**Question 1: A curve passes through the point (1, π/6). Let the slope of the curve at each point (x, y) be y/x + sec (y/x) , x> 0. Then the equation of the curve is**

(a) sin (y/x) = log x + 1/2

(b) cosec (y/x) = log x + 2

(c) sec (2y/x) = log x + 2

(d) cos (2y/x) = log x + 1/2

**Solution:**

Given that dy/dx = y/x + sec (y/x)

This is a homogeneous differential equation.

Let y = vx

dy/dx = v + x (dv/dx)

v + x (dv/dx) = v + sec v

x (dv/dx) = sec v

dv/sec v = dx/x

Integrating

$\int dv/\sec v = \int dx/x$

$\int \cos v \, dv = \int dx/x$

sin v = log |x| + C

sin (y/x) = log |x| + C

which passes through (1, π/6).

So C = sin (π/6) = ½

sin (y/x) = log x + ½ is the required equation.

Hence option a is the answer.

**Question 2: Let y(x) be a solution of the differential equation $(1+e^x)y' + ye^x = 1$. If y(0) = 2, then which of the following statements are true?**

(a) y(-4) = 0

(b) $y(-2) = 0$

(c) $y(x)$ has critical point in the interval $(-1, 0)$

(d) $y(x)$ has no critical point in the interval $(-1, 0)$

**Solution:**

Given that $(1+e^x)y' + ye^x = 1$

Divide by $1+e^x$

$dy/dx + ye^x/(1+e^x) = 1/(1+e^x)$

Integrating factor = $1+e^x$

Hence the solution is

$y(1+e^x) = \int dx + C$

$y(1+e^x) = x + C$

Given $y(0) = 2$

$\Rightarrow x = 0, y = 2$

$2(1+1) = 0+C$

$\Rightarrow C = 4$

$y(1+e^x) = x + 4$

$y = (x + 4)/(1+e^x)$

$y(-4) = 0$

Also $dy/dx = [(1+e^x) - (x+4)e^x]/(1+e^x)^2$

For critical point $dy/dx = 0$

$\Rightarrow (1+e^x) - (x+4)e^x = 0$

Let $g(x) = (1+e^x) - (x+4)e^x$

$g(-1) = 1+(1/e) -(-1+ 4)/e$

$= 1+(1/e) - 3/e$

$= 1 - 2/e >0$

$g(0) = (1+1)- 4 = -2 < 0$

So y(x) has critical point at (-1, 0).

Hence option a and c are correct.

**Question 3: Consider the family of all circles whose centers lie on the straight line y = x. If this family of circles is represented by the differential equation Py" + Qy' +1 = 0, where P, Q are functions of x, y and y' (here y' = dy/dx and y" = $d^2y/dx^2$) then which of the following statements is (are) true?**

(a) P = y + x

(b) P = y - x

(c) $P + Q = 1 - x + y + y' + (y')^2$

(d) $P - Q = x + y - y' - (y')^2$

**Solution:**

Let the equation of circle be $x^2+y^2+2gx+2gy+c = 0$

Differentiating, we get

$2x + 2y y' + 2g + 2gy' = 0$

$\Rightarrow x + yy' + g + gy' = 0$ ..(i)

Again differentiating

$1 + yy'' + (y')^2 + gy'' = 0$

$\Rightarrow g = -(1 + (y')^2 + yy'')/y''$

Put g in (i) and solving we get

$(x-y)y'' - y'(1+y'+(y')^2) = 1$

$(y-x)y'' + (1+y'+(y')^2)y' + 1 = 0$

$\Rightarrow Py'' + Qy' + 1 = 0$

$\Rightarrow P = y-x, Q = 1+y'+(y')^2$

$P+Q = 1 - x + y + y' + (y')^2$

Hence options b and c are correct.

**Question 4: The differential equation representing the family of curves $y^2 = 2c(x+\sqrt{c})$, where c is a positive parameter, is of**

(a) order 1, degree 3

(b) order 2, degree 2

(c) order 2, degree 1

(d) none of the above

**Solution:**

$y^2 = 2c(x+\sqrt{c})$ ..(i)

Differentiating both sides

$2yy' = 2c$

$\Rightarrow c = yy'$

Put c in (i)

$y^2 = 2yy'(x+\sqrt{(yy')})$

$\Rightarrow (y^2-2xyy') = 2(yy')^{3/2}$

$(y^2-2xyy')^2 = 2(yy')^3$

So degree = 3 and order = 1

Hence option a is the answer.

**Question 5: The order of the differential equation whose general solution is given by $y = (c_1+c_2) \cos (x + c_3) - c_4 e^{x + c5}$ where $c_1, c_2, c_3, c_4, c_5$ are arbitrary constants, is**

(a) 5

(b) 4

(c) 3

(d) 2

**Solution:**

Given that the solution of differential equation is $y = (c_1+c_2) \cos (x + c_3) - c_4 e^{x + c5}$

$= (c_1+c_2) \cos (x + c_3) - c_4 e^x e^{c5}$

$= A \cos (x+c_3) - Be^x$

Where $c_1+c_2 = A$ and $c_4e^{c5} = B$

Hence the number of arbitrary constants in the solution is 3.

The order of the differential equation is 3.

Hence option c is the answer.

**Question 6: If $x^2 + y^2 = 1$, then**

(a) $yy'' - 2(y')^2 + 1 = 0$

(b) $yy'' + (y')^2 + 1 = 0$

(c) $yy'' + (y')^2 - 1 = 0$

(d) $yy'' + 2(y')^2 + 1 = 0$

**Solution:**

Given $x^2 + y^2 = 1$

Differentiating we get

$2x + 2yy' = 0$

Again differentiating

$2 + 2(y')^2 + 2yy'' = 0$

=> $1 + (y')^2 + yy'' = 0$

Hence option b is the answer.

**Question 7: The differential equation representing the family of ellipse having foci either on the x axis or on the y axis centre at the origin and passing through the point (0, 3) is**

(a) $xyy'' + x(y')^2 - yy' = 0$

(b) $x + y'' = 0$

(c) $xyy' + y^2 - 9 = 0$

(d) $xyy' - y^2 + 9 = 0$

**Solution:**

General equation of ellipse is given by $x^2/a^2 + y^2/b^2 = 1$ ..(i)

It passes through (0, 3)

=> $0 + 9/b^2 = 1$

=> $b^2 = 9$

so(i) becomes

=> $x^2/a^2 + y^2/9 = 1$

Differentiate w.r.t.x

$2x/a^2 + 2yy'/9 = 0$

$yy'/9 = -x/a^2$

$y' = -(x/y)(9/a^2)$

$(y/x)y' = -9/a^2$

Again differentiate w.r.t.x

$(y/x)y'' + [(xy' - y)/x^2]y' = 0$

=> $xyy'' + x(y')^2 - yy' = 0$

Hence option a is the answer.

**Question 8: Let f: R→ R be a differential function with f(0) = 1 and satisfying the equation f(x+y) = f(x) f'(y) + f'(x) f(y) for all x, y belongs to R. then the value of $\log_e$ (f(4)) is**

**Solution:**

f(x+y) = f(x) f'(y) + f'(x) f(y)

Put x = 0, y = 0

f(0) = 1.f'(0) + f'(0)

=> 2f'(0) = f(0)

=> f'(0) = f(0)/2

=> f'(0) = ½

f(x+0) = f(x) f'(0) + f'(x) f(0)

f(x) = f(x)×½ + f'(x)

=> f'(x) = f(x)/2

$\int f'(x)/f(x) = ½ \int dx$

$\log_e f(x) = ½ x + c$

=> $f(x) = ce^{x/2}$

$\Rightarrow f(x) = e^{x/2}$

$\log_e f(x) = x/2$

$\log_e f(4) = 4/2$

$= 2$

**Question 9: Let y = y(x) be the solution of the differential equation cos x (dy/dx) + 2y sin x = sin 2x, x belongs to (0, π/2). If y(π/3) = 0, then y(π/4) is equal to:**

(a) $2 - \sqrt{2}$

(b) $2 + \sqrt{2}$

(c) $\sqrt{2} - 2$

(d) $1/\sqrt{2} - 1$

**Solution:**

Given that cos x (dy/dx) + 2y sin x = sin 2x

Divide by cos x, we get

(dy/dx) + 2y tan x = 2 sin x

$I.F = e^{\int 2 \tan x\, dx} = \sec^2 x$

The solution of differential equation is

$y. I.F = \int I.F\ 2 \sin x\, dx + c$

$\Rightarrow y \sec^2 x = \int 2 \sin x \sec^2 x\, dx + c$

$\Rightarrow y \sec^2 x = 2 \sec x + c$

When x = π/3, y = 0, then c = -4

From (i), $y \sec^2 x = 2 \sec x - 4$

$\Rightarrow y = (2 \sec x - 4)/\sec^2 x$

$\Rightarrow y(\pi/4) = \sqrt{2} - 2$

Hence option c is the answer.

**Question 10: Let f(x): (0, ∞) → R be a differential function such that f'(x) = 2 - f(x)/x for all x belongs to (0, ∞) and f(1) ≠ 1. Then**

(a) $\lim_{x \to 0+} f'(1/x) = 1$

(b) $\lim_{x\to 0+} xf'(1/x) = 2$

(c) $\lim_{x\to 0+} x^2f'(1/x) = 0$

(d) none of these

**Solution:**

Given $f'(x) = 2 - f(x)/x$

$f'(x) + f(x)/x = 2$

$I.F = e^{\log x} = x$

So $f(x)\, x = \int 2x\, dx$

$= x^2+c$

$\Rightarrow f(x) = x + c/x$ , $c \neq 0$ as $f(x) \neq 1$

Check the options

(a) $\lim_{x\to 0+} f'(1/x) = \lim_{x\to 0+} (1-cx^2) = 1$

(b) $\lim_{x\to 0+} xf'(1/x) = \lim_{x\to 0+} x(1/x + cx) = \lim_{x\to 0+} (1+ cx^2) = 1$

(c) $\lim_{x\to 0+} x^2f'(x) = \lim_{x\to 0+} x^2(1 - c/x^2)$

$= \lim_{x\to 0+} x^2 - c = -c$

Hence option a is the answer.