{2\varepsilon_0}$

$\therefore \quad n = 3$

7. On a frictionless horizontal plane, a bob of mass $m = 0.1$ kg is attached to a spring with natural length $l_0 = 0.1$ m. The spring constant is $k_1 = 0.009$ Nm$^{-1}$ when the length of the spring $l > l_0$ and is $k_2 = 0.016$ Nm$^{-1}$ when $l < l_0$. Initially the bob is released from $l = 0.15$ m. Assume that Hooke's law remains valid throughout the motion. If the time period of the full oscillation is $T = (n\,\pi)$ s, then the integer closest to $n$ is _______.

**Answer (6)**

**Sol.**




$\omega_1 = \sqrt{\dfrac{k_1}{m}}$ and $\omega_2 = \sqrt{\dfrac{k_2}{m}}$

$\therefore \quad$ Time period $= \pi\sqrt{\dfrac{m}{k_1}} + \pi\sqrt{\dfrac{m}{k_2}}$

$= \pi\sqrt{\dfrac{0.1}{0.009}} + \pi\sqrt{\dfrac{0.1}{0.016}}$

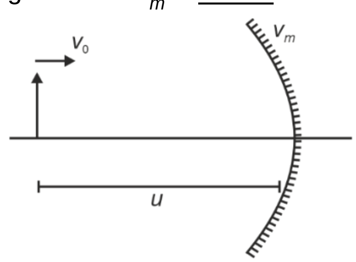
$= \dfrac{\pi}{0.3} + \dfrac{\pi}{0.4}$

$= \pi \times \left(\dfrac{4+3}{12}\right) \times 10$

$= \dfrac{70}{12}\pi$

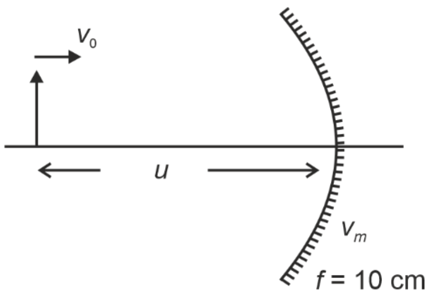
$= 5.83\pi$

8. An object and a concave mirror of focal length $f = 10$ cm both move along the principal axis of the mirror with constant speeds. The object moves with speed $V_0 = 15$ cm s$^{-1}$ towards the mirror with respect to a laboratory frame. The distance between the object and the mirror at a given moment is denoted by $u$. When $u = 30$ cm, the speed of the mirror $V_m$ is such that the image is instantaneously at rest with respect to the laboratory frame, and the object forms a real image. The magnitude of $V_m$ is _____ cm s$^{-1}$.




**Answer (3)**

**Sol.**




$$m = \frac{f}{u-f}$$

$$= \frac{10}{30-10} = \frac{1}{2}$$

$$\therefore \quad (V_0 - V_m) \times m^2 - V_m = 0$$

$$\Rightarrow \quad (V_0 - V_m) \times \frac{1}{4} = V_m$$

$$\Rightarrow \quad V_0 = 5V_m$$

$$\Rightarrow \quad V_m = \frac{V_0}{5}$$

$$= \frac{15}{5}$$

= 3 cm/s

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**SECTION – 2 (Maximum marks : 24)**

- This section contains **SIX (06)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

| | | | |
|---|---|---|---|
| *Full Marks* | : | +4 | **ONLY** if (all) the correct option(s) is(are) chosen; |
| *Partial Marks* | : | +3 | If all the four options are correct but **ONLY** three options are chosen; |
| *Partial Marks* | : | + 2 | If three or more options are correct but **ONLY** two options are chosen, both of which are correct; |
| *Partial Marks* | : | +1 | If two or more options are correct but **ONLY** one option is chosen and it is a correct option; |
| *Zero Marks* | : | 0 | If none of the options is chosen (i.e. the question is unanswered); |
| *Negative Marks* | : | –2 | In all other cases. |

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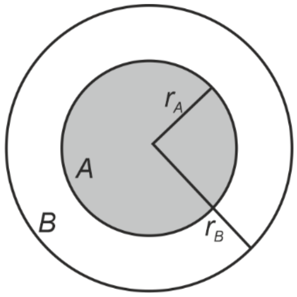
9. In the figure, the inner (shaded) region $A$ represents a sphere of radius $r_A = 1$, within which the electrostatic charge density varies with the radial distance $r$ from the center as $\rho_A = kr$, where $k$ is positive. In the spherical shell $B$ of outer radius $r_B$, the electrostatic charge density varies as $\rho_B = \frac{2k}{r}$. Assume that dimensions are taken care of. All physical quantities are in their SI units.




Which of the following statement(s) is/(are) correct?

(A) If $r_B = \sqrt{\frac{3}{2}}$, then the electric field is zero everywhere outside $B$.

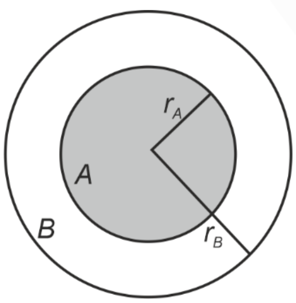
(B) If $r_B = \frac{3}{2}$, then the electric potential just outside $B$ is $\frac{k}{\varepsilon_0}$.

(C) If $r_B = 2$, then the total charge of the configuration is $15\pi k$.

(D) If $r_B = \frac{5}{2}$, then the magnitude of the electric field just outside $B$ is $\frac{13\pi k}{\varepsilon_0}$.

**Answer (B)**

**Sol.**




$$Q_{\text{Total}} = \int_0^{r_A} kr\left(4\pi r^2\right)dr + \int_{r_A}^{r_B} \frac{2k}{r}\left(4\pi r^2\right)dr$$

$$= \frac{4\pi k}{4} r_A^4 + \frac{8\pi k}{2}\left(r_B^2 - r_A^2\right)$$

$$= \pi k + 4\pi k\left(r_B^2 - r_A^2\right)$$

If $r_B = \sqrt{\frac{3}{2}}$

$$Q_{\text{Total}} = \pi k r_A^4 + 4\pi k\left(\frac{3}{2} - r_A^2\right)$$

$$= \pi k + 4\pi k\left(\frac{3}{2} - 1\right)$$

$$= \pi k + 2\pi k = 3\pi k$$

If $r_B = \frac{3}{2}$

$$Q_{\text{Total}} = \pi k + 4\pi k\left(\frac{9}{4} - 1\right)$$

$$= \pi k + 4\pi k\left(\frac{5}{4}\right) = 6\pi k$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{6\pi k}{r_B} = \frac{3k}{2}\frac{2}{3\varepsilon_0} = \frac{k}{\varepsilon_0}$$

(B) is correct

If $r_B = 2$

$$Q_{\text{Total}} = \pi k + 4\pi k(4 - 1)$$

$$= 13\pi k$$

Option (C) is incorrect

If $r_B = \frac{5}{2}$

$$Q_{\text{Total}} = \pi k + 4\pi k\left(\frac{25}{4} - 1\right)$$

$$= \pi k + \pi k(21)$$

$$= 22\pi k$$

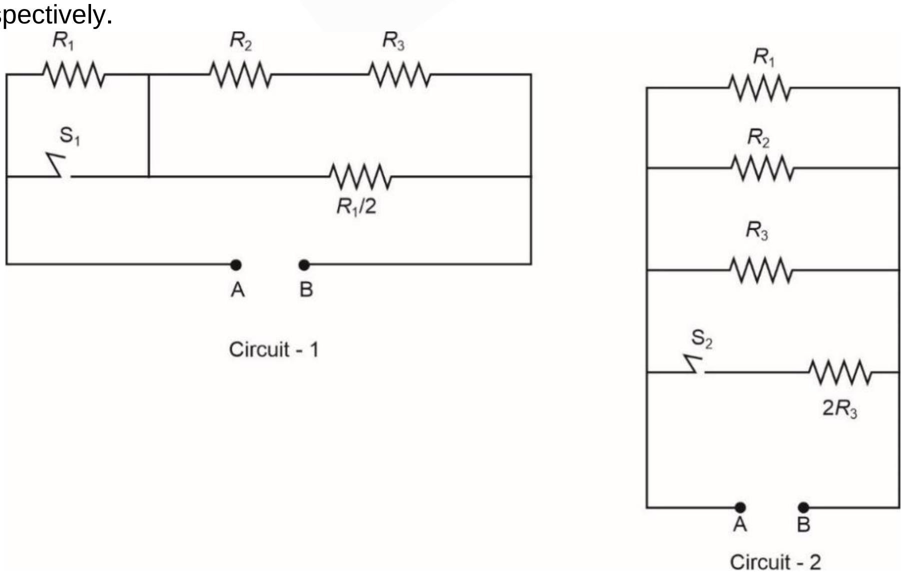
$$E = \frac{1}{4\pi\varepsilon_0}\frac{22\pi k}{25} \times 4$$

$$= \frac{22k}{25\varepsilon_0}$$

10. In Circuit-1 and Circuit-2 shown in the figures, $R_1 = 1\ \Omega$, $R_2 = 2\ \Omega$ and $R_3 = 3\ \Omega$.

$P_1$ and $P_2$ are the power dissipations in Circuit-1 and Circuit-2 when the switches $S_1$ and $S_2$ are in open conditions, respectively.

$Q_1$ and $Q_2$ are the power dissipations in Circuit-1 and Circuit-2 when the switches $S_1$ and $S_2$ are in closed conditions, respectively.




Which of the following statement(s) is(are) correct?

(A) When a voltage source of 6 V is connected across A and B in both circuits, $P_1 < P_2$

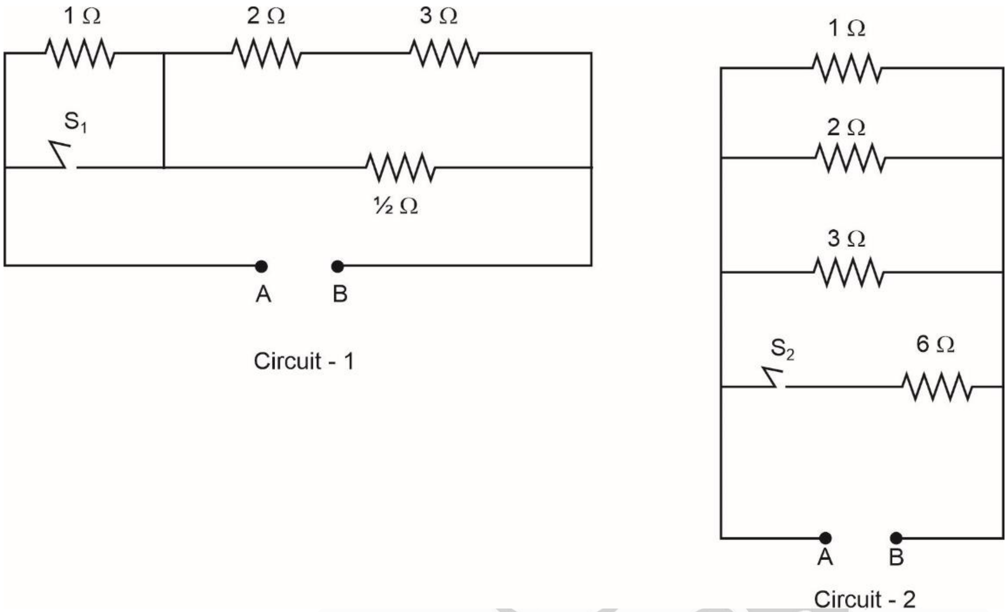
(B) When a constant current source of 2 Amp is connected across A and B in both circuits, $P_1 > P_2$

(C) When a voltage source 6 V is connected across A and B in Circuit-1, $Q_1 > P_1$

(D) When a constant current source of 2 Amp is connected across A and B in both circuits, $Q_2 < Q_1$

**Answer (A, B, C)**

**Sol.**






When $S_1$ and $S_2$ are open

$$(R_{eq})_1 = 1 + \frac{5 \times \frac{1}{2}}{5 + \frac{1}{2}} = 1 + \frac{5}{11} = \frac{16}{11}$$

$$P_1 = \frac{V^2}{R_{eq}} = \frac{(6)^2}{16} \times 11 = \frac{36 \times 11}{16} = 24.75 \text{ W}$$

$$(R_{eq})_2 = \frac{6}{11}\ \Omega$$

$$P_2 = \frac{V^2}{R_{eq}} = \frac{(6)^2}{6} \times 11 = \frac{36 \times 11}{6} = 66 \text{ W}$$

$P_2 > P_1$

Option (A) is correct.

$\Rightarrow$ If 2 A source is used in both the cases.

$$P_1 = i^2 (R_{eq})_1 = (2)^2 \times \frac{16}{11} = \frac{64}{11} = 5.818 \text{ W}$$

$$P_2 = i^2 (R_{eq})_2 = (2)^2 \times \frac{6}{11} = \frac{24}{11} = 2.1818 \text{ W}$$

$P_1 > P_2$

Option (B) is correct

For $Q_1$

$$R_{eq} = \frac{5}{11}\,\Omega$$

$$Q_1 = \frac{V^2}{R_{eq}} = \frac{(6)^2}{\frac{5}{11}} = \frac{36 \times 11}{5} = 79.2\text{ W}$$

$P_1$ = 24.75 W

$Q_1 > P_1$

Option (C) is correct.

For option (D)

$$Q_1 = i^2R_{eq} = (2)^2 \times \frac{5}{11} = \frac{20}{11} = 1.81\text{ W}$$

$$Q_2 = i^2R_{eq} = (2)^2 \times \frac{1}{2} = \frac{4}{2} = 2\text{ W}$$

$Q_2 > Q_1$

Option (D) is incorrect.

11. A bubble has surface tension $S$. The ideal gas inside the bubble has ratio of specific heats $\gamma = \frac{5}{3}$. The bubble is exposed to the atmosphere and it always retains its spherical shape. When the atmospheric pressure is $P_{a1}$, the radius of the bubble is found to be $r_1$ and the temperature of the enclosed gas is $T_1$. When the atmospheric pressure is $P_{a2}$, the radius of the bubble and the temperature of the enclosed gas are $r_2$ and $T_2$, respectively. Which of the following statement(s) is(are) correct?

(A) If the surface of the bubble is a perfect heat insulator, then $\left(\frac{r_1}{r_2}\right)^5 = \frac{P_{a2} + \frac{2S}{r_2}}{P_{a1} + \frac{2S}{r_1}}$

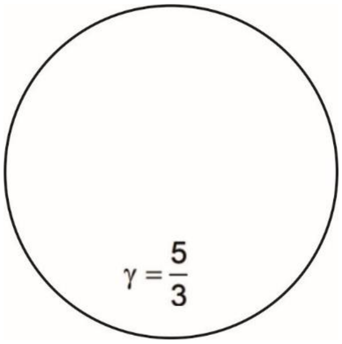
(B) If the surface of the bubble is a perfect heat insulator, then the total internal energy of the bubble including its surface energy does not change with the external atmospheric pressure.

(C) If the surface of the bubble is a perfect heat conductor and the change in atmospheric temperature is negligible, then $\left(\frac{r_1}{r_2}\right)^3 = \frac{P_{a2} + \frac{4S}{r_2}}{P_{a1} + \frac{4S}{r_1}}$

(D) If the surface of the bubble is a perfect heat insulator, then $\left(\frac{T_2}{T_1}\right)^{\frac{5}{2}} = \frac{P_{a2} + \frac{4S}{r_2}}{P_{a1} + \frac{4S}{r_1}}$

**Answer (C, D)**

**Sol.** *S* : Surface tension




| | Pressure | | Radius | | Temperature |
|---|---|---|---|---|---|
| When, | $P_{a1}$ | $\longrightarrow$ | $r_1$ | $\longrightarrow$ | $T_1$ |
| | $P_{a2}$ | $\longrightarrow$ | $r_2$ | $\longrightarrow$ | $T_2$ |

For adiabatic process

$$P_1V_1^{\gamma} = P_2V_2^{\gamma}$$

$$\left(P_{a1} + \frac{4T}{r_1}\right)\left(\frac{4}{3}\pi r_1^3\right)^{\frac{5}{3}} = \left(P_{a2} + \frac{4T}{r_2}\right)\left(\frac{4}{3}\pi r_2^3\right)^{\frac{5}{3}}$$

$$\left(\frac{r_1}{r_2}\right)^5 = \frac{\left(P_{a2} + \frac{4T}{r_2}\right)}{\left(P_{a1} + \frac{4T}{r_1}\right)}$$

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$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = \left(\frac{r_1}{r_2}\right)^{3\left(\frac{2}{3}\right)}$$

$$\left(\frac{T_2}{T_1}\right) = \left(\frac{P_{a2} + \frac{4T}{r_2}}{P_{a1} + \frac{4T}{r_1}}\right)^{\frac{2}{5}}$$

For option (B) Total internal energy + surface energy will not be same as word done by gas will be there.

Option (B) is incorrect.

For option (C)

$$P_1V_1 = P_2V_2$$

$$\left(P_{a1} + \frac{4T}{r_1}\right)\left(\frac{4}{3}\pi r_1^3\right) = \left(P_{a2} + \frac{4T}{r_2}\right)\left(\frac{4}{3}\pi r_2^3\right)$$

$$\left(\frac{r_1}{r_2}\right)^3 = \frac{\left(P_{a2} + \frac{4T}{r_2}\right)}{\left(P_{a1} + \frac{4T}{r_1}\right)}$$

Option (C) is correct

12. A disk of radius $R$ with uniform positive charge density $\sigma$ is placed on the $xy$ plane with its center at the origin. The Coulomb potential along the $z$-axis is

$$V(z)=\frac{\sigma}{2\in_0}\left(\sqrt{R^2+z^2}-z\right)$$

A particle of positive charge $q$ is placed initially at rest at a point on the $z$-axis with $z = z_0$ and $z_0 > 0$. In addition to the Coulomb force, the particle experiences a vertical force $\vec{F}=-c\hat{k}$ with $c > 0$. Let $\beta=\frac{2c\in_0}{q\sigma}$. Which of the following statement(s) is(are) correct?

(A) For $\beta=\frac{1}{4}$ and $z_0=\frac{25}{7}R$, the particle reaches the origin.

(B) For $\beta=\frac{1}{4}$ and $z_0=\frac{3}{7}R$, the particle reaches the origin.

(C) For $\beta=\frac{1}{4}$ and $z_0=\frac{R}{\sqrt{3}}$, the particle returns back to $z = z_0$

(D) For $\beta>1$ and $z_0>0$, the particle always reaches the origin.

**Answer (A, C, D)**

**Sol.** $V(z)=\frac{\sigma}{2\in_0}\left(\sqrt{R^2+z^2}-z\right)$

$U_z=cz$

$\Rightarrow\ U(z)_{net}=\frac{\sigma q}{2\in_0}\left(\sqrt{R^2+z^2}-z\right)+cz$

$=c\left[\frac{\sigma q}{2c\in_0}\left(\sqrt{R^2+z^2}-z\right)+z\right]$

$=c\left[4\sqrt{R^2+z^2}-3z\right]$ at $\beta=\frac{1}{4}$

At $z = 0$, $\beta=\frac{1}{4}$

$U(z)_{net}=c[4R]=4Rc$ …(i)

At $z=z_0=\frac{25}{7}R,\ \beta=\frac{1}{4}$

$U(z)_{net}=c\left[4\times\frac{26R}{7}-3\times\frac{25R}{7}\right]=\frac{29}{7}Rc$ …(ii)

at $z=z_0=\frac{3}{7}R,\ \beta=\frac{1}{4}$

$U(z)_{net}=c\left[4\times\frac{\sqrt{58}}{7}R-\frac{9R}{7}\right]\approx 3Rc$ …(iii)

At $z=\frac{R}{\sqrt{3}},\ \beta=\frac{1}{4}$

$U(z)_{net}=c\left[\frac{8R}{\sqrt{3}}-\frac{3R}{\sqrt{3}}\right]\approx 2.887Rc$

⇒ In option (A) particle reaches at origin with positive kinetic energy

$\frac{dU(z)}{dz} = 0$ at $z = \frac{3R}{\sqrt{7}}$

at $\beta = \frac{1}{4}$ and $z = \frac{3R}{\sqrt{7}}$

$U(z)_{\text{net}} = \sqrt{7}Rc = 2.645$

⇒ In option B at $U(z)_{\text{net}} \approx 3Rc$

⇒ The kinetic energy at origin will become negative

at $z = \frac{R}{\sqrt{3}}$

⇒ In option (C), $U(z)_{net}$ at $z = \frac{R}{\sqrt{3}} < U(z)_{net}$ at $z = 0$,

And $U(z)_{net}$ at $z = \frac{R}{\sqrt{3}} > U(z)_{net}$ at $z = \frac{3R}{\sqrt{7}}$

⇒ Particle will return back to $z_0$.

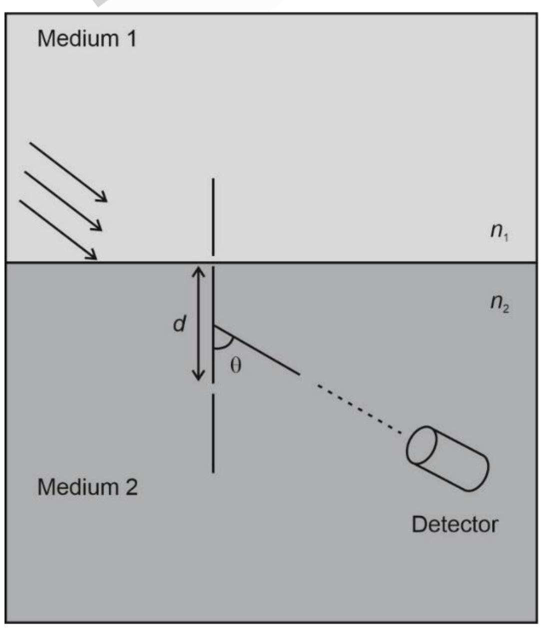
In option (D) $(\beta > 1, z_0 > 0)$

$U(z)_{net}$ will keep on increasing with $z$

⇒ Particle always reaches the origin.

⇒ Answer (A, C, D)

13. A double slit setup is shown in the figure. One of the slits is in medium 2 of refractive index $n_2$. The other slit is at the interface of this medium with another medium 1 of refractive index $n_1$ ($\neq n_2$). The line joining the slits is perpendicular to the interface and the distance between the slits is $d$. The slit widths are much smaller than $d$. A monochromatic parallel beam of light is incident on the slits from medium 1. A detector is placed in medium 2 at a large distance from the slits, and at an angle $\theta$ from the line joining them, so that $\theta$ equals the angle of refraction of the beam. Consider two approximately parallel rays from the slits received by the detector.



Which of the following statement(s) is(are) correct?

(A) The phase difference between the two rays is independent of d.

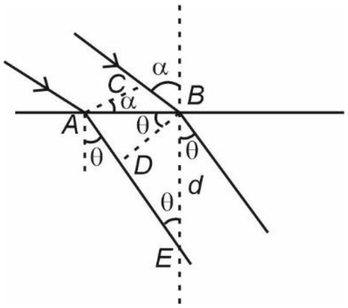
(B) The two rays interfere constructively at the detector.

(C) The phase difference between the two rays depends on $n_1$ but is independent of $n_2$.

(D) The phase difference between the two rays vanishes only for certain values of d and the angle of incidence of the beam, with q being the corresponding angle of refraction.

**Answer (A, B)**

**Sol.**




$AB = (d)(\tan\theta)$

and $BC = AB \sin\alpha = (d)(\tan\theta)(\sin\alpha)$

Also, $AD = AB \sin\theta$

$\Rightarrow$ Path difference (in vaccum)

$= n_1 BC - n_2 AD$

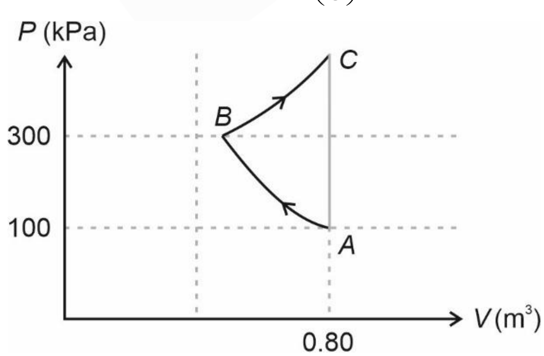
$= n_1(AB)\sin\alpha - n_2(AB\sin\theta)$

$= AB(n_1 \sin\alpha - n_2 \sin\underline{\theta}) = 0$

$\Rightarrow$ (A), (B) are correct

(C), (D) are incorrect.

14. In the given *P*-*V* diagram, a monoatomic gas $\left(\gamma = \frac{5}{3}\right)$ is first compressed adiabatically from state *A* state *B*. Then it expands isothermally from state *B* to state *C*. [Given : $\left(\frac{1}{3}\right)^{0.6} \simeq 0.5, \ln 2 \simeq 0.7$].




Which of the following statement(s) is(are) correct?

(A) The magnitude of the total work done in the process $A \to B \to C$ is 144 kJ.

(B) The magnitude of the work done in the process $B \to C$ is 84 kJ.

(C) The magnitude of the work done in the process $A \to B$ is 60 kJ.

(D) The magnitude of the work done in the process $C \to A$ is zero.

**Answer (C, D)**

**Sol.** $Pv^{\gamma} = c$

$$\Rightarrow \quad 100(0.8)^{5/3} = 300(v)^{5/3}$$

$$\Rightarrow \quad V_B = \frac{0.8}{3^{3/5}}$$

$$\Rightarrow \quad W_{AB} = \frac{P_AV_A - P_BV_B}{\frac{5}{3}-1} = \frac{80 - 300 \times \frac{0.8}{3^{3/5}}}{2/3} \text{ kJ}$$

$$= \frac{80 - 240(0.5)}{2/3} \text{ kJ}$$

$$= -60 \text{ kJ}$$

$\Rightarrow$ (C) is correct

$C \to A$ is isochoric $\Rightarrow$ (D) is correct

$$BC : W_{BC} = nRT \ln \frac{V_2}{V_1} = PV \ln \frac{V_2}{V_1}$$

$$= 300 \times \frac{0.8}{3^{3/5}} \ln \left[ \frac{0.8}{\frac{0.8}{3^{3/5}}} \right]$$

$$= \frac{240}{3^{3/5}} \ln \left(3^{3/5}\right)$$

$$= 120 \times \frac{3}{5} \ln 3 \text{ kJ}$$

$$= 79 \text{ kJ}$$

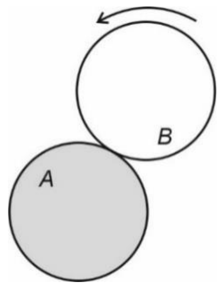
$\Rightarrow$ (A) & (B) are not correct.

**SECTION – 3 (Maximum marks : 12)**

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

| | | | |
|---|---|---|---|
| Full Marks | : | +3 | If **ONLY** the correct option is chosen; |
| Zero Marks | : | 0 | If none of the options is chosen (i.e. the question is unanswered); |
| Negative Marks | : | −1 | In all other cases. |

15. A flat surface of a thin uniform disk $A$ of radius $R$ is glued to a horizontal table. Another thin uniform disk $B$ of mass $M$ and with the same radius $R$ rolls without slipping on the circumference of $A$, as shown in the figure. A flat surface of $B$ also lies on the plane of the table. The center of mass of $B$ has fixed angular speed $\omega$ about the vertical axis passing through the center of $A$. The angular momentum of $B$ is $nM\omega R^2$ with respect to the center of $A$. Which of the following is the value of $n$?




(A) 2 (B) 5

(C) $\frac{7}{2}$ (D) $\frac{9}{2}$

**Answer (B)**

**Sol.** Angular momentum of $B$ with respect to center of $A$

$$\vec{L} = \vec{L}_{CM} + \vec{L}_{\text{Body about CM}}$$

$$= M(2R)^2\omega\hat{k} + \frac{MR^2}{2}(\omega_{\text{body}})\hat{k}$$

$$= M(2R)^2\omega\hat{k} + \frac{MR^2}{2}(2\omega)\hat{k}$$

$$= 5MR^2\omega\hat{k}$$

Comparing the magnitude with $nM\omega R^2$

$n = 5$

16. When light of a given wavelength is incident on a metallic surface, the minimum potential needed to stop the emitted photoelectrons is 6.0 V. This potential drops to 0.6 V if another source with wavelength four times that of the first one and intensity half of the first one is used. What are the wavelength of the first source and the work function of the metal, respectively? [Take $\frac{hc}{e} = 1.24 \times 10^{-6}$ J m $C^{-1}$.]

(A) $1.72 \times 10^{-7}$ m, 1.20 eV

(B) $1.72 \times 10^{-7}$ m, 5.60 eV

(C) $3.78 \times 10^{-7}$ m, 5.60 eV

(D) $3.78 \times 10^{-7}$ m, 1.20 eV

**Answer (A)**

**Sol.** $h\nu - \phi = 6$ eV

$$\frac{hc}{\lambda} - \phi = 6 \text{ eV} \quad \ldots\text{(i)}$$

$$\frac{hc}{4\lambda} - \phi = 0.6 \text{ eV}$$

$$\frac{3hc}{4\lambda} = 5.4 \text{ eV}$$

$\therefore \quad \lambda = \frac{3hc}{4 \times 5.4 \text{ eV}} = \frac{3 \times 1.24 \times 10^{-6}}{4 \times 5.4}$

$= 1.72 \times 10^{-7}$ m

$\Rightarrow$ from equation (i)

$$\frac{hc}{1.72 \times 10^{-7}} \times \frac{1}{1.6 \times 10^{-19}} - \phi = 6 \text{ eV}$$

$$\frac{2 \times 10^{-25}}{2.75 \times 10^{-26}} - \phi = 6$$

$\Rightarrow \quad \phi = (7.27 - 6) \cong 1.2$ eV

17. Area of the cross-section of a wire is measured using a screw gauge. The pitch of the main scale is 0.5 mm. The circular scale has 100 divisions and for one full rotation of the circular scale, the main scale shifts by two divisions. The measured readings are listed below.

| Measurement condition | Main scale reading | Circular scale reading |
|---|---|---|
| Two arms of gauge touching each other without wire | 0 division | 4 divisions |
| Attempt-1: With wire | 4 divisions | 20 divisions |
| Attempt-2: With wire | 4 divisions | 16 divisions |

What are the diameter and cross-sectional area of the wire measured using the screw gauge?

(A) 2.22 ± 0.02 mm, $\pi$ (1.23 ± 0.02) mm$^2$

(B) 2.22 ± 0.01 mm, $\pi$ (1.23 ± 0.01) mm$^2$

(C) 2.14 ± 0.02 mm, $\pi$ (1.14 ± 0.02) mm$^2$

(D) 2.14 ± 0.01 mm, $\pi$ (1.14 ± 0.01) mm$^2$

**Answer (D)**

**Sol. Attempt-1**

MSR = 4 × 0.5 = 2 mm

$\text{CSR} = \frac{1}{100} \times 20 = 0.20 \text{ mm}$

Zero error = $\frac{1}{100} \times 4 = 0.04 \text{ mm}$

$(\text{Reading})_1$ = MSR + CSR – Zero error

= 2 + 0.20 – 0.04 mm

= 2.16 mm

**Attempt-2**

MSR = 4 × 0.5 = 2 mm

$\text{CSR} = \frac{1}{100} \times 16 = 0.16 \text{ mm}$

Zero error $= \frac{1}{100} \times 4 = 0.04$ mm

$(\text{Reading})_2$ = MSR + CSR – Zero error

= 2 + 0.16 – 0.04 mm

= 2.12 mm

Reading $= \frac{(\text{Reading})_1 + (\text{Reading})_2}{2}$

= 2.14 mm

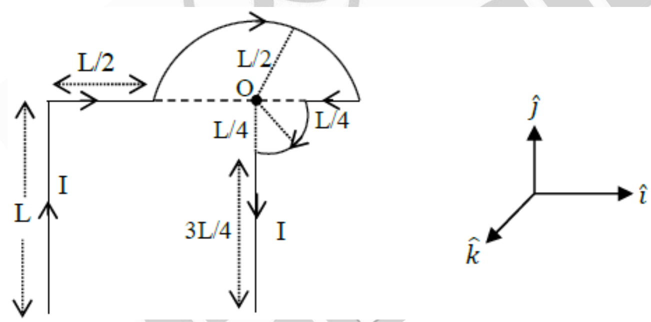
Error $= \frac{1}{100} = 0.01$ mm so diameter $= 2.14 \pm 0.01$ mm

Area $= \frac{\pi d^2}{4}$

$$dA = \left(\frac{2\pi d}{4}\right)(\Delta d)$$

$$\Rightarrow \quad A = \pi\left[1.14 \pm 0.01\right] \text{ mm}^2$$

18. Which one of the following options represents the magnetic field $\vec{B}$ at $O$ due to the current flowing in the given wire segments lying on the $xy$ plane?




(A) $\vec{B} = \frac{-\mu_0 I}{L}\left(\frac{3}{2} + \frac{1}{4\sqrt{2}\,\pi}\right)\hat{k}$

(B) $\vec{B} = \frac{-\mu_0 I}{L}\left(\frac{3}{2} + \frac{1}{2\sqrt{2}\,\pi}\right)\hat{k}$

(C) $\vec{B} = \frac{-\mu_0 I}{L}\left(1 + \frac{1}{4\sqrt{2}\,\pi}\right)\hat{k}$

(D) $\vec{B} = \frac{-\mu_0 I}{L}\left(1 + \frac{1}{4\pi}\right)\hat{k}$

**Answer (C)**

**Sol.** $B_{net} = B_{semicircle} + B_{quarter} + B_{straight}$

$$= \frac{\mu_0 I}{4\left(\frac{L}{2}\right)} + \frac{\mu_0 I}{8 \times \left(\frac{L}{4}\right)} + \frac{\mu_0 I}{4\pi \times L}\left(\frac{1}{\sqrt{2}}\right)\left(-\hat{k}\right)$$

$$= \left(\frac{\mu_0 I}{2L} + \frac{\mu_0 I}{2L} + \frac{\mu_0 I}{4\sqrt{2}\pi L}\right)\left(-\hat{k}\right)$$

$$= \frac{\mu_0 I}{L}\left(1 + \frac{1}{4\sqrt{2}\,\pi}\right)\left(-\hat{k}\right)$$