Question 1) A cylindrical vessel of height 500 mm has an orifice (small hole) at its bottom. The orifice is initially closed and water is filled in it up to height H. Now the top is completely sealed with a cap and the orifice at the bottom is opened. Some water comes out from the orifice and the water level in the vessel becomes steady with a height of the water column being 200 mm. Find the fall in height (in mm) of water level due to the opening of the orifice. [Take atmospheric pressure = 1.0×10^5 N/m^2 [density of water = 1000 kg/m^3 and g = 10 m/s^2. Neglect any effect of surface tension.]

Answer: 6

Solution:

$P = P_0 - \rho gh = 98 \times 10^3$ N/m^2

$P_0V_0 = PV$

$10^5[(500 - H) A] = 98 \times 10^3 [A(500 - 200)]$

H = 206 mm

Level fall = 206 - 200 = 6mm

Question 2) Two soap bubbles A and B are kept in a closed chamber where the air is maintained at a pressure of 8 N/m^2. The radii of bubbles A and B are 2 cm and 4cm, respectively. The surface tension of the soap-water used to make bubbles is 0.04 N/m. Find the ratio n_B/n_A where n_A and n_B are the number of moles of air in bubbles A and B, respectively. [Neglect the effect of gravity]

Answer: 6

Solution:

$(P_{in})_A = (4S/r_A) + P_0 = (4 \times 0.04/0.02) + 8 = 16$ N/m^2

$(P_{in})_B = (4S/r_B) + P_0 = (4 \times 0.04/0.04) + 8 = 12$ N/m^2

$n_A = (P_{in})_A V_A/RT$

$$\Rightarrow \frac{n_B}{n_A} = \frac{(P_{in})_B}{(P_{in})_A} \times \left(\frac{r_B}{r_A}\right)^3 = 6$$

Question 3) An ideal fluid flows (laminar flow) through a pipe of non-uniform diameter. The maximum and minimum diameters of the pipes are 6.4 cm and 4.8 cm, respectively. The ratio of the minimum and maximum velocity of the fluid in this pipe is

(A) 9/16

(B) $\sqrt{3}/2$

(C) ¾

(D) 81/256

Answer: (A) 9/16

Solution:

From the equation of continuity

$A_1V_1 = A_2V_2$

Here V_1 and V_2 are the velocities at two ends of the pipe.

A_1 and A_2 are the area of pipe at two ends

$$\Rightarrow \frac{V_1}{V_2} = \frac{A_2}{A_1} = \frac{\pi(4.8)^2}{\pi(6.4)^2} = \frac{9}{16}$$

Question 4) A large open tank has two holes in the wall. One is a square hole of side L at a depth y from the top and the other is a circular hole of radius R at a depth 4 y from the top. When the tank is completely filled with water, the quantities of water flowing out per second from both holes are the same. Then, R is equal to

(A) $L/\sqrt{2\pi}$

(B) $2\pi L$

(C) L

(D) $L/2\pi$

Answer: (A) $L/\sqrt{2\pi}$

Solution:

As we know,

Velocity, $V = \sqrt{2gh}$

From the equation of continuity $A_1V_1 = A_2V_2$

$$\sqrt{(2gy)} \times L^2 = \sqrt{(2g \times 4y)}\pi R^2$$

$\Rightarrow L^2 = 2\pi R^2$

Therefore, $R = L/\sqrt{2\pi}$

Question 5) The top of a water tank is open to the air and its water level is maintained. It is giving out 0.74 m^3 water per minute through a circular opening of 2 cm radius in its wall. The depth of the centre of the opening from the level of water in the tank is close to

(A) 6.0 m

(B) 4.8 m

(C) 9.6 m

(D) 2.9 m

Answer: (B) 4.8 m

Solution

In flow volume = Outflow volume

$\Rightarrow (0.74/60) = (\pi \times 4 \times 10^{-4}) \times \sqrt{2gh}$

$\sqrt{2gh} = (74 \times 100)/240\pi$

$\sqrt{2gh} = 740/24\pi$

$2gh = (740 \times 740)/(24 \times 24 \times 10)$ [Since, $\pi^2 = 10$]

$\Rightarrow h \approx 4.8$ m

Question 6) If the piston is pushed at a speed of 5 mm/s, the air comes out of the nozzle with a speed of

(A) 0.1 m/s

(B) 1 m/s

(C) 2 m/s

(D) 8 m/s

Answer: (C) 2 m/s

Solution:

Here, the piston is pushed at a speed, $v_1 = 5$ m/s

Let air comes out of the nozzle with a speed v_2

From the principle of continuity

$a_1v_1 = a_2v_2$

$\Rightarrow \pi r_1^2 v_1 = \pi r_2^2 v_2$

$\Rightarrow r_1^2 v_1 = r_2^2 v_2$

$\Rightarrow (20)^2 \times 5 = (1)^2 \times v_2$

v_2 = 2000 mm/s

= 2 m/s

Question 7) A boat floating in a water tank is carrying a number of large stones. If the stones are unloaded into the water, what will happen to the water level?

Answer: When stones are put in water, the level of water falls

Solution:

When the stones were in the boat, the weight of the stones was balanced by the buoyant force.

$V_s d_s = V_l d_l$

V_l, V_s= volume of liquid and stone respectively

d_l, d_s= density of liquid and stone respectively

Since $d_s > d_l$: $V_s < V_l$

Therefore when stones are put in water, the level of waterfalls.

Question 8) A cube of wood supporting 200 gm mass just floats in water. When the mass is removed, the cube rises by 2cm. What is the size of the cube?

Answer: 10 cm

Solution:

Let the size or edge of the cube be l. When mass m = 200 g is on the cube of wood

$200g + l^3\, d_{wood}\, g = l^3 d_{water}\, g$

$\Rightarrow l^3 d_{wood} = l^3 d_{water} - 200$ ——-(i)

When the mass m = 200 g is removed

$l^3 d_{wood} = (l - 2)l^2 d_{water}$ ——-(ii)

From equation (i) and (ii)

$l^3 d_{water} - 200 = (l-2)l^2 d_{water}$

(since $d_{water} = 1$)

Therefore, $l^3 - 200 = l^2(l-2)$

$\Rightarrow l = 10$ cm

Question 9) A vessel contains oil (density = 0.8 gm/cm^3) over mercury (density = 13.6 g/cm^3). A

homogeneous sphere floats with half its volume immersed in mercury and the other half in oil. The density of the material of the sphere in gm/cm^3 is

(A) 3.3

(B) 6.4

(C) 7.2

(D) 12.8

Answer: (C) 7.2

Solution:

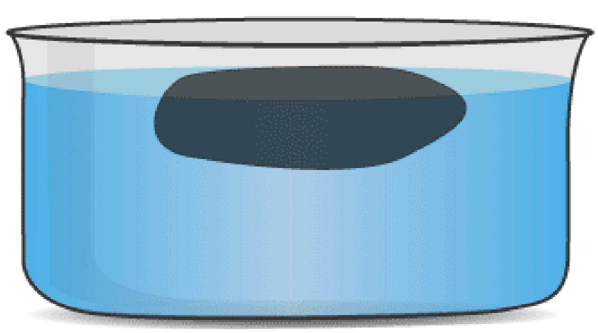
Weight of sphere = upthrust due to Hg = upthrust due to oil

$Vdg = (V/2)\, d_{Hg}\, g + (V/2) d_{oil} g$

$\Rightarrow d = (d_{Hg} + d_{oil})/2$

$= (13.6 + 0.8)/2 = 7.2$ g/cm^3

Question 10) A body floats in a liquid contained in a beaker. The whole system as shown in the figure falls freely under gravity. The upthrust on the body is/are



(A) Zero

(B) Equal to the weight of the liquid displaced

(C) Equal to the weight of the body in air

(D) Equal to the weight of the immersed portion of the body

Answer: (A) Zero

Solution:

The whole system falls freely under gravity, so g = 0

According to Archimedes principle

Upthrust = weight of fluid displaced

= (mass of fluid displaced) x g

Therefore, upthrust = 0