Question 1: If $3/(2+\cos\theta+i\sin\theta) = a+ib$, then $[(a-2)^2+b^2]$ is

(1) 0

(2) 1

(3) -1

(4) 2

Solution:

Given $3/(2+\cos\theta+i\sin\theta) = a+ib$

$3((2+\cos\theta) - i\sin\theta)/(2+\cos\theta+i\sin\theta)((2+\cos\theta)- i\sin\theta)$

$= ((6+3\cos\theta) - i\,3\sin\theta)/((2+\cos\theta)^2+\sin^2\theta)$

$= ((6+3\cos\theta) - i(3\sin\theta))/(5+4\cos\theta)$

Comparing with a+ib, we get

$a = (6+3\sin\theta)/(5+4\cos\theta)$

$b = -3\sin\theta/(5+4\cos\theta)$

$(a-2)^2+b^2 = [(6+3\sin\theta)/(5+4\cos\theta)]-2)^2+(-3\sin\theta/(5+4\cos\theta))^2$

$= (-4-5\cos\theta)^2+9\sin^2\theta)/(5+4\cos\theta)^2$

$= ((4+5\cos\theta)^2+9\sin^2\theta)/(5+4\cos\theta)^2$

$= (16+40\cos\theta+25\cos^2\theta+9\sin^2\theta)/(5+4\cos\theta)^2$

$= (16+40\cos\theta+16\cos^2\theta+9(\sin^2\theta+\cos^2\theta))/(5+4\cos\theta)^2$

$= (16+40\cos\theta+16\cos^2\theta+9)/(5+4\cos\theta)^2$

$= (16\cos^2\theta+40\cos\theta+25)/(5+4\cos\theta)^2$

$= (4\cos\theta+5)^2/(5+4\cos\theta)^2$

$= 1$

Hence option (2) is the answer.

Question 2: If $z = x + iy$ is a complex number where x and y are integers. Then, the area of the rectangle whose vertices are the roots of the equation

$$z\overline{z}^3 + \overline{z}z^3 = 350$$

(1) 48

(2) 32

(3) 40

(4) 80

Solution:

Given

$$z\overline{z}^3 + \overline{z}z^3 = 350$$
$$z\overline{z}(\overline{z}^2 + z^2) = 350$$

Take z = x+iy

$(x+iy)(x-iy)[(x-iy)^2+(x+iy)^2] = 350$

$(x^2+y^2)(2x^2-2y^2) = 350$

$(x^2+y^2)(x^2-y^2) = 175 = (25)\times7$

$x^2+y^2 = 25$ (i)

$x^2-y^2 = 7$ ..(ii)

Adding (i) and (ii), we get

$2x^2 = 32$

$x = \pm4$

And $y = \pm3$

Hence the vertices of the triangle are (4,3), (4, -3), (-4, -3) and (-4,3).

Area of rectangle = length × breadth

= 8×6

= 48

Hence option (1) is the answer.

Question 3: If 1, a_1, a_2,...a_{n-1} are the nth roots of unity, then the value of $(1-a_1)(1 - a_2)(1 - a_3)...(1 - a_{n-1})$ is

(1) √3

(2) 1/2

(3) n

(4) 0

Solution:

Given 1, a_1, a_2,...a_{n-1} are the nth roots of unity.

So $x^n-1 = (x-1)(x-a_1)...(x-a_{n-1})$

$(x^n-1)/(x-1) = (x-a_1)(x-a_2)...(x-a_{n-1})$

$x^{n-1}+x^{n-2}+...x^2+x+1 = (x-a_1)(x-a_2)...(x-a_{n-1})$

Substitute x = 1

$(1-a_1)(1-a_2)...(1-a_{n-1})$

= 1+1+1+...n times

= n

Hence option (3) is the answer.

Question 4: If $|(z+i)/(z-i)| = \sqrt{3}$, then radius of the circle is

(1) 2/√21

(2) 1/√21

(3) √3

(4) √21

Solution:

Take z = x+iy

$|z| = \sqrt{(x^2+y^2)}$

Given $|(z+i)/(z-i)| = \sqrt{3}$

$|(z+i)| = \sqrt{3}|(z-i)|$

Put z = x+iy

$|(x+(y+1)i)| = \sqrt{3}|(x+(y-1)i)|$

Squaring we get

$x^2 + (y+1)^2 = 3(x^2+(y-1)^2)$

$x^2 + y^2 + 2y + 1 = 3x^2 + 3y^2 - 6y + 3$

$2x^2 + 2y^2 - 8y + 2 = 0$

$x^2 + y^2 - 4y + 1 = 0$

Equation of circle is $(x-h)^2 + (y-k)^2 = r^2$

Where (h, k) is the centre and radius is r.

$(x-0)^2 + (y-2)^2 = \sqrt{3}^2$

Radius is $\sqrt{3}$.

Hence option (3) is the answer.

Question 5: If α and β ∈ C are the distinct roots of the equation $x^2 - x + 1 = 0$, then $\alpha^{101} + \beta^{107}$ is equal to:

(1) 1

(2) 2

(3) -1

(4) 0

Solution:

$x^2 - x + 1 = 0$

Solve by using the quadratic formula, we get

$x = (1 \pm i\sqrt{3})/2$

$= (1+i\sqrt{3})/2 , (1-i\sqrt{3})/2$

$= -(-1-i\sqrt{3})/2, -(-1+i\sqrt{3})/2$

$= -\omega^2, -\omega$

$\alpha = -\omega^2$

$\beta = -\omega$

$\alpha^{101} + \beta^{107} = (-\omega^2)^{101} + (-\omega)^{107}$

$= -(\omega^{202} + \omega^{107})$

$= -[(\omega^3)^{67}\,\omega + (\omega^3)^{35}\,\omega^2]$

$= -[\omega + \omega^2]$

$= 1$

Hence option (1) is the answer.

Question 6: The region represented by {z = x+iy ∈ C : z -Re(z) ≤1} is also given by the inequality: {z = x + iy ∈ C : z -Re(z) ≤ 1}

(1) $y^2 \leq 2(x + 1/2)$

(2) $y^2 \leq x + (1/2)$

(3) $y^2 \geq 2(x + 1)$

(4) $y^2 \geq (x + 1)$

Solution:

{z = x + iy ∈ C : z -Re(z) ≤ 1}

$|z| = \sqrt{(x^2+y^2)}$

Re(z) = x

z- Re(z) ≤1

$\Rightarrow \sqrt{(x^2+y^2)} - x \leq 1$

$\Rightarrow \sqrt{(x^2+y^2)} \leq 1+x$

$\Rightarrow (x^2+y^2) \leq (1+x^2+2x)$

$\Rightarrow y^2 \leq 2(x + 1/2)$

Hence option (1) is the answer.

Question 7: If $((1+i)/(1-i))^{m/2} = ((1+i)/(i-1))^{n/3} = 1$, (m, n∈N) then the greatest common divisor of the least values of m and n is :

Solution:

$[(1+i)(1+i)/(1+i)(1-i)]^{m/2} = [(1+i)(-1-i)/(-1+i)(-1-i)]^{n/3} = 1$ $(i = \sqrt{-1})$

Solving LHS of above equation, we get

$(2i/2)^{m/2} = 1$

=> m = 8

Solving RHS,

$[(-1-i-i+1)/(1+1)]^{n/3} = 1$

$(-2i/2)^{n/3} = 1$

$(-i)^{n/3} = 1$

=> n = 12

Greatest common divisor of m and n is 4.

Question 8: If z is any complex number satisfying $|z - 3 - 2i| \leq 2$, then the minimum value of $|2z - 6 + 5i|$ is

(1) 2

(2) 1

(3) 3

(4) 5

Solution:

$|z - 3 - 2i| \leq 2$

$|2z - 6 + 5i| = 2|z - 3 + (5/2)i|$

$|z - 3 + (5/2)i|$

$=|z - 3 -2i+2i+(5/2)i|$ (using triangle inequality)

$= |(z - 3 -2i)+(9/2)i|$

$\geq ||z - 3 -2i)|-(9/2)|$

$\geq |2-9/2|\geq 5/2$

=> $|z - 3 + (5/2)i|\geq 5/2$

=> $|2z - 6+ 5i|\geq 5$

Hence option (4) is the answer.

Question 9: If z is a complex number such that the imaginary part of z is non-zero and $a = z^2 + z + 1$ is real. Then, a cannot take the value

(1) -1

(2)1/3

(3) 1/2

(4) ¾

Solution:

Given a = z^2+z+1

$z^2+z+1-a = 0$

If solution is not real then $b^2-4ac<0$.

1 - 4(1-a) <0

1 - 4 + 4a <0

-3 + 4a <0

4a <3

a< 3/4

Values of a less than 3/4 will give non real solutions.

Hence option (4) is the answer.

Question 10: Let a, b, x and y be real numbers such that a-b = 1 and y ≠ 0. If the complex number z = x + iy satisfies Im((az +b)/(z+1)) = y, then which of the following is(are) possible value(s) of x?

(1) $-1 + \sqrt{(1 - y^2)}$

(2) $-1 - \sqrt{(1 - y^2)}$

(3) $1 + \sqrt{(1 + y^2)}$

(4) $1 - \sqrt{(1 + y^2)}$

Solution:

a - b = 1

y ≠ 0

Im[(az +b)/(z+1)] = y

Let z = x+iy

=> Im [(a(x + iy) + b)/(x + iy + 1)][((x + 1) - iy)/((x + 1) - iy)] = y

=> $[-(ax + b)y + ay(x+1)]/(x+1)^2 + y^2 = y$

=> $(-axy - by + axy + ay)/(x+1)^2 + y^2 = y$

=> $a - b = (x+1)^2 + y^2$

=> $1 = (x+1)^2 + y^2$

So $x = -1 \pm\sqrt{(1-y^2)}$

Hence option (1) and (2) are correct.