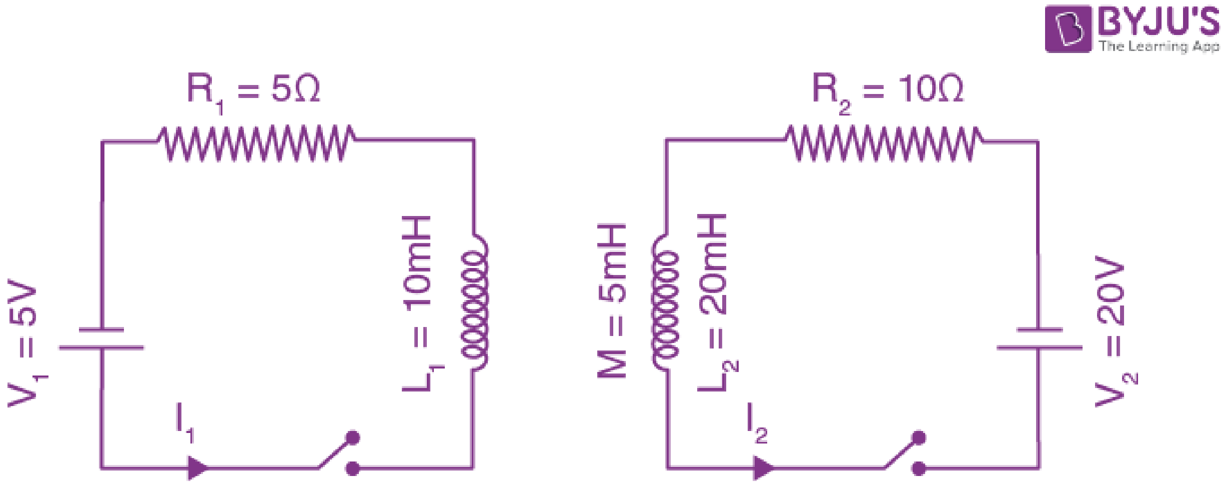
Question 1) The inductance of two LR circuits are placed next to each other, as shown in the figure. The value of the self-inductance of the inductors, resistance, mutual-inductance and applied voltages are specified in the given circuit. After both the switches are closed simultaneously the total work done by the batteries against the induced EMF in the inductors by the time the currents reach their steady-state values is ______ mJ.



Answer: 55 mJ

Solution:

Given,

Mutual Inductance, M = 5mH

L_1 = 10 mH

V_1 = 5 V

L_2 = 20 mH

V_2 = 20 V

$I_1 = V_1/R_1 = 5/5 = 1A$

$I_2 = V_2/R_2 = 20/10 = 2A$

After both the switches are closed simultaneously, the total work done by the batteries against the induced EMF = Increase in the magnetic energy

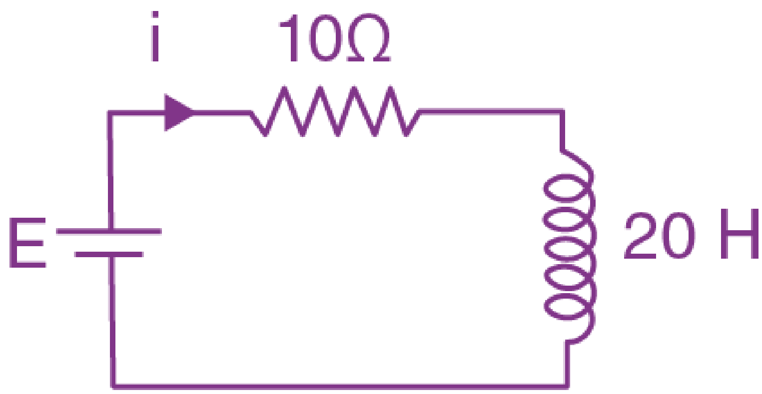
Therefore, $W = \Delta U = (½)\, L_1I_1^2 + (½)\, L_2I_2^2 + MI_1I_2$

$= (½)(10 \times 10^{-3})1^2 + (½)(20 \times 10^{-3})(2^2) + (5 \times 10^{-3}) \times 1 \times 2$

$= (5 + 40 + 10) \times 10^{-3}$

$= 55$ mJ

Question 2) A 20 Henry inductor coil is connected to a 10 ohm resistance in series as shown in the figure. The time at which rate of dissipation of energy (Joule's heat) across the resistance is equal to the rate at which magnetic energy is stored in the inductor, is



(A) 2/ln 2

(B) (½) ln2

(C) 2ln 2

(D) ln 2

Answer: (C) 2ln 2

Solution:

$i = i_0 (1 - e^{-t/\tau})$

$di/dt = (i_0/\tau) e^{-t/\tau}$

$di/dt = [E/(10 \times 2)] e^{-t/2}$ --------(1) [since $\tau = L/R = 20/10 = 2$]

$[L(di/dt)i] = i^2R$

$\Rightarrow L(di/dt) = iR$ ---------(2)

From equation (1) and (2) we get

$L [E/(10 \times 2)] e^{-t/2} = iR$

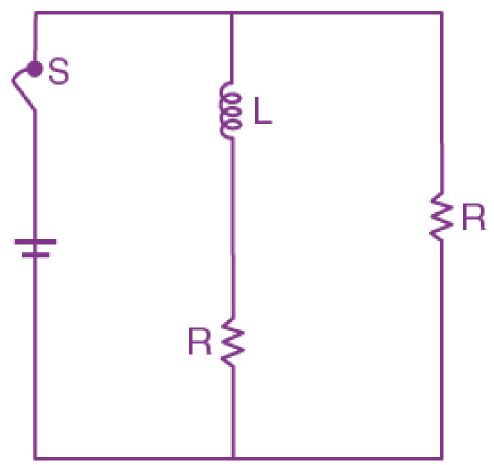
$(L/R)[E/(20)]\ e^{-t/2} = i_0\ (1 - e^{-t/\tau})$ [since $\tau = L/R = 20/10 = 2$]

$[E/10]\ e^{-t/2} = [E/10)\ (1 - e^{-t/\tau})$

$e^{-t/2} = ½$

$t/2 = \ln 2$

Question 3) In the figure shown, a circuit contains two identical resistors with resistance R = 5 ohm and inductance with L = 2 mH. An ideal battery of 15 V is connected in the circuit. What will be the current through the battery long after the switch is closed?



(A) 5.5 A

(B) 7.5 A

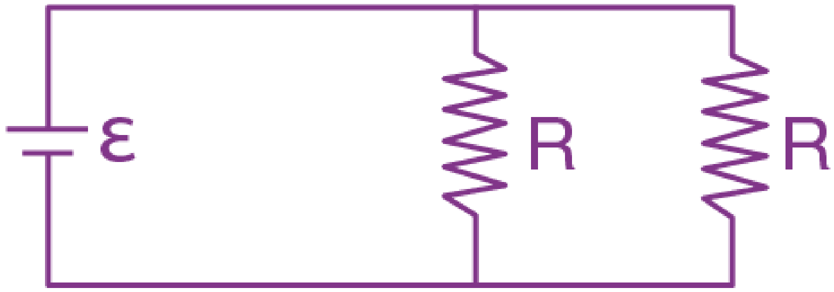
(C) 3 A

(D) 6 A

Answer: (D) 6 A

Solution:

For a long time after the switch is closed, the inductor will be idle so the equivalent diagram will be



$I = \varepsilon/[(R \times R)/(R + R)]$

$= 2\varepsilon/R$

$= (2 \times 15)/5$

$= 6$ A

Question 4)An AC voltage source of variable angular frequency and fixed amplitude V_0 is connected in series with a capacitance C and an electric bulb of resistance R (inductance zero). When ω is increased

(A) the bulb glows dimmer

(B) the bulb glows brighter

(C) total impedance of the circuit is unchanged

(D) total impedance of the circuit increases

Answer: (B) the bulb glows brighter

Solution:

Impedance $Z = \sqrt{\frac{1}{(\omega C)^2} + R^2}$

As ω increases, Z decreases

Hence, the bulb will glow brighter

Question 5) A series R-C combination is connected to an AC voltage of angular frequency ω=500 radian/s. If the impedance of the R-C circuit is R√1.25 the time constant (in millisecond) of the circuit is?

(A) 1

(B) 2

(C) 3

(D) 4

Answer: (D) 4

Solution:

Given, ω=500 radian/s

The capacitance of the capacitor is C

$X_c = 1/\omega C = 1/500C$

Impedance of the circuit, $Z = R\sqrt{1.25}$

Using $Z^2 = R^2 + X_c^2$

$1.25R^2 = R^2 + 1/(500)^2C^2$

$0.25R^2 = 1/(0.25 \text{ x } 10^6)C^2$

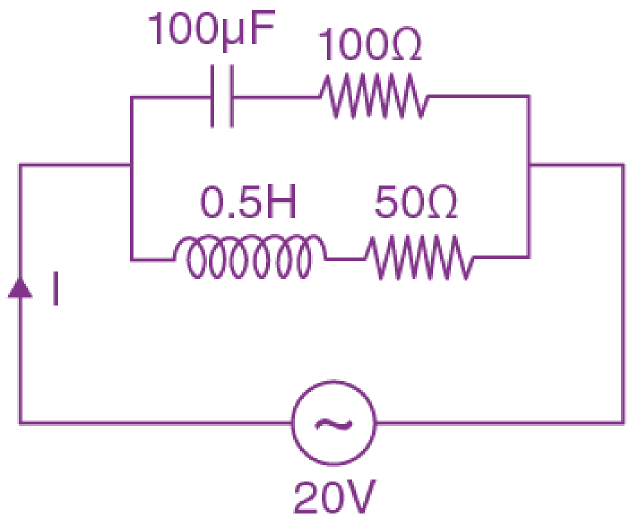
$R^2C^2 = 10^{-6}/(0.25)^2$

$\Rightarrow RC = 10^{-3}/0.25$

$= 0.004$ s

$= 4$ ms

Question 6) In the given circuit, the AC source has ω = 100 rad/s. Considering the inductor and capacitor to be ideal, the correct choice (s) is(are)



(A) The current through the circuit, I is 0.3 A.

(B) The current through the circuit, is 0.3 √2A

(C) The voltage across 100Ω resistor 10√2 V

(D) The voltage across 50Ω resistor 10 V

Answer: (A) and (C)

Solution:

In the upper branch the net impedance, (C = 100μF, R = 100Ω)

Therefore, the net impedance will be

$$Z_1 = \sqrt{\frac{1}{(\omega C)^2} + R^2}$$
$$Z_1 = \sqrt{\frac{1}{(100\times100\times10^{-6})^2} + 100^2}$$

= 100√2

Current $I_1 = V/Z_1 = 20/(100\sqrt{2})$

$\cos \Phi_1 = R/(100\sqrt{2})$

= 100/(100√2)

= 1/√2

$\Rightarrow \Phi_1 = 45^0$

In the lower branch the net impedance (L = 0.5 H, R = 50 Ω)

Therefore, the net impedance will be

$$Z_2 = \sqrt{(\omega L)^2 + R^2}$$
$$Z_2 = \sqrt{(0.5 \times 100)^2 + 100^2}$$

= 50√2

Current, $I_2 = V/Z_2$

= 20/(50√2)

cos Φ_1 = R/(50√2)

= 50/(50√2)

= 1/√2

$\Rightarrow \Phi_1 = 45^0$

Thus the total current I is given by the summation of I_1 and I_2 which differ by 90^0 in phase and hence

$$I = \sqrt{I_1^2 + I_2^2}$$

= (1/√10) A ≈ 0.3 A

Voltage across 100 Ω = I_1R_1 = [20/(100√2)] x 100 = 10√2 V

Voltage across 50 Ω = I_2R_2 = [20/(50√2)] x 50 = 10√2 V

Question 7) An alternating voltage v(t) = 220 sin 100 πt volt is applied to a purely resistive load of 50 Ω. The time taken for the current to rise from half of the peak value to the peak value is

(A) 5 ms

(B) 2.2 ms

(C) 7.2 ms

(D) 3.3 ms

Answer: (D) 3.3 ms

Solution:

Given,

$V(t) = 220 \sin 100\pi t$

$I(t) = (220/50) \sin 100\pi t$

Phase to be covered $\theta = 60^0 = \pi/3$

Time taken, $t = \theta/\omega$

$= (\pi/3)/100\pi$

$= (1/300)$ sec

$= 3.3$ ms

Question 8) A sinusoidal voltage V(t) = 100 sin (500t) is applied across a pure inductance of L= 0.02 H. The current through the coil is

(A) 10 cos (500 t)

(B) -10 cos (500 t)

(C) 10 sin (500 t)

(D) - 10 sin (500 t)

Answer: (B) -10 cos (500 t)

Solution:

In a pure inductive circuit current always lags behind the emf by $\pi/2$

If $v(t) = v_0 \sin\omega t$

Then, $I(t) = I_0 \sin(\omega t - \pi/2)$

Given, $V(t) = 100 \sin (500t)$

And $I_0 = E_0/\omega L$

$= 100/ (500 \times 0.02) = 10\sin(500t - \pi/2)$

$I_0 =- 10 \cos (500 t)$

Question 9) An AC circuit has R = 100 Ω, C= 2μF and L= 80 mH, connected in series. The quality factor of the circuit is

(A) 2

(B) 0.5

(C) 20

(D) 400

Answer: (A) 2

Solution:

Quality factor

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$Q = \frac{1}{100}\sqrt{\frac{80\times10^{-3}}{2\times10^{-6}}}$$

$$Q = \frac{1}{100}\sqrt{40\times10^{3}}$$

= 200/100 = 2

Question 10) When the rms voltage V_L, V_C and V_R are measured respectively across the inductor L, the capacitor C and the resistor R in a series LCR circuit connected to an AC source, it is found that the ratio V_L: V_C: V_R = 1: 2: 3. If the rms voltage of the AC sources is 100 V, The V_R is close to

(A) 50 V

(B) 70 V

(C) 90 V

(D) 100 V

Answer: (C) 90 V

Solution:

Given,

V_L: V_C: V_R = 1: 2: 3

V = 100 V

⇒ $V_R = 3K$, $V_L = K$, $V_C = 2K$

We know,

$$V=\sqrt{V_R^2+(V_L-V_C)^2}$$
$$100=\sqrt{9K^2+K^2}$$

100 = √10 K

K =100/√10

V_R = 3K = (3 x 100)/√10

= 94.86 volts

So, V_R is close to 90 V