Question 1: If $\alpha = \cos^{-1}(3/5)$, $\beta = \tan^{-1}(1/3)$, where $0 < \alpha, \beta < \pi/2$, then $\alpha - \beta$ is

(a) $\tan^{-1}(9/5\sqrt{10})$

(b) $\cos^{-1}(9/5\sqrt{10})$

(c) $\tan^{-1}(9/14)$

(d) $\sin^{-1}(9/5\sqrt{10})$

Solution:

Given $\alpha = \cos^{-1}(3/5)$

$\cos \alpha = 3/5$

$\sin \alpha = 4/5$

$\tan \alpha = 4/3$

Also $\beta = \tan^{-1}(1/3)$

So $\tan \beta = 1/3$

$\tan(\alpha-\beta) = (\tan \alpha - \tan \beta)/(1 + \tan \alpha \tan \beta)$

$= (4/3 - 1/3)/(1 + 4/9)$

$= 1/(13/9)$

$= 9/13$

So $(\alpha-\beta) = \tan^{-1}(9/13)$

$= \sin^{-1}(9/5\sqrt{10})$

Hence option d is the answer.

Question 2: The principal value of $\tan^{-1}(\cot 43\pi/4)$ is

(a) $-3\pi/4$

(b) $3\pi/4$

(c) $-\pi/4$

(d) π/4

Solution:

$\tan^{-1}(\cot 43\pi/4) = \tan^{-1}[\cot(10\pi + 3\pi/4)]$

$= \tan^{-1}[\cot 3\pi/4]$ (since $\cot(2n\pi + x = \cot x)$]

$= \tan^{-1}(\tan(\pi/2 - 3\pi/4))$

$= (\pi/2 - 3\pi/4))$

$= (2\pi - 3\pi)/4$

$= -\pi/4$

Hence option c is the answer.

Question 3: If $\alpha = 3\sin^{-1}(6/11)$ and $\beta = 3\cos^{-1}(4/9)$, where the inverse trigonometric functions take only the principal values, then the correct option(s) is (are)

(a) $\cos\beta > 0$

(b) $\sin\beta < 0$

(c) $\cos(\alpha + \beta) > 0$

(d) $\cos\alpha < 0$

Solution:

$\alpha = 3\sin^{-1}(6/11) > 3\sin^{-1}1/2 > \pi/2$

$\Rightarrow \alpha > \pi/2$

So $\cos\alpha < 0$

$\beta = 3\cos^{-1}(4/9) > 3\cos^{-1}(½) > \pi$

$\Rightarrow \beta > \pi$

So $\cos\beta < 0$ and $\sin\beta < 0$

Now $\alpha + \beta > 3\pi/2$

$\cos(\alpha + \beta) > 0$

Hence option b, c and d are correct.

Question 4: The principal value of $\sin^{-1}(\sin 2\pi/3)$ is

(a) $-2\pi/3$

(b) $2\pi/3$

(c) $4\pi/3$

(d) none of these

Solution:

The principal value of $\sin^{-1}(\sin 2\pi/3) = \sin^{-1}(\sin \pi - \pi/3)$

$= \sin^{-1} \sin \pi/3$

$= \pi/3$

Hence option d is the answer.

Question 5: A possible value of $\tan(\frac{1}{4} \sin^{-1} \sqrt{63}/8)$ is:

(a) $1/(2\sqrt{2})$

(b) $1/\sqrt{7}$

(c) $\sqrt{7} - 1$

(d) $2\sqrt{2} - 1$

Solution:

We have to find $\tan(\frac{1}{4} \sin^{-1} \sqrt{63}/8)$.

Let $\sin^{-1}(\sqrt{63}/8) = \theta$

$\sin \theta = \sqrt{63}/8$

$\cos \theta = 1/8$

$2 \cos^2(\theta/2) - 1 = 1/8$

$\Rightarrow \cos^2 \theta/2 = 9/16$

$\cos \theta/2 = 3/4$

=> $(1 - \tan^2 \theta/4)/(1 + \tan^2 \theta/4) = 3/4$

$\tan \theta/4 = 1/\sqrt{7}$

Hence option b is the answer.

Question 6: If S is the sum of the first 10 terms of the series $\tan^{-1}(⅓) + \tan^{-1}(1/7) + \tan^{-1}(1/13) + \tan^{-1}(1/21) + \ldots$, then tan S is equal to

(a) 5/6

(b) 5/11

(c) -6/5

(d) 10/11

Solution:

$S = \tan^{-1}(1/3) + \tan^{-1}(1/7) + \tan^{-1}(1/13) + \ldots$upto 10 terms

$= \tan^{-1}(2-1)/(1 + 2.1) + \tan^{-1}(3-2)/(1 + 3.2) + \tan^{-1}(4-3)/(1 + 3.4) + \ldots + \tan^{-1}(11 - 10)/(1 + 11.10)$

$= (\tan^{-1} 2 - \tan^{-1} 1) + (\tan^{-1} 3 - \tan^{-1} 2) + (\tan^{-1} 4 - \tan^{-1} 3) + \ldots + (\tan^{-1} 11 - \tan^{-1} 10)$

$= (\tan^{-1} 11 - \tan^{-1} 1)$

$= \tan^{-1}(5/6)$

$\tan S = 5/6$

Hence option a is the answer.

Question 7: The value of $\sin^{-1}(12/13) - \sin^{-1}(3/5)$ is equal to

(a) $\pi - \sin^{-1}(63/65)$

(b) $\pi/2 - \sin^{-1}(56/65)$

(c) $\pi/2 - \cos^{-1}(56/65)$

(d) $\pi - \cos^{-1}(33/65)$

Solution:

$\sin^{-1}(12/13) - \sin^{-1}(3/5)$

$= \sin^{-1}((12/13)\times(4/5) - (3/5)\times(5/13))$

Since $\sin^{-1}x - \sin^{-1}y = \sin^{-1}(x\sqrt{(1-y^2)} - y\sqrt{(1-x^2)})$

$= \sin^{-1}(33/65)$

$= \cos^{-1}(56/65)$

$= \pi/2 - \sin^{-1}(56/65)$

Hence option b is the answer.

Question 8: Considering only the principal values of inverse functions, the set A = $\{x \geq 0: \tan^{-1}(2x) + \tan^{-1}(3x) = \pi/4\}$

(a) contains two elements

(b) contains more than two elements

(c) is a singleton set

(d) is an empty set

Solution:

Consider $\tan^{-1}(2x) + \tan^{-1}(3x) = \pi/4$

$\Rightarrow \tan^{-1}(5x/(1-6x^2)) = \pi/4$

$\Rightarrow 5x/(1-6x^2) = 1$

$\Rightarrow 5x = 1-6x^2$

$\Rightarrow 6x^2 + 5x - 1 = 0$

$\Rightarrow (6x - 1)(x + 1) = 0$

$\Rightarrow x = 1/6$ (since $x \geq 0$, $x = -1$ is rejected)

So A is a singleton set.

Hence option c is the answer.

Question 9: The value of $\tan^{-1}[(\sqrt{(1+x^2)} + \sqrt{(1-x^2)})/(\sqrt{(1+x^2)} - \sqrt{(1-x^2)})]$, $|x| < ½$, $x \neq 0$, is equal to

(a) $\pi/4 + ½ \cos^{-1}x^2$

(b) $\pi/4 + \cos^{-1}x^2$

(c) $\pi/4 - ½ \cos^{-1}x^2$

(d) $\pi/4 - \cos^{-1}x^2$

Solution:

Let $x^2 = \cos 2\theta$

=> $\theta = ½ \cos^{-1} x^2$

=> $\tan^{-1}[(\sqrt{(1+x^2)} + \sqrt{(1-x^2)})/(\sqrt{(1+x^2)} - \sqrt{(1-x^2)})] = [\sqrt{(1 + \cos 2\theta)} + \sqrt{(1 - \cos 2\theta)}]/[\sqrt{(1 + \cos 2\theta)} - \sqrt{(1 - \cos 2\theta)}]$

$= \tan^{-1}(1 + \tan \theta)/(1 - \tan \theta)$

$= \tan^{-1}(\tan (\pi/4 + \theta))$

$= \pi/4 + \theta$

$= \pi/4 + ½ \cos^{-1} x^2$

Hence option a is the answer.

Question 10: If $f(x) = 2 \tan^{-1}x + \sin^{-1}(2x/(1+x^2))$, $x>1$, then $f(5)$ is equal to

(a) $\tan^{-1} 65/156$

(b) $\pi/2$

(c) π

(d) $4 \tan^{-1}(5)$

Solution:

Given $f(x) = 2 \tan^{-1}x + \sin^{-1}(2x/(1+x^2))$

When $x> 1$, $\sin^{-1}(2x/(1+x^2)) = \pi - 2 \tan^{-1}x$

So $f(x) = 2 \tan^{-1}x + \pi - 2 \tan^{-1}x$

$= \pi$

$f(5) = \pi$

Hence option c is the answer.