Question 1: A circle S passes through the point (0, 1) and is orthogonal to the circles $(x-1)^2 + y^2 = 16$ and $x^2 + y^2 = 1$. Then

(a) radius of S is 8

(b) radius of S is 7

(c) centre of S is (-7, 1)

(d) centre of S is (-8, 1)

Solution:

Let the equation of the circles be

$x^2+y^2+2gx+2fy+c = 0$ ..(i)

It passes through (0, 1)

=> $1 + 2f + c = 0$ …(ii)

Since circle (i) is orthogonal to $(x-1)^2 + y^2 = 16$

=> $x^2 + y^2 - 2x - 15 = 0$

and $x^2 + y^2 - 1 = 0$

$2g\times(-1) + 2f\times 0 = c-15$

$2g + c - 15 = 0$ …(iii)

$2g\times 0 + 2f\times 0 = c-1$

=> $c = 1$ …(iv)

Solving (ii), (iii) and (iv)

=> $c = 1$, $g = 7$ and $f = -1$

The required circle is $x^2 + y^2 + 14x - 2y + 1 = 0$, with centre (-7, 1) and radius = 7.

Hence option b and c are correct.

Question 2: Let O be the centre of the circle $x^2 + y^2 = r^2$, where $r > \sqrt{5}/2$. Suppose PQ is a chord of this circle and the equation of the line passing through P and Q is $2x + 4y = 5$. If the centre of the circumcircle of the triangle OPQ lies on the line $x + 2y = 4$, then the value of r is

(a) 1

(b) 4

(c) 6

(d) 2

Solution:

S1: $x^2 + y^2 = r^2$ where $r > \sqrt{5}/2$

$C_1 = (0,0)$

let S_2: $x^2 + y^2 + ax + by = 0$

$C_2 = (-a/2, -b/2)$

PQ: $S_1 - S_2 = 0$

PQ: $ax + by + r^2 = 0$ ………………… (1)

Given PQ: $2x + 4y - 5 = 0$ ………………… (2)

comparing equation (1) and (2)

$(a/2) = (b/4) = (r^2/-5)$…………..…..(3)

Also, centre of S_2 lies on $x + 2y = 4$

$(-a/2) - b = 4$ …(4)

From equation (3) and (4)

$r = 2$

Hence option d is the answer.

Question 3: The circle passing through the point (-1, 0) and touching the y-axis at (0, 2) also passes through the point

(a) (-3/2, 0)

(b) (-5/2, 2)

(c) (-3/2, 5/2)

(d) (-4, 0)

Solution:

Equation of circle passing through a point (x_1, y_1) and touching the straight line L, is given by

$(x - x_1)^2 + (y - y_1)^2 + \lambda L = 0$

The circle is passing through the point (0, 2)

=> $(x - 0)^2 + (y - 2)^2 + \lambda L = 0$

=> $x^2 + (y - 2)^2 + \lambda x = 0$ …(i)

Circle passes through (-1, 0).

=> $1 + 4 - \lambda = 0$

=> $\lambda = 5$

Put $\lambda = 5$ in (i)

$x^2 + (y - 2)^2 + 5x = 0$

=> $x^2 + y^2 - 4y + 4 + 5x = 0$

=> $x^2 + y^2 + 5x - 4y + 4 = 0$

Put y = 0

=> x = -4, x = -1

Check the options.

Hence option d is the answer.

Question 4: The number of common tangents to the circles $x^2 + y^2 = 4$ and $x^2 + y^2 - 6x - 8y = 24$ is

(a) 1

(b) 2

(c) 0

(d) 3

Solution:

Given circle $x^2 + y^2 = 4$ with centre $C_1(0, 0)$ and $R_1 = 2$

Also $x^2 + y^2 - 6x - 8y - 24 = 0$ with centre $C_2(3, 4)$ and $R_2 = 7$

Distance between centres = $C_1C_2 = 5 = R_2 - R_1$

So the circles touch internally and they can have just one common tangent at the point of contact.

Hence option a is the answer.

Question 5: The points of intersection of the line $4x - 3y - 10 = 0$ and the circle $x^2 + y^2 - 2x + 4y - 20 = 0$ are

(a) (4, 2)

(b) (-2, -6)

(c) (2, 2)

(d) (-2, -4)

Solution:

Given equation of the line $4x - 3y - 10 = 0$

=> $x = (3y+10)/4$ …(i)

Equation of the circle $x^2 + y^2 - 2x + 4y - 20 = 0$ ..(ii)

Put (i) in (ii)

=> $[(3y+10)^2/4^2] + y^2 - 2(3y+10)/4 + 4y - 20 = 0$

=> $y^2 + 4y - 12 = 0$

=> $y = 2, -6$

Put y in (i)

=> $x = 4, -2$

So the points are (4, 2) and (-2, -6).

Hence option a and b are correct.

Question 6: If the tangent at (1,7) to the curve $x^2 = y-6$ touches the circle $x^2 + y^2 + 16x + 12y + c = 0$, then the value of c is

(a) 185

(b) 85

(c) 195

(d) 95

Solution:

Given curve is $x^2 = y - 6$

Differentiate w.r.t.x

$2x = dy/dx$

$(dy/dx)_{(1,7)} = 2$

Equation of tangent at (1, 7) to $x^2 = y - 6$ is

$y-y_1 = m(x-x_1)$

Here m = 2

$(y-7) = 2 (x - 1)$

$2x - y + 5 = 0$

The perpendicular from the centre (-8, -6) to 2x - y + 5 = 0 is equal to radius of circle.

So $|(-16+6+5)/\sqrt{5}| = \sqrt{(64 + 36 - c)}$

=> 5 = 100 - c

=> c = 95

Hence option d is the answer.

Question 7: The circle passing through the intersection of the circles, $x^2 + y^2 - 6x = 0$ and $x^2 + y^2 - 4xy = 0$, having its centre on the line, 2x - 3y + 12 = 0, also passes through the point

(a) (-1, 3)

(b) (-3, 6)

(c) (-3, 1)

(d) (1, -3)

Solution:

Let the family of circles be $S_1 + \lambda S_2 = 0$

$x^2 + y^2 - 6x + \lambda(x^2 + y^2 - 4y) = 0$

=> $(1+\lambda)x^2 + (1+\lambda)y^2 - 6x - 4\lambda y = 0$ …(i)

Centre $(-g, -f) = (3/(1+\lambda), 2\lambda/(1+\lambda))$

Centre lies on 2x - 3y + 12 = 0

Then $(6/(\lambda+1)) - (6\lambda/(\lambda+1)) + 12 = 0$

=> $\lambda = -3$

Equation of circle (i)

$-2x^2 - 2y^2 - 6x + 12y = 0$

$\Rightarrow x^2 + y^2 + 3x - 6y = 0$

Check options.

(-3, 6) satisfy equation (ii).

Hence option b is the answer.

Question 8: Let the tangents drawn from the origin to the circle, $x^2 + y^2 - 8x - 4y + 16 = 0$ touch it at points A and B. Then $(AB)^2$ is equal to

(a) 52/5

(b) 56/5

(c) 64/5

(d) 32/5

Solution:

Length of tangent, $L = \sqrt{S_1} = \sqrt{16} = 4$

$R = \sqrt{(16 + 4 - 16)}$

$= 2$

Length of chord ofcontact $= 2LR/\sqrt{(L^2+R^2)}$

$= 16/\sqrt{20}$

Square of length of chord of contact = 64/5

Hence option c is the answer.

Question 9: The centre of the circle inscribed in the square formed by the lines $x^2 - 8x + 12 = 0$ and $y^2 - 14y + 45 = 0$

(a) (4, 7)

(b) (7, 4)

(c) (9, 4)

(d) (4, 9)

Solution:

The centre is given by the intersection of the diagonals (the mid-point of a diagonal).

$x^2 - 8x + 12 = 0$

=> $(x - 6)(x-2) = 0$

=> $x = 6 , 2$

$y^2 - 14y + 45 = 0$

=> $(y - 5)(y-9) = 0$

=> $y = 5, 9$

So A(2, 5), B(2, 9), C(6, 9), D(6, 5) form the square.

Therefore the centre of circle inscribed in square will be

((2+6)/2, (5+9)/2) = (4, 7)

Hence option a is the answer.

Question 10: If the tangent at the point P on the circle is $x^2 + y^2 + 6x + 6y = 2$ meets a straight line $5x - 2y + 6 = 0$ at a point Q on the y-axis, then the length of PQ is

(a) 4

(b) 5

(c) $2\sqrt{5}$

(d) $3\sqrt{5}$

Solution:

Given that the line $5x - 2y + 6 = 0$ is intersected by tangent at P to the circle $x^2 + y^2 + 6x + 6y - 2 = 0$ on y ais at Q.

On y axis, x = 0

=> 2y = 6

=> y = 3

So Q is (0, 3)

Tangent passes throught (0, 3).

PQ = length of tangent to the circle from (0, 3)

= √(0+9+0+18-2)

= 5

Hence option b is the answer.