ISC SEMESTER 2 EXAMINATION
SPECIMEN QUESTION PAPER
MATHEMATICS

Maximum Marks: 40

Time allowed: One and a half hour

*Candidates are allowed an additional **10 minutes** for **only** reading the paper.*

*They must **Not** start writing during this time.*

The Question Paper consists of three sections A, B and C.

*Candidates are required to attempt all questions from **Section A** and all questions **EITHER** from **Section B OR Section C***

All working, including rough work, should be done on the same sheet as, and adjacent to the rest of the answer.

The intended marks for questions or parts of questions are given in brackets [ ].

Mathematical tables and graph papers are provided.

SECTION A - 32 MARKS

Question 1

Choose the correct option for the following questions.

(i) If $\int \frac{(\log x)^2}{x}\, dx = \frac{(\log x)^k}{k} + c$, then the value of k is: **[1]**

(a) 3

(b) 2

(c) 1

(d) None of the above options

(ii) $\int_0^{2a} f(x)\, dx = \int_0^a f(x)\, dx + \int_0^a f(k-x)\, dx$, then the value of k is: **[1]**

(a) a

(b) $2a$

(c) Independent of a

(d) None of the above options

(iii) The degree of the differential equation $\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^2 = x^2\left(\frac{d^2y}{dx^2}\right)^2$ is: **[1]**

(a) 1

(b) 2

(c) 3

(d) 4

(iv) Given $\int e^x\left(\frac{x-1}{x^2}\right)dx = e^x f(x) + c$. Then $f(x)$ satisfying the equation is: **[1]**

(a) x

(b) x^2

(c) $\frac{1}{x}$

(d) None of the above options

(v) Two cards are drawn out randomly from a pack of 52 cards one after the other, without replacement. The probability of first card being a king and second card not being a king is: **[1]**

(a) $\frac{48}{663}$

(b) $\frac{24}{663}$

(c) $\frac{12}{663}$

(d) $\frac{4}{663}$

(vi) If two balls are drawn from a bag containing 3 white, 4 black and 5 red balls. Then, the probability that the drawn balls are of different colours is: **[1]**

(a) $\frac{1}{66}$

(b) $\frac{3}{66}$

(c) $\frac{19}{66}$

(d) $\frac{47}{66}$

Question 2 **[2]**

(a) Evaluate : $\int \frac{x^3-x^2+x-1}{x-1}\, dx$

OR

(b) Evaluate : $\int \log_{10} x\, dx$

Question 3 **[2]**

(a) Solve the differential equation :
$cosec^3 x dy - \text{cosec}\, y\, dx = 0$

OR

(b) Solve the differential equation :
$\frac{dy}{dx} = 2^{-y}$

Question 4 **[2]**

Evaluate : $\int_2^8 \frac{\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}} dx$

Question 5 **[4]**

(a) A bag contains 6 red and 5 blue balls and another bag contains 5 red and 8 blue balls. A ball is drawn from the first bag and without noticing its colour is placed in the second bag. If a ball is drawn from the second bag, then find the probability that the drawn ball is red in colour.

OR

(b) A bag contains 3 red and 4 white balls and another bag contains 2 red and 3 white balls. If one ball is drawn from the first bag and 2 balls are drawn from the second bag, then find the probability that all three balls are of the same colour.

Question 6 **[4]**

Evaluate : $\int \frac{1+\sin x}{1-sin\, x} dx$

Question 7 **[6]**

In a bolt factory, machines X, Y and Z manufacture 20%, 35% and 45% respectively of the total output. Of their output 8%, 6% and 5% respectively are defective bolts. One bolt is drawn at random from the product and is found to be defective. What is the probability that it was manufactured in the machine Y?

Question 8 **[6]**

(a) Evaluate: $\int \frac{dx}{\sin x + \sin 2x}$

OR

(b) Evaluate : $\int_0^{\frac{\pi}{4}} \log(1 + tanx)\, dx$

SECTION B - 8 MARKS

Question 9 **[2]**

Choose the correct option for the following questions.

(i) The equation of the plane which is parallel to $2x - 3y + z = 0$ and which passes through $(1, -1, 2)$ is:

(a) $2x - 3y + z - 7 = 0$

(b) $2x - 3y + z + 7 = 0$

(c) $2x - 3y + z - 8 = 0$

(d) $2x - 3y + z + 6 = 0$

(ii) The intercepts made on the coordinate axes by the plane $2x + y - 2z = 3$ are:

(a) $\frac{-3}{2}, -3, \frac{-3}{2}$

(b) $\frac{3}{2}, 3, \frac{-3}{2}$

(c) $\frac{3}{2}, -3, \frac{-3}{2}$

(d) $\frac{3}{2}, 3, \frac{3}{2}$

Question 10 **[2]**

Find the equation of the plane passing through the point $(1, 1, 1)$ and is perpendicular to the line $\frac{x-1}{3} = \frac{y-2}{0} = \frac{z-3}{4}$. Also, find the distance of this plane from the origin.

Question 11 **[4]**

Using integration, find the area of the region bounded between the line $x = 4$ and the parabola $y^2 = 16x$.

SECTION C - 8 MARKS

Question 12 **[2]**

Choose the correct option for the following questions.

(i) If the regression line of $x\ on\ y$ is, $9x + 3y - 46 = 0$ and $y\ on\ x$ is, $3x + 12y - 7 = 0$, then the correlation coefficient 'r' is equal to:

(a) $\frac{-1}{12}$

(b) $\frac{1}{12}$

(c) $\frac{-1}{2\sqrt{3}}$

(d) $\frac{1}{2\sqrt{3}}$

(ii) If $\bar{X} = 40, \bar{Y} = 6, \sigma_x = 10, \sigma_y = 1.5\ and\ r = 0.9$ for the two sets of data X and Y, then the regression line of X on Y will be :

(a) $x - 6y - 4 = 0$

(b) $x + 6y - 4 = 0$

(c) $x - 6y + 4 = 0$

(d) $x + 6y + 4 = 0$

Question 13 **[2]**

For 5 observations of pairs (x, y) of variables X and Y, the following results are obtained: $\sum x = 15, \sum y = 25, \sum x^2 = 55, \sum y^2 = 135, \sum xy = 83.$

Calculate the value of $b_{xy}\ and\ b_{yx}$.

Question 14 **[4]**

A manufacturer wishes to produce two commodities A and B. The number of units of material, labour and equipment needed to produce one unit of each commodity is shown in the table given below. Also shown is the available number of units of each item, material, labour, and equipment.

Items	Commodity A	Commodity B	Available no. of Units
Material	1	2	8
Labour	3	2	12
Equipment	1	1	10

Find the maximum profit if each unit of commodity A earns a profit of ₹ 2 and each unit of B earns a profit of ₹ 3.